From electron-positron photoproduction in strong laser fields to QED cascades

Workshop on High Energy Density Physics with BELLA-i

21.1.2016

Sebastian Meuren

Karen Z. Hatsagortsyan, Christoph H. Keitel, Antonino Di Piazza

MAX-PLANCK-GESELLSCHAFT

MPI for Nuclear Physics, Heidelberg (Germany)

- **Introduction to strong-field QED**
  - Why should we study Strong-Field QED?
  - Intuitive explanation of the QED critical field
  - Phenomena related to the nonlinear regime of QED
  - From a single vertex to a QED cascade
- **First part: nonlinear Breit-Wheeler pair production (SM)**
  - Semiclassical description
  - Difference between classical and quantum absorption
  - Initial conditions for the classical propagation
  - Momentum distribution of the created pairs
  - Importance of interference effects
- **Second part: QED cascades (Matteo Tamburini)**
  - Why are QED cascades interesting?
  - How to seed a QED cascade: high-Z vs. low-Z gas
  - Importance of the laser-field structure: ponderomotive pressure
  - Numerical results

Motivation: Why do we want to test nonlinear QED?

QED: eletrons, positrons and photons

$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}\left(\mathrm{i}\not{\partial} - e\not{\mathcal{A}} - m\right)\psi - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}, \qquad \mathcal{F}^{\mu\nu} = \partial^\mu\mathcal{A}^\nu - \partial^\nu\mathcal{A}^\mu$$

- Here, ϵ_0, $\hbar$ and c are set to unity (sometimes restored for clarity)
- The characteristic scales of atomic physics and QED are determined by the electron mass (m) and charge ($e < 0$)

QED

$$\mathcal{E} = mc^2 \sim 10^6\,\mathrm{eV}$$

$$\lambda\!\!\!^{-}_C = \hbar c/(mc^2) \sim 10^{-13}\,\mathrm{m}$$

$$E_{\mathsf{cr}} = (mc^2)^2/(|e|\,\hbar c) \sim 10^{16}\,\mathrm{V/cm}$$

Atomic physics

$$\mathcal{E}_H = (Z\alpha)^2\mathcal{E}/2 \sim Z^2 \times 10\,\mathrm{eV}$$

$$a_B = \lambda\!\!\!^{-}_C/(Z\alpha) \sim Z^{-1} \times 10^{-10}\,\mathrm{m}$$

$$E_{\mathsf{eff}} = (Z\alpha)^3 E_{\mathsf{cr}} \sim Z^3 \times 10^{10}\,\mathrm{V/cm}$$

$\alpha = e^2/(4\pi\epsilon_0\hbar c) \approx 1/137$: fine-structure constant, Z: atomic number

Conceptual changes

Energy	$\mathcal{E}$	nonrelativistic vs. relativistic description
Length	$\lambda\!\!\!^{-}_C$	classical vs. quantum field theory
Field	E_{cr}	vacuum vs. nonlinear QED

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Main motivation for strong-field QED:

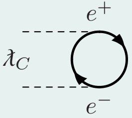
- E_{cr} is a fundamental scale of the theory
- Probing $E \gtrsim E_{\mathrm{cr}}$ means testing the theory in a new regime
- Maybe our current understanding is insufficient $\longrightarrow$ new physics!

Sauter-Schwinger vacuum instability

- A pure electric field $E \geq E_{\mathrm{cr}}$ is unstable, it decays spontaneously

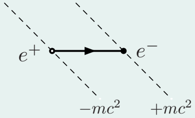
  First observation: Sauter (1931), First modern calculation: Schwinger (1951)

Vacuum fluctuations



Instead of being empty, the vacuum is filled with quantum fluctuations

Heuristic tunneling picture



"Tilted" energy levels $\longrightarrow$ tunneling

Probability: $\sim \exp\left(-\pi E_{\mathrm{cr}}/E\right)$

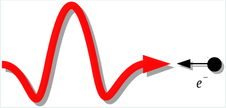
- Heuristic derivation of the critical field $E_{\mathrm{cr}} = 1.3 \times 10^{16}$ V/cm:
  - Spatial extend of the fluctuations (Heisenberg): $\sim \lambda\!\!\!^{-}_C = \hbar/(mc)$
  - Energy gap between virtual and real (Einstein): $\sim mc^2$
  - Work by the field (Lorentz force): $\sim E\,|e|\,\lambda\!\!\!^{-}_C \longrightarrow E_{\mathrm{cr}} = mc^2/(|e|\,\lambda\!\!\!^{-}_C)$
- In vacuum $I_{\mathrm{cr}} = 4.6 \times 10^{29}\ \mathrm{W/cm^2}$ is not achievable in the near future:

	$\sim \hbar\omega$	Future facilities	I (intensity)	current
optical	$1\,\mathrm{eV}$	CLF, ELI, XCELS,...	$10^{24-25}\ \mathrm{W/cm^2}$	$10^{22}\ \mathrm{W/cm^2}$
x-ray	$10\,\mathrm{keV}$	LCLS-II, XFEL,...	$10^{27}\ \mathrm{W/cm^2}$ (goal)	$10^{18}\ \mathrm{W/cm^2}$

How to reach the critical field with existing technology

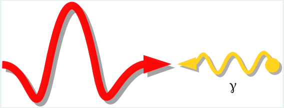
- The laser intensity I is not a Lorentz scalar ($I' \sim \gamma^2 I$, $\gamma = \epsilon/m$)
- Critical intensity $I_{\mathrm{cr}} = 4.6 \times 10^{29}\,\mathrm{W/cm^2}$ is obtainable in the boosted frame if $\gamma \sim 10^3 - 10^4$ even if $I \lesssim 10^{22}\,\mathrm{W/cm^2}$ (optical Petawatt system)

Electron-Laser interactions



Electrons with an energy $\epsilon \gtrsim \mathrm{GeV}$ are obtainable via laser-wakefield acceleration

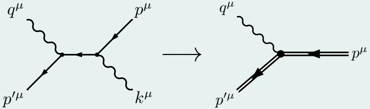
Light-by-light scattering



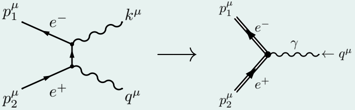
Photons with an energy $\hbar\omega_\gamma \gtrsim \mathrm{GeV}$ are obtainable via Compton backscattering

- For very strong fields the simultaneous interaction with several laser photons becomes important – describable using "dressed" states:

Compton scattering



Breit-Wheeler pair production



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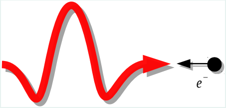

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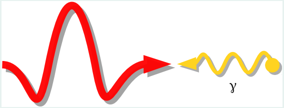


Photons with an energy $\hbar\omega_\gamma \gtrsim \mathrm{GeV}$ are obtainable via Compton backscattering

- For very strong fields the simultaneous interaction with several laser

Current laser-plasma wakefield acceleration record:

$\epsilon = 4\,\mathrm{GeV}$ ($\gamma \sim 10^4$) W. P. Leemans, et al., PRL **113**, 245002 (2014)

Current Compton-backscattering photon energy record:

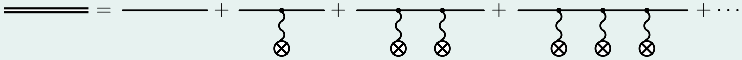
$\hbar\omega_\gamma = 2.9\,\mathrm{GeV}$ N. Muramatsu, et al., NIMA **737**, 184–194 (2014)

$\longrightarrow$ **Investigate the fundamental processes using an all-optical setup!**

Dressed states and the classical intensity parameter

- Dressed states are solutions of the interacting Dirac equation:

$$(\mathrm{i}\not{\partial} - m)\,\psi_p = 0,\;\; \psi_p = \qquad\qquad (\mathrm{i}\not{\partial} - e\not{A} - m)\,\Psi_p = 0,\;\; \Psi_p =$$



The dressed propagator/external line includes an arbitrary number of interactions with the classical background field

- A single interaction with the background scales as $\sim \xi$ $(\xi = a_0)$

$$\xi \sim \frac{|e|\sqrt{\langle -A^2\rangle}}{mc} \sim \frac{|e|E}{mc\,\omega},$$

Intensity parameter

$$= \frac{\not{p} + m}{p^2 - m^2} \sim \frac{1}{m},$$

Free propagator

$$= -\mathrm{i}e\not{A} \sim |e|\sqrt{-A^2}$$

Coupling vertex

(E, ω: field strength and angular frequency of the laser field, respectively)

Perturbative regime $\xi \ll 1$	Each coupling suppressed by ξ^2 (probability) n-photon absorption scales as ξ^{2n}
Nonperturbative regime $\xi \gtrsim 1$	Dressing becomes important $[I \gtrsim 10^{18}\,\mathrm{W/cm^2}$ for optical lasers $(\hbar\omega \sim 1\,\mathrm{eV})]$
Semiclassical regime $\xi \gg 1$	Probability amplitude is highly oscillating, classical interpretation of stationary points

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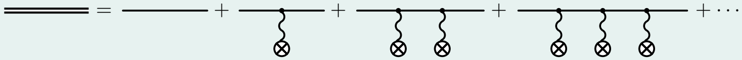

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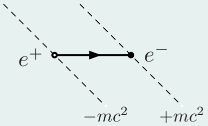
With BELLA-i the unexplored regime $\xi \gg 1$ could be studied!

Pair production and the quantum nonlinearity parameter

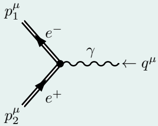
Sauter-Schwinger effect



Spontaneous decay of the vacuum

- Sizable if $E \gtrsim E_{\mathrm{cr}} = m^2c^3/(\hbar\,|e|)$ (at the QED critical field)
- Probability: $\sim \exp\left(-\pi E_{\mathrm{cr}}/E\right)$ (for a pure electric field)

Breit-Wheeler pair production



Decay of an incoming photon

- Sizable if $\chi \gtrsim 1$ (critical field reached in the boosted frame)
- Probability: $\sim \exp\left[-8/(3\chi)\right]$ (if $\chi \ll 1$ and $\xi \gg 1$)

Electron-positron photoproduction depends crucially on the quantum nonlinearity parameter

$$\chi \sim \frac{|e|\,\hbar}{m^3c^4}\sqrt{\langle q^\mu F^2_{\mu\nu} q^\nu \rangle} \sim (2\hbar\omega_\gamma/mc^2)(E/E_{cr})$$

[$\hbar\omega_\gamma$: energy of the incoming photon; last relation assumes a head-on collision]

- The photon four-momentum is transfered at the vertex
- Pair is produced ultra relativistic, background field is boosted

Sauter-Schwinger effect

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Breit-Wheeler pair production

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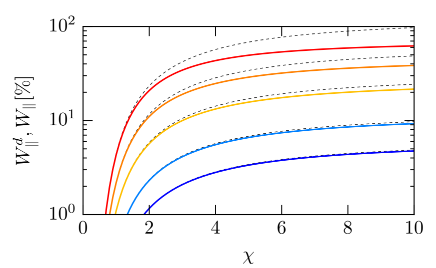
BELLA-i facility:

$\epsilon = 8\,\mathrm{GeV}$ electrons $\longrightarrow \hbar\omega_\gamma \approx 3\,\mathrm{GeV}$ photons

$\chi = 1 \longrightarrow \xi = 30$ ($\lesssim 10^{21}\,\mathrm{W/cm^2}$ for $\hbar\omega = 1.55\,\mathrm{eV}$)

Total pair-creation probability:



$\xi = 10, 20, 50, 100, 200$ ($N = 5$)

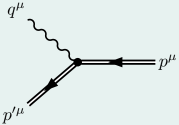
$F^{\mu\nu}(x) = f^{\mu\nu} \sin^2[\phi/(2N)] \sin(\phi)$, $\phi = kx$

(dashed lines: without wave-function decay)

SM, et al., PRD **91**, 013009 (2015)

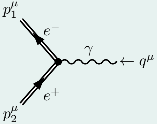
From a single vertex to a QED cascade

Photon emission



In general an electron can radiate more than only once

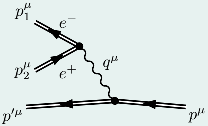
Pair production



The survival probability of a photon can become exponentially small

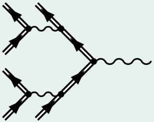
- The total probability $P \sim \alpha\xi N$ for the fundamental processes can become very large [$\alpha \approx 1/137$, N: number of laser cycles]
- At a certain point processes with many vertices become important
- Starting from a single particle a cascade developes

Trident pair production



Simplest cascade process

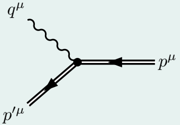
QED cascade



Exponential increase of particles

From a single vertex to a QED cascade

Photon emission



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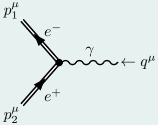

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Trident pair production

QED cascade

Seminal SLAC E-144 experiment:

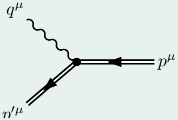
$\epsilon = 46.6\,\mathrm{GeV}$ ($\gamma \sim 10^5$), $\hbar\omega = 2.4\,\mathrm{eV}$, $I \sim 10^{18}\,\mathrm{W/cm^2}$ ($\xi \approx 1$, $\chi \approx 1$)

Nonlinear Compton scattering: C. Bula, et al. PRL **76**, 3116 (1996)

Trident pair production: D. L. Burke, et al. PRL **79**, 1626 (1997)

From a single vertex to a QED cascade

Photon emission



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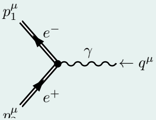


Many recent papers on QED cascades:

Grismayer, Vranic, Martins, Fonseca, and Silva, arXiv:1511.07503 (2015)
Tamburini, Di Piazza, and Keitel, arXiv:1511.03987 (2015)
Gelfer, Mironov, Fedotov, Bashmakov, Nerush, Kostyukov, and Narozhny, PRA (2015)
Gonoskov, Bastrakov, Efimenko, Ilderton, Marklund, Meyerov, et al., PRE (2015)
Green and Harvey, CPC (2015)
Lobet, Ruyer, Debayle, d'Humières, Grech, Lemoine, and Gremillet, PRL (2015)
Vranic, Grismayer, Martins, Fonseca, and Silva, CPC (2015)
Bashmakov, Nerush, Kostyukov, Fedotov, and Narozhny, POP (2014)
Mironov, Narozhny, and Fedotov, PLA (2014)
Narozhny and Fedotov, EPJST (2014)
Ridgers, Kirk, Duclous, Blackburn, Brady, Bennett, Arber, and Bell, JCP (2014)
Tang, Bake, Wang, and Xie, PRA (2014)
...

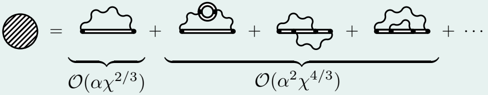
From radiative corrections to full nonperturbativity

Furry-picture approach to strong-field QED:

- Strong background fields ($\xi \gtrsim 1$) are included exactly (dressed states)
- The radiation field (non-occupied modes) is treated perturbatively

$\longrightarrow$ QED becomes a nonperturbative theory (like QCD?) for $\alpha\chi^{2/3} \gtrsim 1$

Full breakdown of perturbation theory



Mass operator: perturbation theory with respect to the radiation field

Different regimes for strong background fields ($\xi \gg 1$):

1. $\chi \ll 1$: **classical regime**
   Quantum effects are very small, pair production is exponentially suppressed
2. $\chi \gtrsim 1, \alpha\chi^{2/3} \ll 1$: **quantum regime**
   Recoil and pair production are important, but the radiation field is a perturbation
3. $\alpha\chi^{2/3} \gtrsim 1$: **fully nonperturbative regime**
   Perturbative treatment of the radiation field breaks down

V. I. Ritus, J. Sov. Laser Res. **6**, 497–617 (1985)

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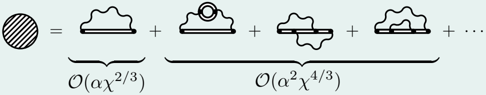

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V.

To study the fully nonperturbative regime one needs both highly energetic particles and strong (optical) lasers to reach $\chi \gg 1$!

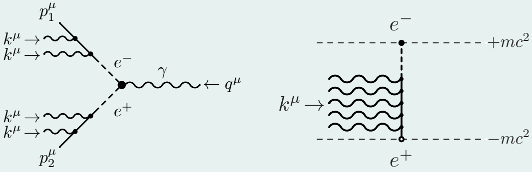
- **Multiphoton vs. tunneling regime**
  - Keldysh parameter
- **Semiclassical description**
  - Difference between classical and quantum absorption
  - Initial conditions for the classical propagation
  - Importance of interference effects

More details can be found in:

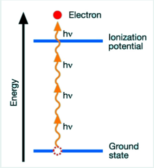
SM, C. H. Keitel, and A. Di Piazza, arXiv:1503.03271 (2015)
SM, K. Z. Hatsagortsyan, C. H. Keitel, and A. Di Piazza, PRL **114**, 143201 (2015)
SM, K. Z. Hatsagortsyan, C. H. Keitel, and A. Di Piazza, PRD **91**, 013009 (2015)

Mulitphoton vs. tunneling pair production

Multiphoton pair production ($\xi \ll 1$)



Multiphoton ionization



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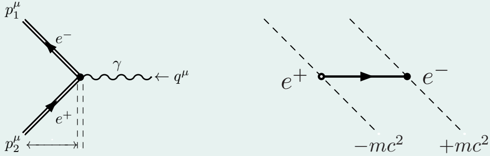
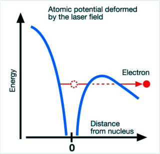
Tunneling pair production ($\xi \gg 1$)



Tunnelionization



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- Pair production is similar to ionization in atomic physics
- The Keldysh parameter distinguishes the two regimes:
  AP: $\gamma = \omega\sqrt{2mI_p}/(|e|E)$, SFQED: $1/\xi = \omega mc/(|e|E)$ ($I_p = 2mc^2$)
  [ω, E: laser angular frequency/field strength, I_p: atomic ionization potential]

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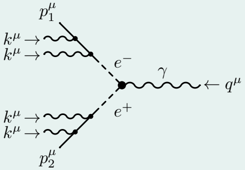
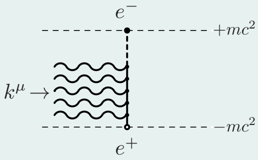
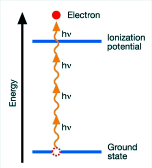




Multiphoton ionization



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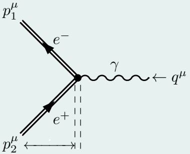
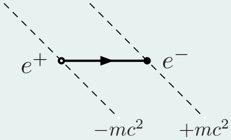
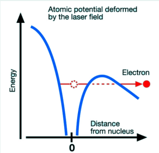
Tunneling pair production ($\xi \gg 1$)





Tunnelionization



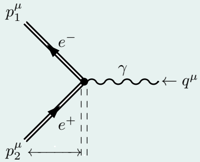
© DESY

- Pair production is similar to ionization in atomic physics

The Breit-Wheeler process was only studied in the multi-photon regime.
BELLA-i could study Breit-Wheeler in the tunneling regime!

Nonlinear Breit-Wheeler process

Leading-order Feynman diagram



- Photon: four-momentum q^μ $(q^2 = 0)$
- Electron: four-momentum p_1^μ $(p^2 = m^2)$
- Positron: four-momentum p_2^μ $(p'^2 = m^2)$

(we do *not* introduce dressed momenta!)

Semiclassical approximation

- We assume a strong plane-wave laser pulse $[F^{\mu\nu} = F^{\mu\nu}(\phi),\ \xi \gg 1]$
- The S-matrix is solvable analytically (to leading order)
- Stationary-phase analysis: main contribution to the process at $\phi = \phi_k$
- We propagate the final momenta back in time $p_{1,2}^\mu \longrightarrow p_{1,2}^\mu(\phi)$

$$p_1^\mu(\phi) + p_2^\mu(\phi) = q^\mu + n(\phi)k^\mu$$

- At the stationary phases ϕ_k $n(\phi) > 0$ is minimal
- Process happens where the pair becomes real as easy as possible!

SM, C. H. Keitel, and A. Di Piazza, arXiv:1503.03271 (2015)

Classical vs. quantum absorption

Global conservation law

$$p_1^\mu + p_2^\mu = q^\mu + nk^\mu$$

Local conservation law

$$p_1^\mu(\phi) + p_2^\mu(\phi) = q^\mu + n(\phi)k^\mu$$

Classical absorption

$$n_{\mathrm{cl}}k^\mu = p_1^\mu + p_2^\mu - [p_1^\mu(\phi_k) + p_2^\mu(\phi_k)]$$

Propagation from the stationary point

Quantum absorption

$$n_{\mathrm{q}}k^\mu = p_1^\mu(\phi_k) + p_2^\mu(\phi_k) - q^\mu$$

Absorption during the creation

- Pair production at ϕ: $n(\phi)k^\mu$ must be absorbed "non-classically"
  $\longrightarrow$ $n(\phi)k^\mu$ is a measure for the effective tunneling distance
- Stationary-phase condition obeyed at $\phi = \phi_k$:
  $\longrightarrow$ $n(\phi_k)$: minimum laser four-momentum needed to be on shell

Implications for the QED-PIC community

- We obtain the scaling laws: $n_{\mathrm{q}} \sim \xi/\chi$ and $n_{\mathrm{cl}} \sim \xi^3/\chi$, respectively
- The energy transver from the laser to the particles is dominated by classical physics (taken into account self-consistently in a PIC code)
- The quantum absorption is not taken into account in a PIC code
  $\longrightarrow$ We have a definite error estimate now!

Characteristic four-vectors of the problem

- The Breit-Wheeler process is characterized by the quantitites:

$$\underbrace{q^\mu,}_{\text{gamma photon}} \qquad \underbrace{k^\mu,}_{\text{laser photons}} \qquad \underbrace{f_1^{\mu\nu}, f_2^{\mu\nu}}_{\text{laser polarizations}} \quad \longrightarrow \quad \Lambda_i^\mu = \frac{f_i^{\mu\nu} q_\nu}{(q f_i^2 q)^{1/2}}$$

- They allow us to construct a canonical light-cone basis:

$$k^\mu, \quad \bar{k}^\mu = q^\mu / kq, \quad e_1^\mu = \Lambda_1^\mu, \quad e_2^\mu = \Lambda_2^\mu, \quad (q^2 = 0,\ kq \neq 0,\ \Lambda_i^2 = -1)$$

Invariant momentum parameters

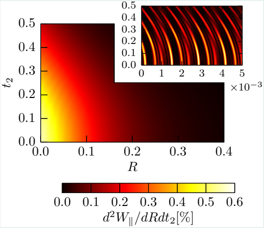
- We define the Lorentz-invariant momentum parameters R, t_1 and t_2:

$$p_1^\mu = (1/2 + R) q^\mu + s' k^\mu + t_1 m \Lambda_1^\mu + t_2 m \Lambda_2^\mu, \qquad p_1^2 = m^2,$$
$$p_2^\mu = (1/2 - R) q^\mu - s k^\mu - t_1 m \Lambda_1^\mu - t_2 m \Lambda_2^\mu, \qquad p_2^2 = m^2$$

- From the on-shell conditions we obtain the relations ($n = s' - s$):

$$s = \frac{1}{(2R-1)} \frac{m^2}{kq} (1 + t_1^2 + t_2^2), \quad s' = \frac{1}{(2R+1)} \frac{m^2}{kq} (1 + t_1^2 + t_2^2)$$

Momentum distribution of the created pairs



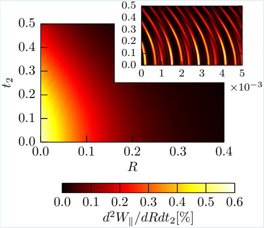
- Both R and t_2 are constants of motion for a plane-wave/constant-crossed field
- The corresponding distributions are not changed by the classical propagation

Parameters: $\chi = 1$, $\xi = 10$, $N = 5$, $\phi_0 = \pi/2$,
Pulse: $\psi_1'(\phi) = \sin^2[\phi/(2N)]\sin(\phi+\phi_0)$, $\psi_2'(\phi) = 0$

- To include quantum processes into a PIC code, the **initial conditions for the classical propagation** of the created particles must be known
- Approach so far:
  - Ignore the transverse degree of freedom
  - All particles move initially into the forward direction
- Aim: full 3D simulation
  - We need to provide initial values for R, t_1 and t_2
  - Constant-crossed field rate: distribution for R and t_2
- **Question: which initial value for $\mathbf{t_1}$?** Our answer: $t_1 = 0$

  $\longrightarrow$ 3D simulations possible for the first time!

Momentum distribution of the created pairs



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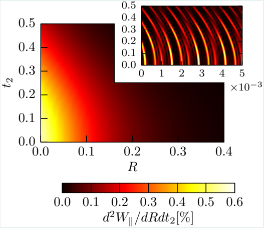
Initial conditions for the classical propagation

Momentum distribution of the created pairs



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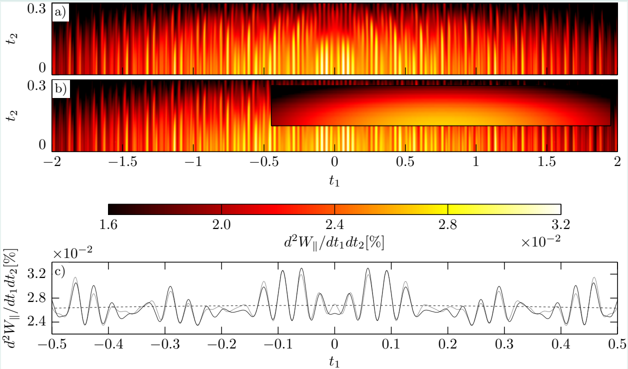
  - We need to provide initial values for R, t_1 and t_2

BELLA-i could investigate the validity of the SF-QED framework and the semiclassical approximation $\longrightarrow$ crucial for PIC codes!

Local constant-crossed field approximation



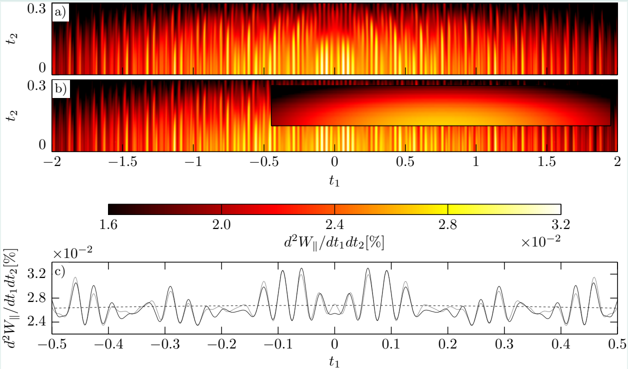
$\chi = 1$, $\xi = 5$, $N = 5$ cycle, e.g., $\omega_\gamma = 17\,\mathrm{GeV}$ and $10^{20}\ \mathrm{W/cm^2}$

- Stationary-phase approximation possible for $\xi \gg 1$
- Location of the stationary points: classical equation of motion
- Probability amplitude: pair-creation inside a constant-crossed field
- However: interference between different formation regions important

SM, C. H. Keitel, and A. Di Piazza, arXiv:1503.03271 (2015)

Pair production: Importance of interference effects

Local constant-crossed field approximation



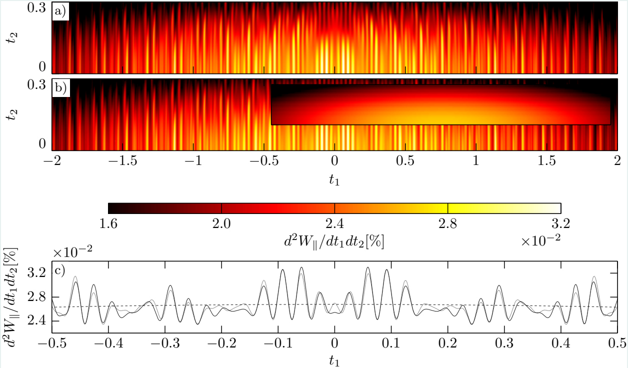
$\chi = 1$, $\xi = 5$, $N = 5$ cycle, e.g., $\omega_\gamma = 17\,\mathrm{GeV}$ and $10^{20}\ \mathrm{W/cm^2}$

- Stationary-phase approximation possible for $\xi \gg 1$
- Loca… …n
- Prob… …field
- How… …portant

SM, C. H. …

> Applied on the probability level, the local constant-crossed field approximation cannot reproduce the substructure!
> This was observed for Compton scattering in:
> Harvey, Ilderton, King, PRA **91** 013822 (2015)

Local constant-crossed field approximation



$\chi = 1$, $\xi = 5$, $N = 5$ cycle, e.g., $\omega_\gamma = 17\,\mathrm{GeV}$ and $10^{20}\ \mathrm{W/cm^2}$

- Stationary-phase approximation possible for $\xi \gg 1$
- Loca

Applied on the probability level, the local constant-crossed field

High repetition rate: BELLA-i could study the spectrum in detail!
(highly monochromatic photons are required to resolve interferences)

t

SM

Summary: main topics of the talk

- **Why should we study Strong-Field QED?**
  - Intuitive explanation of the QED critical field
  - Phenomena related to the nonlinear regime of QED
- **Lasers as a tool to study the critical field**
  - Nonlinear Compton scattering
  - Nonlinear Breit-Wheeler pair production
  - From a single vertex to a QED cascade
  - Fully nonperturbative regime of QED
- **Nonlinear Breit-Wheeler process**
  - Multiphoton vs. tunneling pair production
  - Semiclassical description
  - Difference between classical and quantum absorption
  - Initial conditions for the classical propagation
  - Importance of interference effects

Thank you for your attention and your questions!

Laser-pulse-shape control of seeded QED cascades

BELLA-i workshop

21 January 2016

Matteo Tamburini

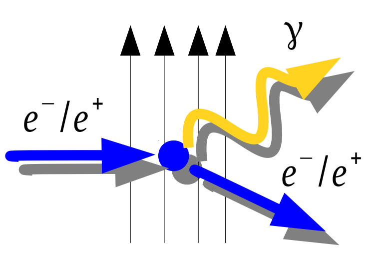
Antonino Di Piazza   Christoph H. Keitel

MAX-PLANCK-GESELLSCHAFT

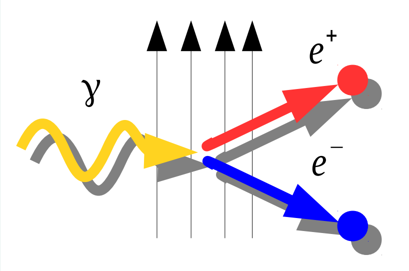
MPI for Nuclear Physics, Heidelberg

The basic QED processes in a strong electromagnetic field

Photon emission by an e^-/e^+



Photon conversion into an e^-e^+ pair



Strong-field QED processes are controlled by the quantum parameter χ

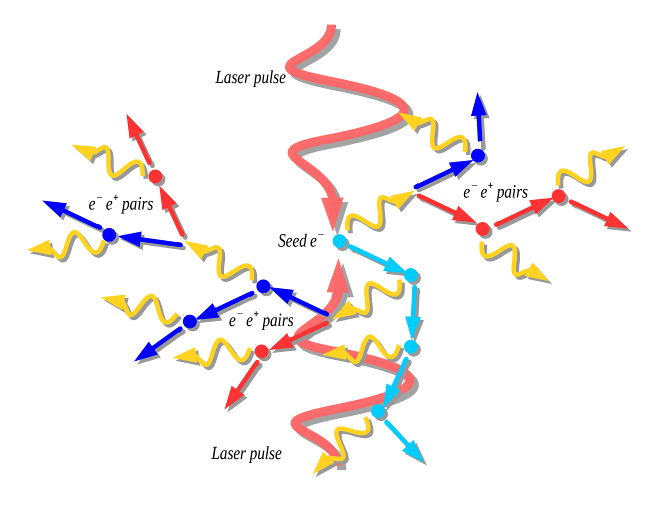
$\chi_{e/\gamma} = |F_{\mu\nu} p^{\nu}_{e/\gamma}|/E_{cr} mc$

For $\chi_e \ll 1$ the typical energy of the emitted photons is $\varepsilon_\gamma \approx \chi_e \varepsilon_e$, ε_e being the electron energy. Single photon emission recoil dominates when $\chi_e \gtrsim 1$.

$e E_{cr} \lambda_C = mc^2$; $\quad E_{cr} \approx 1.3 \times 10^{16}$ V/cm

For $\chi_\gamma \ll 1$ the probability of photon conversion into an e^-e^+ pair is suppressed as $e^{-8/3\chi_\gamma}$. Photon conversion is important when $\chi_\gamma \gtrsim 1$.

Seeded QED cascades



1) Seed e^- are violently accelerated by the laser fields and emit large amounts of γ which, in turn, convert into e^-e^+ pairs.

2) The generated e^-e^+ pairs are then accelerated by the laser fields and originate a new generation of particles.

3) QED cascades were predicted to develop in the collision of two laser pulses each with an intensity around 10^{24} W/cm^2 (Bell *et al.* PRL 2008, Kirk *et al.* PPCF 2009, Nerush *et al.* PRL 2011).

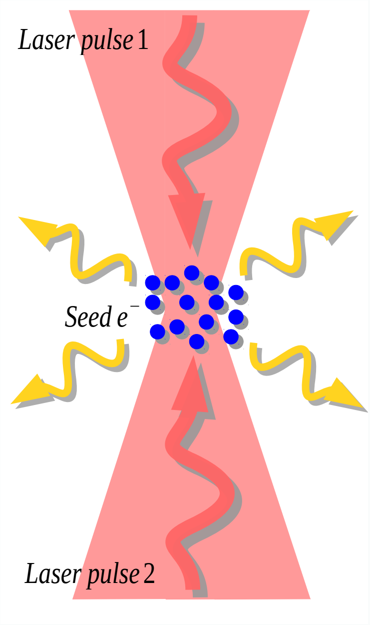
Introduction 3/3

Why are laser-driven QED cascades interesting?

- QED cascades open up the investigation of a novel regime dominated by the interplay between strong-field QED and multiparticle processes.
- QED cascades play a fundamental role in astrophysical environments such as the magnetosphere of pulsars, rendering an earth based implementation with intense lasers attractive.
- QED cascades were predicted to limit the attainable intensity of extreme laser sources due to the depletion of the laser pulse energy (Fedotov *et al.* PRL 2010, Bulanov *et al.* PRL 2010, Nerush *et al.* PRL 2011).

Our findings

- To date, the research has been focused on the intensity required to trigger QED cascades. The implications of the strong field gradients associated with tightly focused laser pulses have been neglected.
- We have shown (Tamburini *et al.* arXiv:1511.03987) the essential role played by the laser field shape on the onset of seeded QED cascades when accounting for realistic laser pulse structures. Tight focusing may prevent the formation of QED cascades even at intensities around 10^{26} W/cm^2, while moderate focusing allows to trigger QED cascades at intensities below 10^{24} W/cm^2.

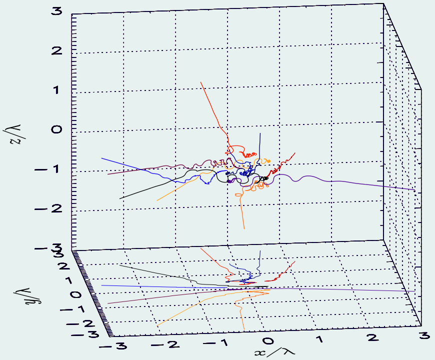


Parameters

- Two ultraintense linearly polarized laser pulses collide head-on in a tenuous gas (such as the residual gas of a vacuum chamber).
- Their transverse spatial profile is Gaussian, with $\lambda = 0.8\,\mu$m wavelength, $T = \lambda/c \approx 2.67$ fs period, and hyperbolic secant temporal field envelope with 20 fs duration FWHM of the intensity.
- A fully 3D description of the laser pulse fields with terms up to the fifth order in the diffraction angle $\epsilon = \lambda/\pi w_0$, w_0 being the waist radius, is employed.
- Initially, 10^3 seed electrons are located at rest within a λ^3 volume at the laser pulse focus with uniform random distribution (electron density $n = 2 \times 10^{15}\mathrm{cm}^{-3}$, while the critical density is $n_c = m\omega^2/4\pi e \approx 1.1 \times 10^{21}\lambda_{\mu\mathrm{m}}^{-2}\mathrm{cm}^{-3}$
- Electrons originate from the ionization of different atomic species (e.g. H, O), and go into the continuum at different values of the laser field at the focus.

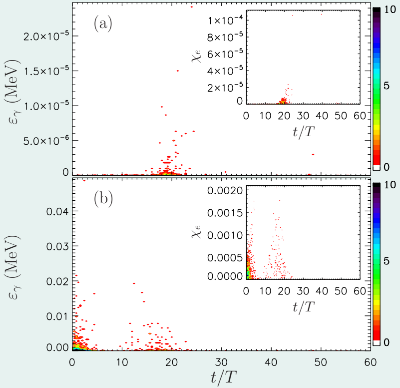
Tight focusing: 1λ waist radius, 10^{26} W/cm^2 intensity

Trajectories of ten seed electrons



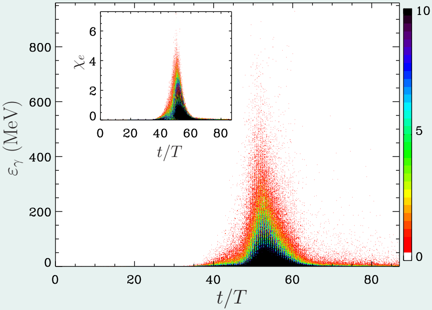
The trajectories for $0 < t < 24\ T$ of ten seed electrons. At $t = 24T$ all seed electrons are outside the focal spot and the laser pulse peaks are still at $z = \pm 35\,\lambda$ ($\approx 2.9 \times 10^{19}$ W/cm^2 intensity at the focus). The projection of the trajectories on the focal plane xy is also reported.

Emitted photon energies



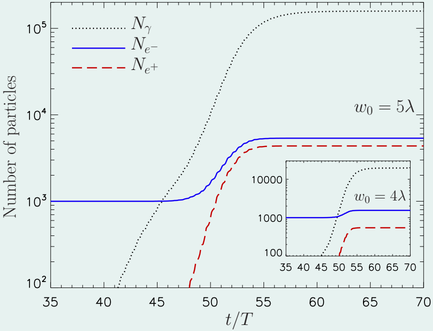
The photon energy ε_γ and the χ_e parameter (inset) at each photon emission event as function of time. Initially, the peaks of the two laser pulses are located at:
(a) $z_0 = \pm 59\lambda$ (H). (b) $z_0 = \pm 32.4\lambda$ (O^{+7}).
The colors correspond to the number of events (black means ≥ 10 events).

Emitted photon energies



The photon energy ε_γ and the χ_e parameter (inset) at each photon emission event. Initially, the peaks of the laser pulses are located at $z_0 = \pm 49.2\,\lambda$ (H). The colors correspond to the number of events (black means ≥ 10 events).

The number of particles



The evolution of the number of electrons N_{e-}, positrons N_{e+} and photons N_γ with energy $\varepsilon_\gamma > 25$ MeV. The inset displays the results with the same parameters but $w_0 = 4\,\lambda$ waist radius.

Although the laser pulse intensity decreases from 10^{26} W/cm^2 to 10^{24} W/cm^2 here χ_e exceeds unity and copious emission of photons with several hundreds MeV energy occurs.

$e^- e^+$ pair creation regimes as function of the waist radius w_0 and either the intensity I or the power P per laser pulse

I (W/cm^2)	w_0 (λ)	Regime
1×10^{24}	$\lesssim 2$	No $e^- e^+$ pairs
	3	Transition region: no pairs/$e^- e^+$ gas
	4	$e^- e^+$ gas
	$\gtrsim 5$	$e^- e^+$ cascade
$1 \times 10^{25-26}$	$\lesssim 2$	No $e^- e^+$ pairs
	3	Transition region: no pairs/$e^- e^+$ cascade
	$\gtrsim 4$	$e^- e^+$ cascade

P (PW)	w_0 (λ)	Regime
200	$\lesssim 3$	No $e^- e^+$ pairs
	4	Transition region: $e^- e^+$ cascade/$e^- e^+$ gas
	5 - 8	$e^- e^+$ gas
	$\gtrsim 9$	$e^- e^+$ yield $< 1\%$ of the initial e^-
500	$\lesssim 2$	No $e^- e^+$ pairs
	3	Transition region: no pairs/$e^- e^+$ cascade
	4 - 9	$e^- e^+$ cascade ($w_0 \approx 4$ max. growth)
	10 - 12	$e^- e^+$ gas
	$\gtrsim 13$	$e^- e^+$ yield $< 1\%$ of the initial e^-

M. Tamburini, A. Di Piazza, and C. H. Keitel, arXiv:1511.03987 (2015).

Summary and conclusions

Our main findings on the onset of QED cascades

- Our results show that the laser field structure may dominate the onset of e^-e^+ cascades even with respect to the laser intensity, and must be considered in the design and interpretation of experiments with tightly focused laser pulses.

- We have highlighted the importance of the nature of the gas. Inner shell electrons of high-Z elements may go into the continuum only when the peak of the laser pulses reach the focus. In this case the power required to initiate a QED cascade falls to 11 PW per pulse (Tamburini *et al.* arXiv:1511.03987).

- These findings open up the possibility of controlling the onset of QED cascades via, e.g., the laser pulse waist radius or choosing suitable high-Z gases.

Thank you for your attention!