

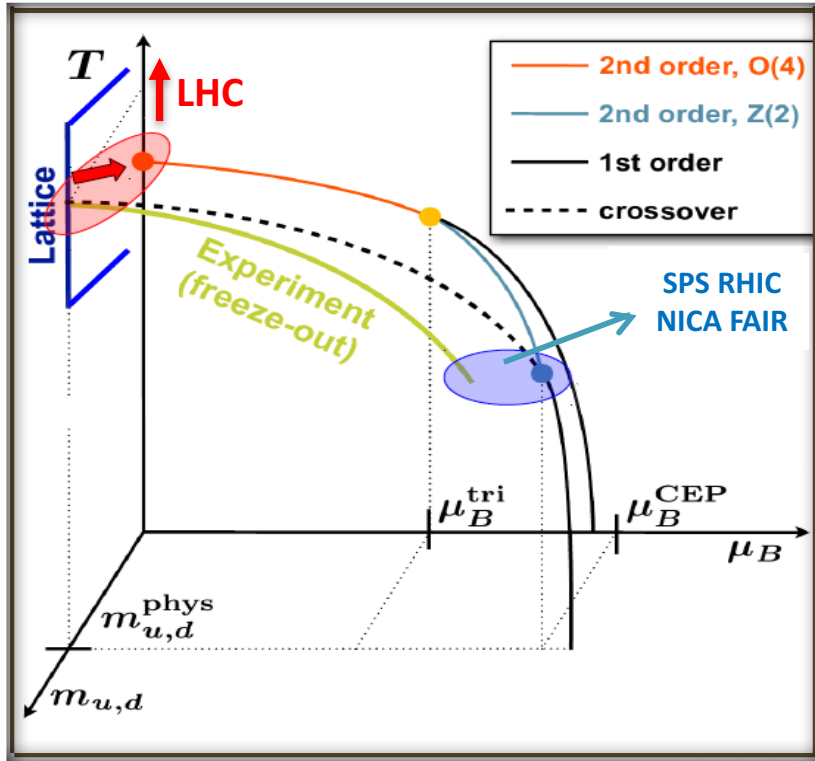
High Moments of Particle Distributions from Heavy-Ion Collisions at LHC

Mesut Arslandok
(Yale University, CERN)

- ✓ Why fluctuations?
- ✓ Results: Conserved charge fluctuations
- ✓ Experimental challenges
- ✓ Future plans

RHIC Beam Energy Scan and Beyond – Online Workshop
Lawrence Berkeley National Laboratory, California, August 16, 2021

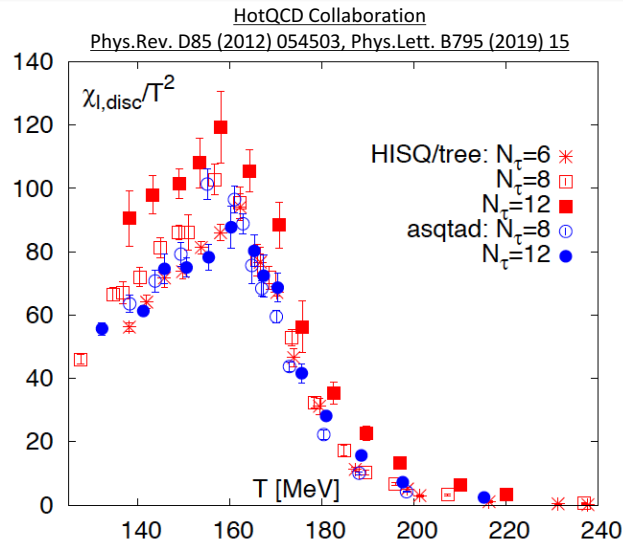
Why fluctuations?: Nature of chiral phase transition



F. Karsch, Schleching 2016

small u, d quark masses
 \leftrightarrow
vicinity to 2nd order $O(4)$ criticality
 \downarrow
pseudocritical features possible

Criticality at Crossover

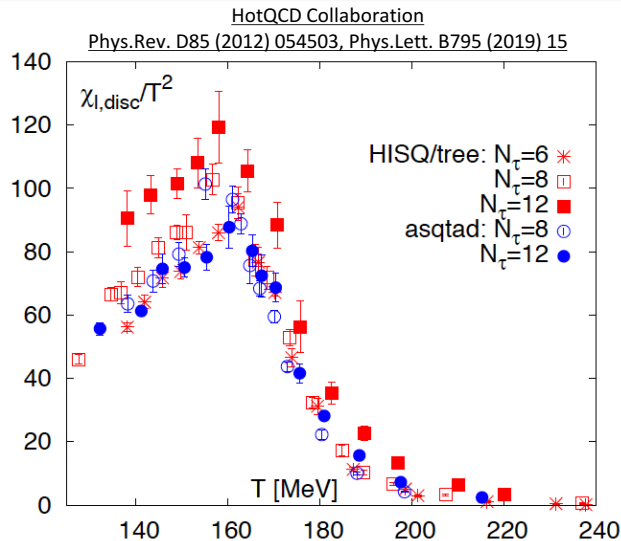


$$T_{pc} = 156.5 \pm 1.5 \text{ MeV}$$

$$\langle \bar{\psi}\psi \rangle_l^{n_f=2} = \frac{T}{V} \frac{\partial \ln Z}{\partial m_l}$$

$$\chi_{m,l} = \frac{\partial}{\partial m_l} \langle \bar{\psi}\psi \rangle_l^{n_f=2}$$

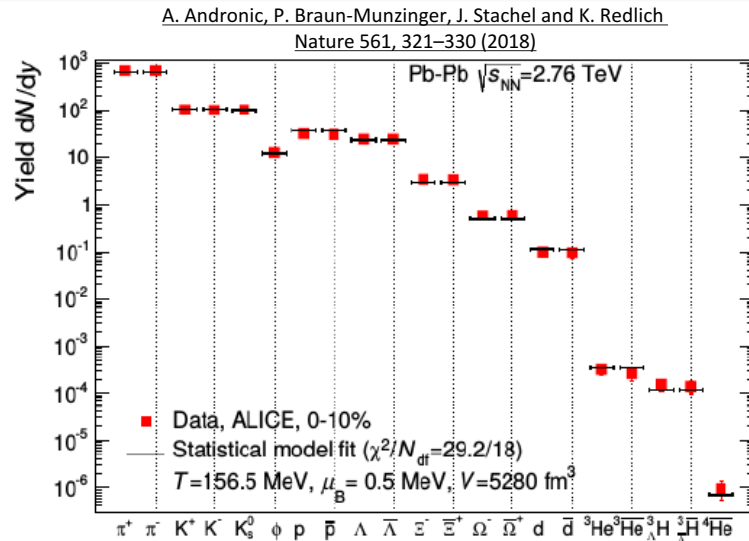
Criticality at Crossover



$$T_{pc} = 156.5 \pm 1.5 \text{ MeV}$$

$$\langle \bar{\psi}\psi \rangle_l^{n_f=2} = \frac{T}{V} \frac{\partial \ln Z}{\partial m_l}$$

$$\chi_{m,l} = \frac{\partial}{\partial m_l} \langle \bar{\psi}\psi \rangle_l^{n_f=2}$$



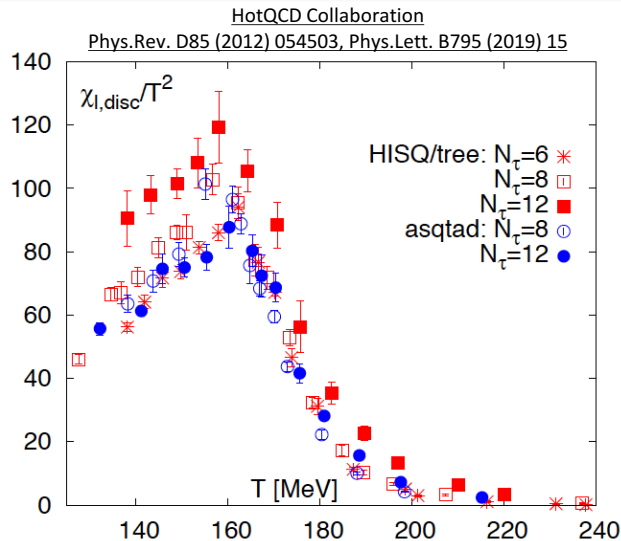
$$T_{fo}^{ALICE} = 156.5 \pm 3 \text{ MeV}$$

$$\langle N_i \rangle = V \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1}$$

$$\mu_i = \mu_B B_i + \mu_s S_i + \mu_I I_i$$

$$\chi^2 = \sum_{k=1}^n \frac{(\langle N_k^{\text{exp}} \rangle - \langle N_k^{\text{HRG}} \rangle)^2}{\sigma_k^2}$$

Criticality at Crossover

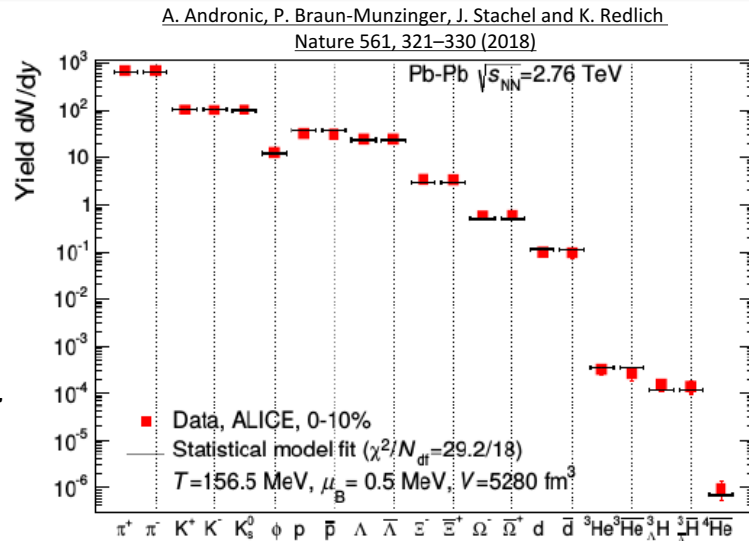


$$T_{pc} = 156.5 \pm 1.5 \text{ MeV}$$

$$\langle \bar{\psi}\psi \rangle_l^{n_f=2} = \frac{T}{V} \frac{\partial \ln Z}{\partial m_l}$$

$$\chi_{m,l} = \frac{\partial}{\partial m_l} \langle \bar{\psi}\psi \rangle_l^{n_f=2}$$

**Chemical freeze-out
at the
phase boundary!**



$$T_{fo}^{ALICE} = 156.5 \pm 3 \text{ MeV}$$

$$\langle N_i \rangle = V \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1}$$

$$\mu_i = \mu_B B_i + \mu_s S_i + \mu_I I_i$$

$$\chi^2 = \sum_{k=1}^n \frac{(\langle N_k^{\text{exp}} \rangle - \langle N_k^{\text{HRG}} \rangle)^2}{\sigma_k^2}$$

Chemical freeze-out near $T_{pc} \rightarrow$ motivation to look for higher order moments

Link to LQCD: Fluctuations of conserved charges

For a thermal system within the **Grand Canonical Ensemble**

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \mu_{B,Q,S}) \Rightarrow \hat{\chi}_n^{N=B,S,Q} = \frac{\partial^n P/T^4}{\partial (\mu_N/T)^n}$$

Susceptibilities



$$\hat{\chi}_2^B = \frac{\kappa_2(\Delta N_B)}{VT^3}$$

Cumulants

Link to LQCD: Fluctuations of conserved charges

For a thermal system within the **Grand Canonical Ensemble**

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \mu_{B,Q,S}) \Rightarrow \hat{\chi}_n^{N=B,S,Q} = \frac{\partial^n P / T^4}{\partial (\mu_N / T)^n}$$

Susceptibilities



$$\hat{\chi}_2^B = \frac{\kappa_2(\Delta N_B)}{VT^3}$$

Cumulants



$$\frac{\kappa_4(\Delta N_B)}{\kappa_2(\Delta N_B)} = \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B}$$

Higher orders

P. Braun-Munzinger, A. Rustamov, J. Stachel, Nucl. Phys. A960 (2017) 114

Link to LQCD: Fluctuations of conserved charges

For a thermal system within the **Grand Canonical Ensemble**

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \mu_{B,Q,S}) \Rightarrow \hat{\chi}_n^{N=B,S,Q} = \frac{\partial^n P/T^4}{\partial (\mu_N/T)^n}$$

Susceptibilities



$$\hat{\chi}_2^B = \frac{\kappa_2(\Delta N_B)}{VT^3}$$

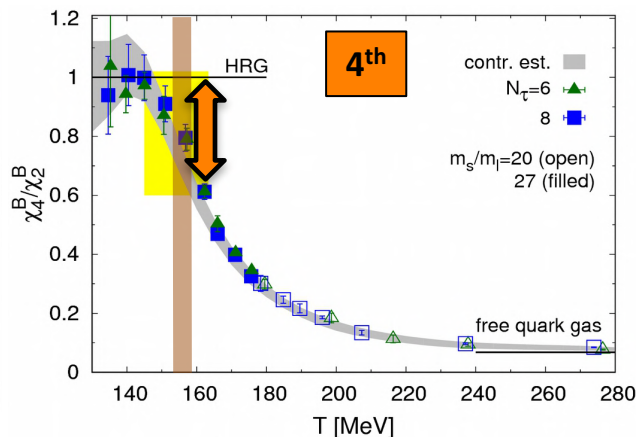
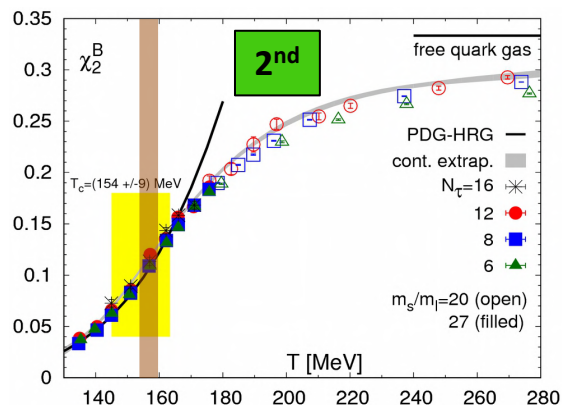
Cumulants



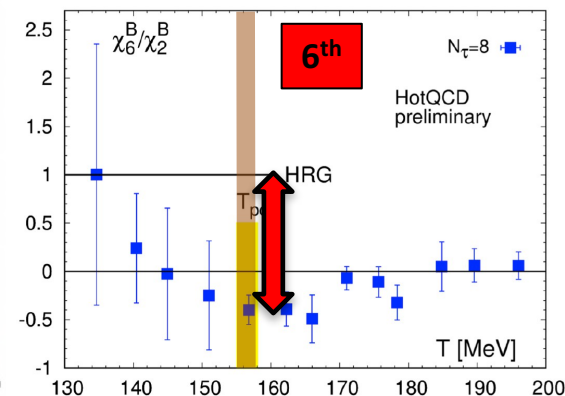
$$\frac{\kappa_4(\Delta N_B)}{\kappa_2(\Delta N_B)} = \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B}$$

Higher orders

P. Braun-Munzinger, A. Rustamov, J. Stachel, Nucl. Phys. A960 (2017) 114



Phys. Rev. D 95 (2017), 054504, Phys. Rev. D 96, 074510 (2017)



- Up to 3rd order LQCD agrees with Hadron Resonance Gas (HRG)
- At 4th order (~30%) and 6th order (~150%) deviation from HRG

$$X = N_B - N_{\bar{B}}$$

➤ **rth central moment:**

$$\mu_r \equiv \langle (X - \langle X \rangle)^r \rangle = \sum_X (X - \langle X \rangle)^r P(X)$$

➤ **First four cumulants**

$$\begin{aligned} \kappa_1 &= \langle X \rangle, & \kappa_2 &= \mu_2, \\ \kappa_3 &= \mu_3, & \kappa_4 &= \mu_4 - 3\mu_2^2 \end{aligned}$$

➤ **Uncorrelated Poisson limit:**

$$\langle N_B N_{\bar{B}} \rangle = \langle N_B \rangle \langle N_{\bar{B}} \rangle$$

Baseline: Skellam distribution

$$X = N_B - N_{\bar{B}}$$

➤ **rth central moment:**

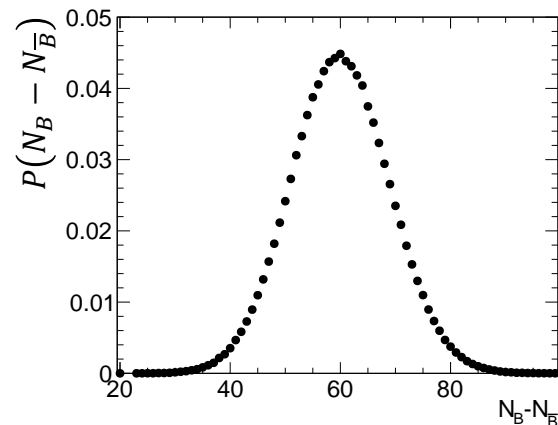
$$\mu_r \equiv \langle (X - \langle X \rangle)^r \rangle = \sum_X (X - \langle X \rangle)^r P(X)$$

➤ **First four cumulants**

$$\begin{aligned} \kappa_1 &= \langle X \rangle, & \kappa_2 &= \mu_2, \\ \kappa_3 &= \mu_3, & \kappa_4 &= \mu_4 - 3\mu_2^2 \end{aligned}$$

➤ **Uncorrelated Poisson limit:**

$$\langle N_B N_{\bar{B}} \rangle = \langle N_B \rangle \langle N_{\bar{B}} \rangle$$



Difference between two independent Poissonian distributions

$$\kappa_n = \langle N_B \rangle + (-1)^n \langle N_{\bar{B}} \rangle$$

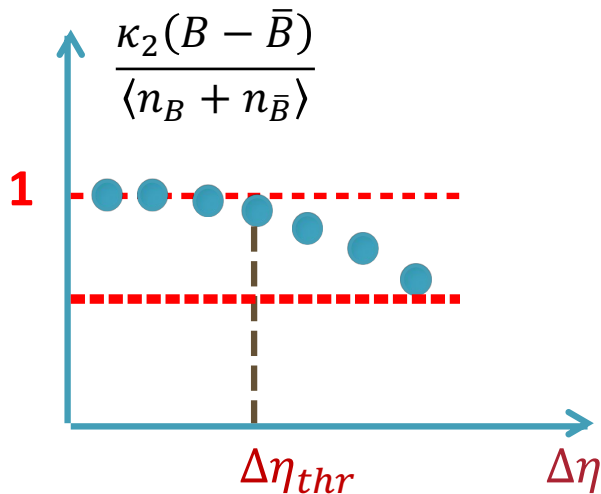


$$\frac{\kappa_{2n+1}}{\kappa_{2k}} = \frac{\langle n_B \rangle - \langle n_{\bar{B}} \rangle}{\langle n_B \rangle + \langle n_{\bar{B}} \rangle}$$

Odd cumulants vanish for $\langle N_B \rangle = \langle N_{\bar{B}} \rangle$

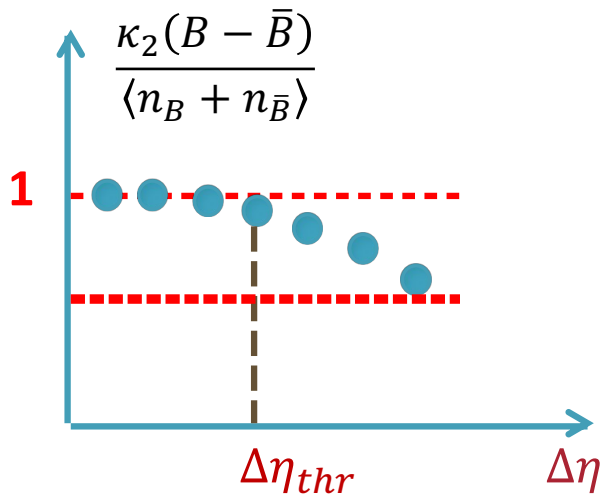
Importance of acceptance and baryon number conservation

- Fluctuations of conserved charges appear **only inside finite acceptance**
- **In the limit of very small acceptance**
→ only Poissonian fluctuations



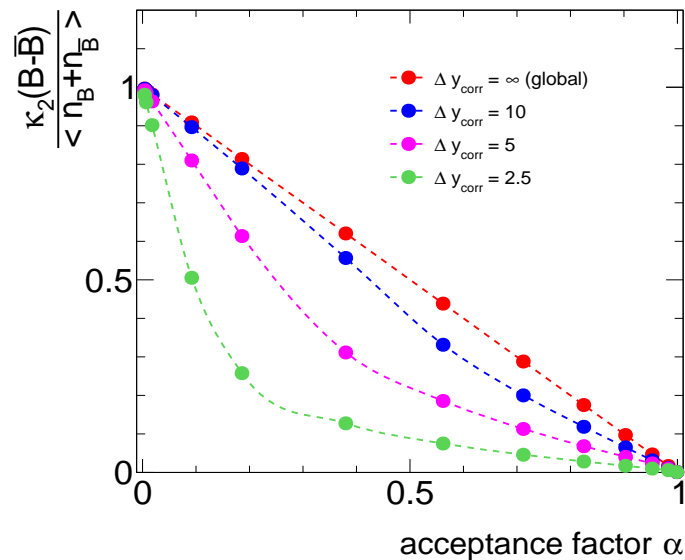
Importance of acceptance and baryon number conservation

- Fluctuations of conserved charges appear **only inside finite acceptance**
- **In the limit of very small acceptance**
→ only Poissonian fluctuations



- **Baryon number conservation** imposes subtle correlations

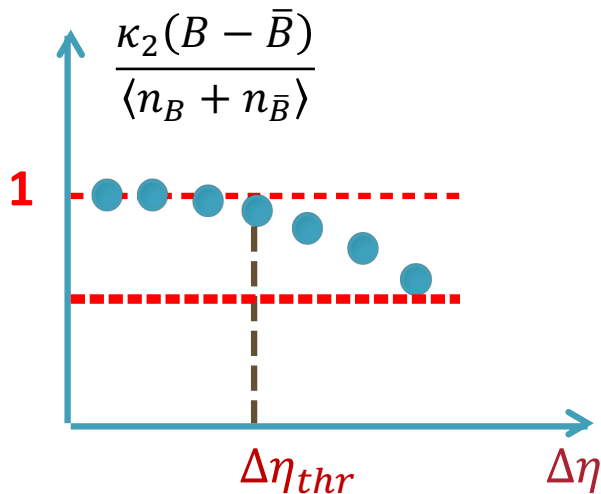
$$\alpha = \frac{\langle N_B^{acc} \rangle}{\langle N_B^{4\pi} \rangle} \quad |y_{\bar{B}} - y_B| < \frac{\Delta y_{corr}}{2}$$



P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1907.03032
 A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901
 V. Vovchenko, V. Koch Phys. Rev. C 103, 044903 (2021)

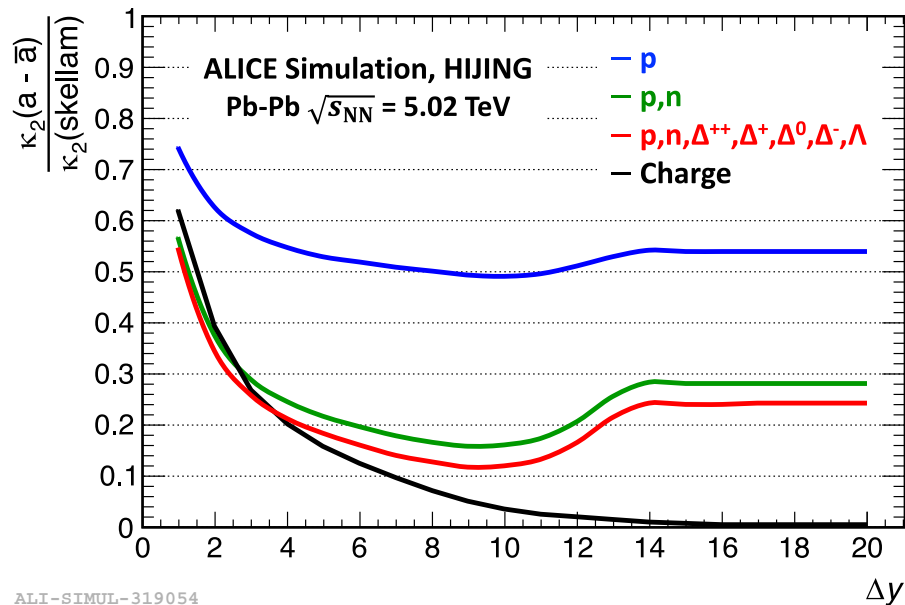
Importance of acceptance and baryon number conservation

- Fluctuations of conserved charges appear **only inside finite acceptance**
- **In the limit of very small acceptance**
→ only Poissonian fluctuations



- **Baryon number conservation** imposes subtle correlations

$$\alpha = \frac{\langle N_B^{acc} \rangle}{\langle N_B^{4\pi} \rangle} \quad |y_{\bar{B}} - y_B| < \frac{\Delta y_{corr}}{2}$$



Results

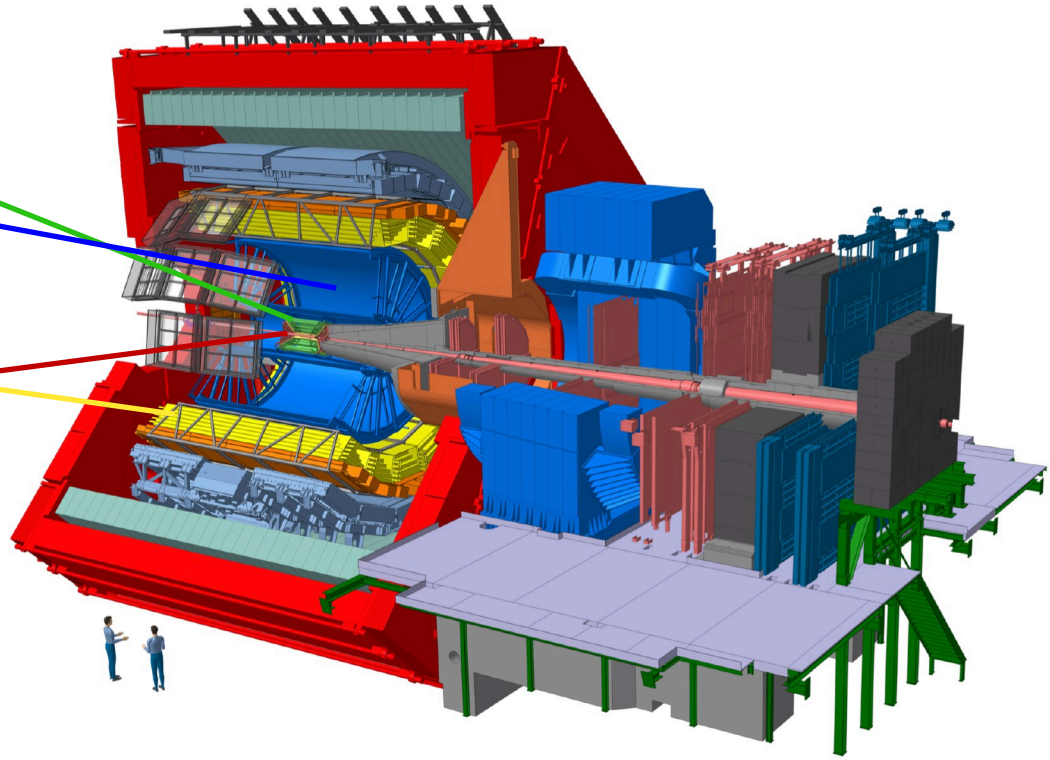
A Large Ion Collider Experiment

Main detectors used:

- Inner Tracking System (ITS) → Tracking and vertexing
- Time Projection Chamber (TPC) → Tracking and Particle Identification (PID)
- Time Of Flight (TOF) → Tracking and PID
- Vertex 0 (V0) → Centrality determination

Data Set:

- $\sqrt{s_{NN}} = 5.02 \text{ TeV}$, ~150 M events
- $\sqrt{s_{NN}} = 2.76 \text{ TeV}$, ~12 M events



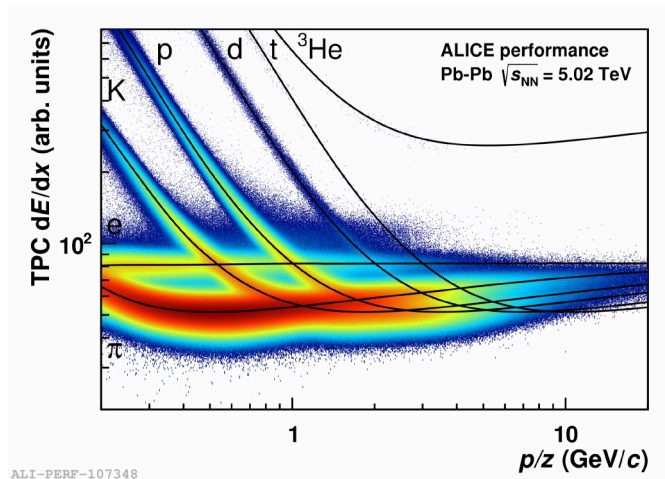
Methods

➤ Cut-based approach:

- Count tracks of a given particle type

➤ Identity method:

- Count probabilities to be of a given particle type
- Gives folded multiplicity distribution
- Allows for **larger efficiencies**
→ smaller correction needed



A. Rustamov, M. Gazdzicki, M. I. Gorenstein, PRC 86, 044906 (2012),
M. Arslanodk, A. Rustamov, NIM A 946 (2019) 162622

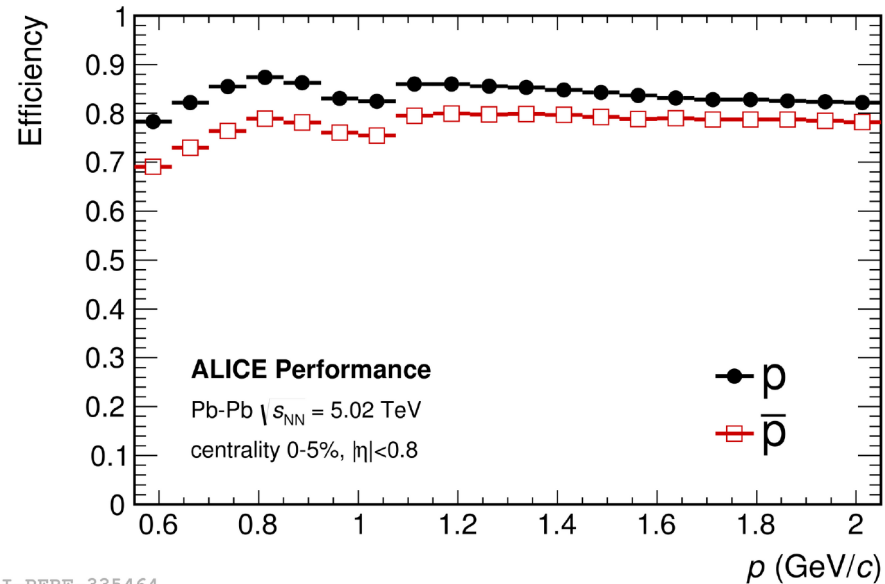
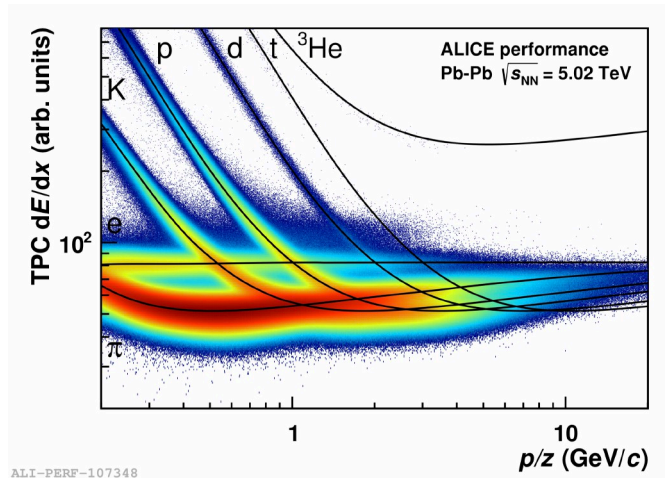
Methods

➤ Cut-based approach:

- Count tracks of a given particle type

➤ Identity method:

- Count probabilities to be of a given particle type
- Gives folded multiplicity distribution
- Allows for **larger efficiencies**
→ smaller correction needed



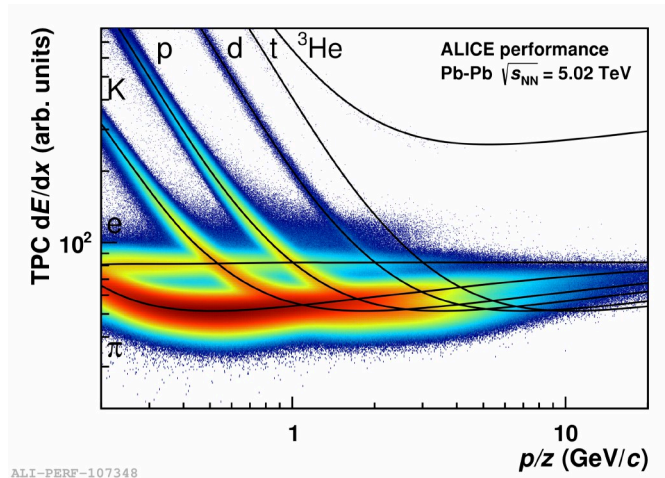
Methods

➤ Cut-based approach:

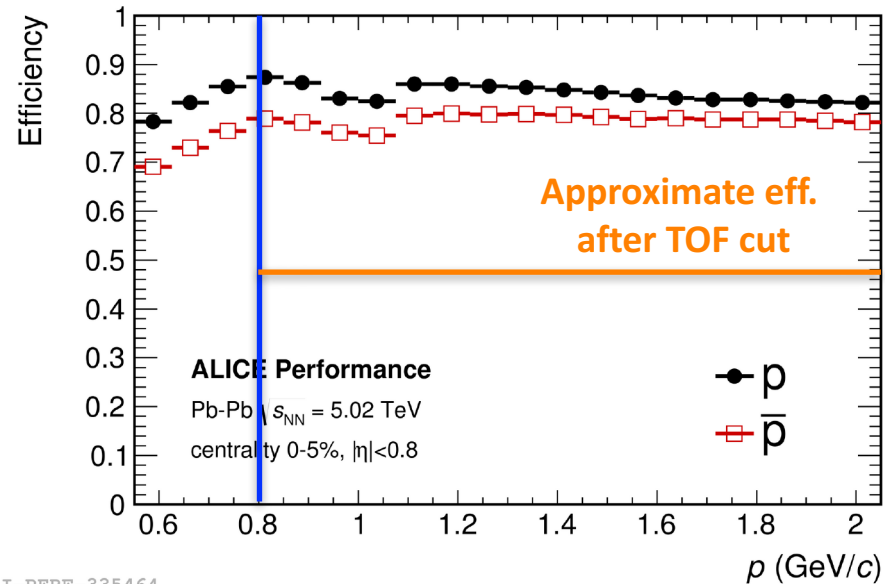
- Count tracks of a given particle type

➤ Identity method:

- Count probabilities to be of a given particle type
- Gives folded multiplicity distribution
- Allows for **larger efficiencies**
→ smaller correction needed

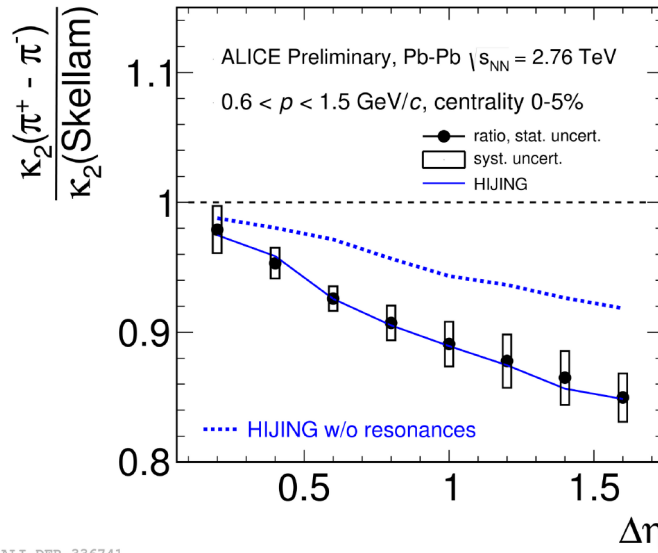


TOF cut boundary



ALI-PERF-335464

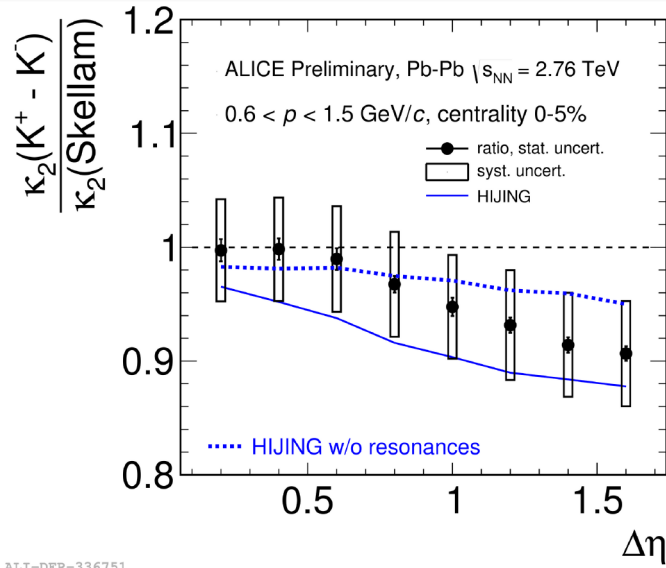
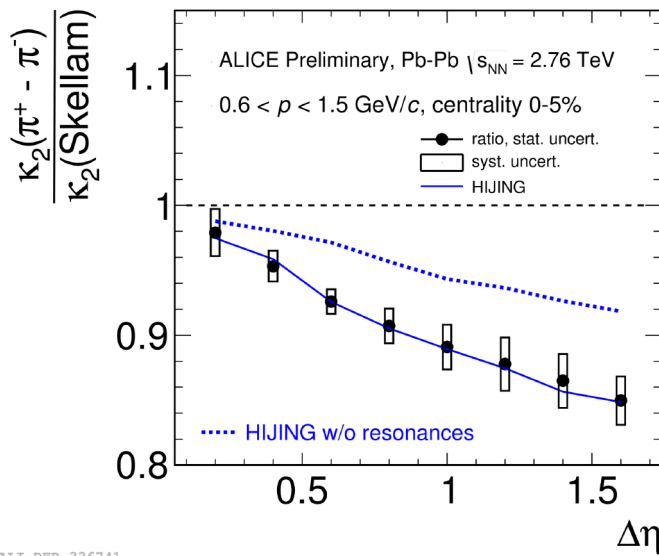
Net-(global)charge fluctuations



ALI-DER-336741

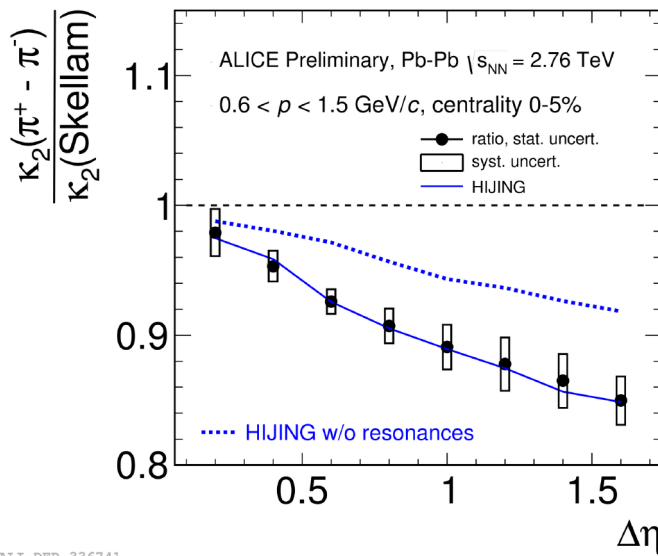
- **Net-electric-charge:** → Strongly dominated by **resonance contributions**

Net-(global)charge fluctuations

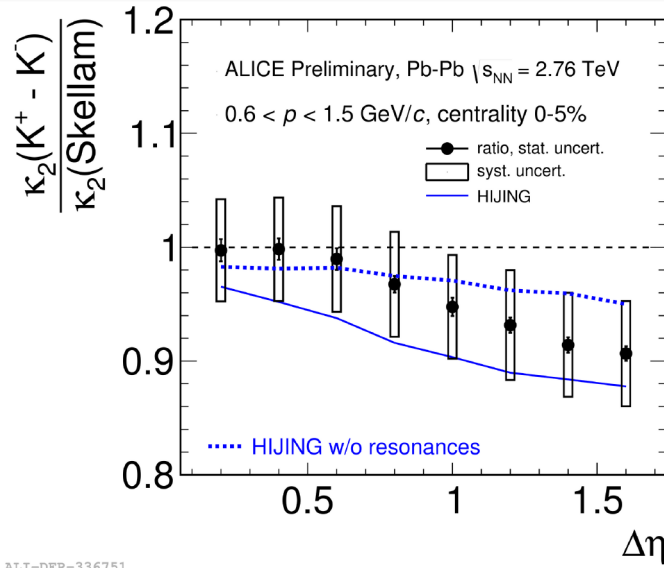


- **Net-electric-charge:** → Strongly dominated by **resonance contributions**
- **Net-strangeness:** → Kaons are dominated by **ϕ -decay**

Net-(global)charge fluctuations



ALI-DER-336741



ALI-DER-336751

➤ **Net-electric-charge:** → Strongly dominated by **resonance contributions**

➤ **Net-strangeness:** → Kaons are dominated by **ϕ -decay**

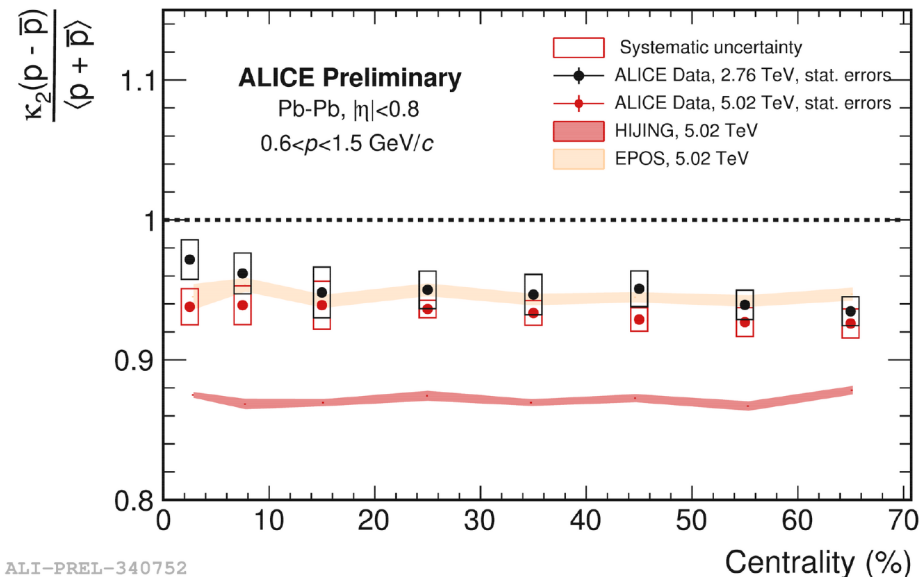
➤ **Net-baryon:**

→ Due to **isospin randomization**, at $\sqrt{s_{NN}} > 10$ GeV **net-baryon** fluctuations can be obtained from corresponding **net-proton** measurements (M. Kitazawa, and M. Asakawa, Phys. Rev. C 86, 024904 (2012))

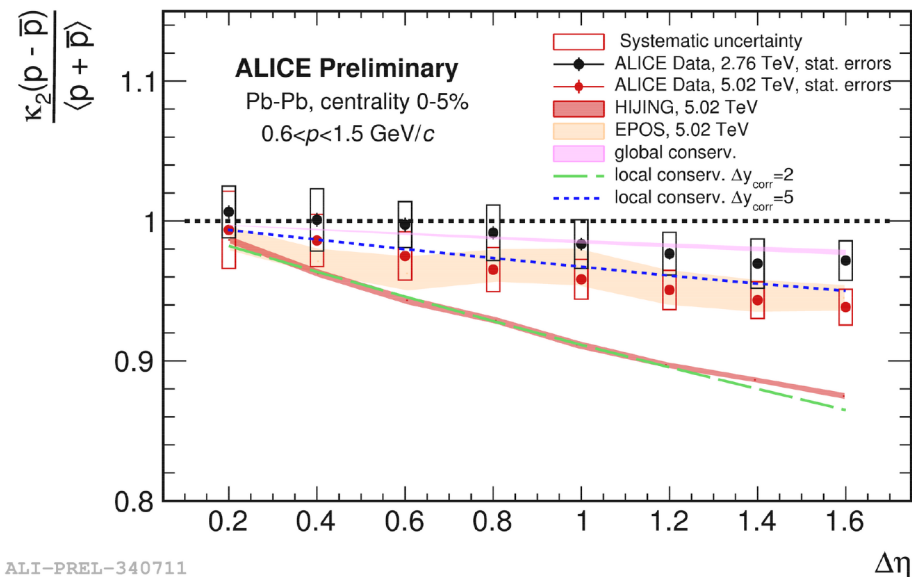
→ No resonance feeding $p + \bar{p}$

→ **Best candidate for measuring charge susceptibilities**

2nd order cumulants of net-p



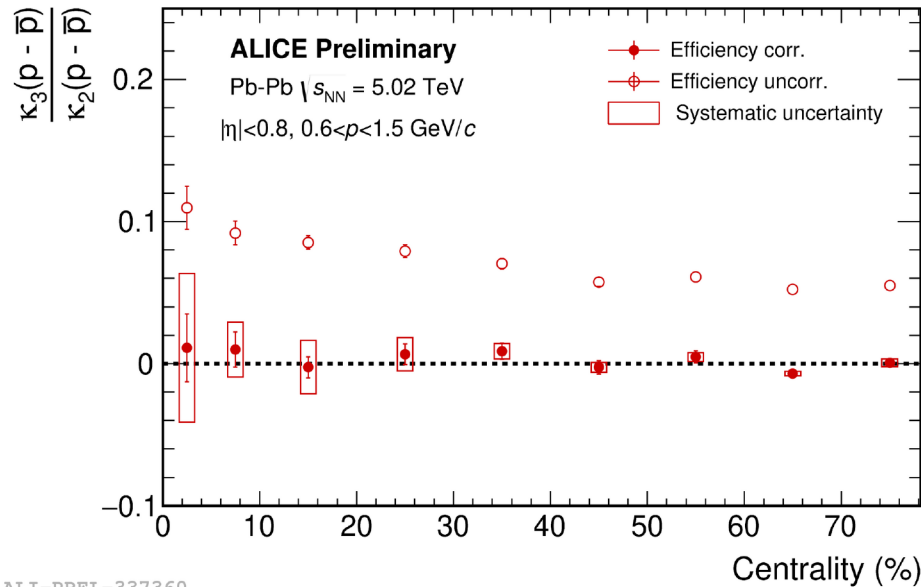
ALI-PREL-340752



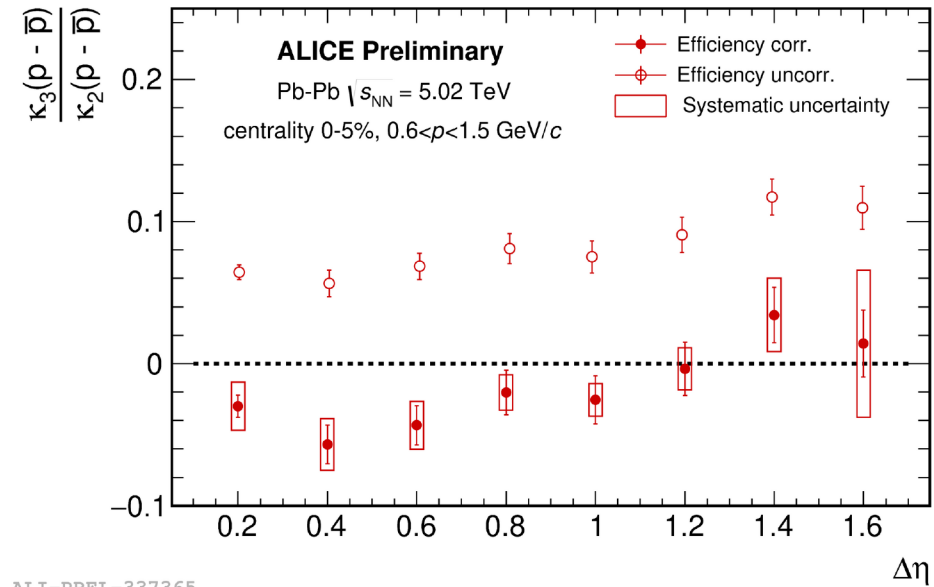
ALI-PREL-340711

- Deviation from Skellam baseline is due to **baryon number conservation**
- ALICE data suggest **long range correlations**, $\Delta y = \pm 2.5$ unit or longer
- EPOS agrees with ALICE data but HIJING deviates significantly
 - Event generators based on string fragmentation (HIJING) conserve baryon number over $\Delta y = \pm 1$ unit

3rd order cumulants of net-p



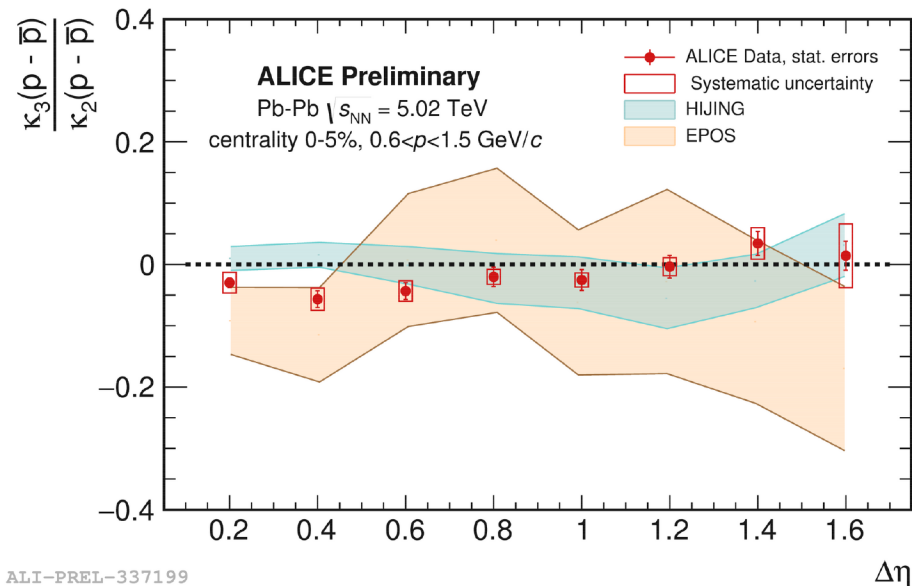
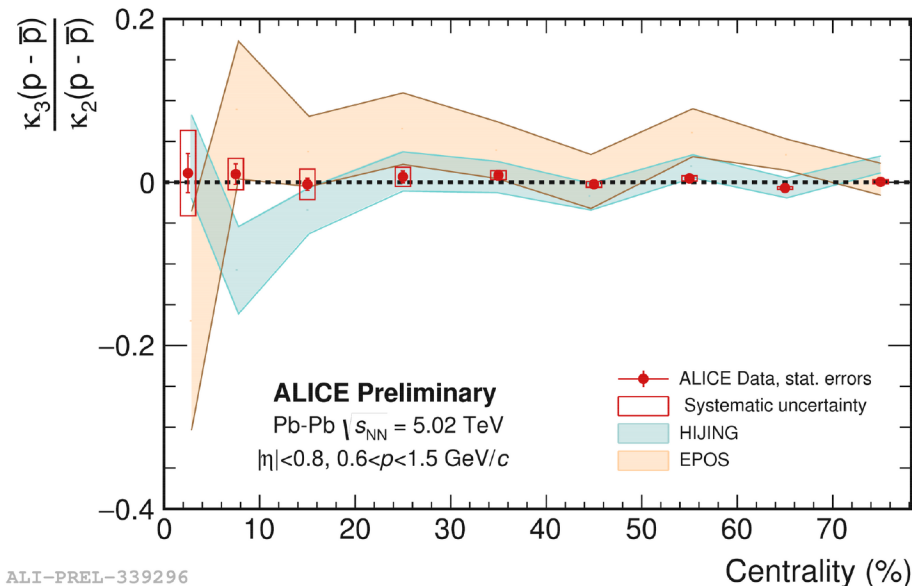
ALI-PREL-337360



ALI-PREL-337365

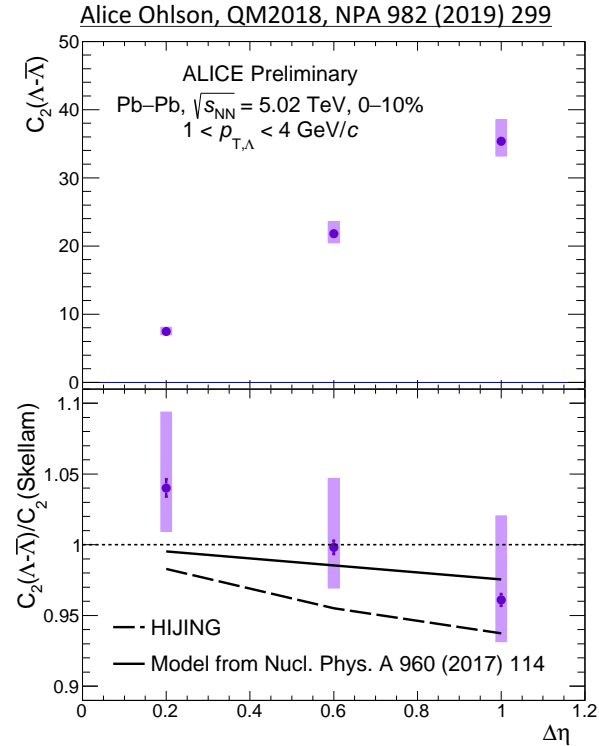
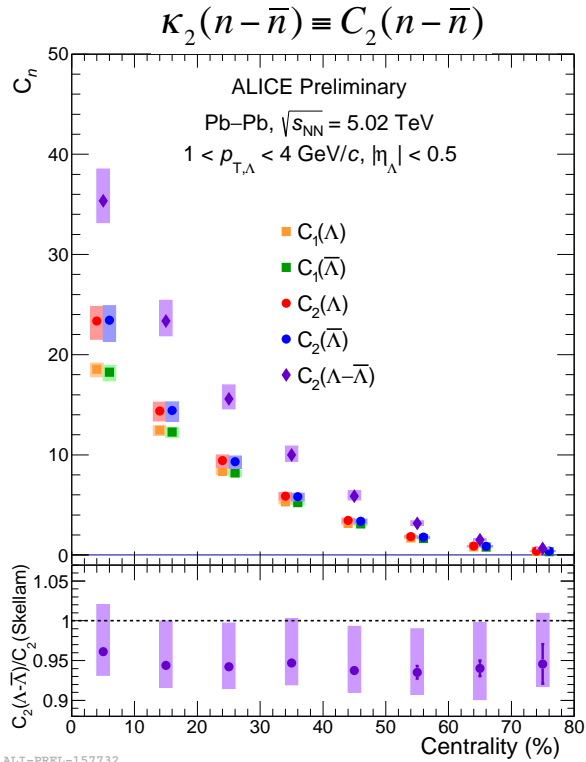
- **Data** agree with Skellam baseline “0” as a function of centrality and pseudorapidity
- Achieved precision of **better than 5%**

3rd order cumulants of net-p



- **Data** agree with Skellam baseline “0” as a function of centrality and pseudorapidity
- Achieved precision of **better than 5%**
- **EPOS and HIJING in agreement with data**
 - Both models conserve global charge → net-p within acceptance is ~ 0

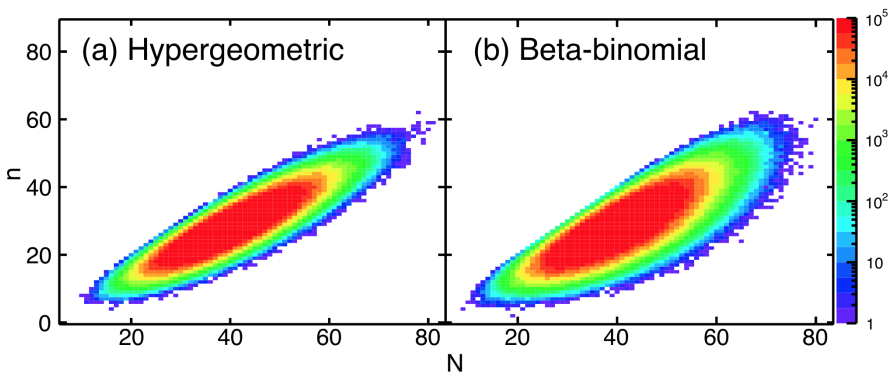
2nd order cumulants of net- Λ



- Similar trend as for **net-p**
- **Better precision** is needed to see the impact of strangeness conservation

Experimental Challenges

What if efficiency loss is not binomial?



Draw N balls from the urn
without returning balls to
the urn

In each draw, when one draws
a white ball, two white balls are
returned to the urn

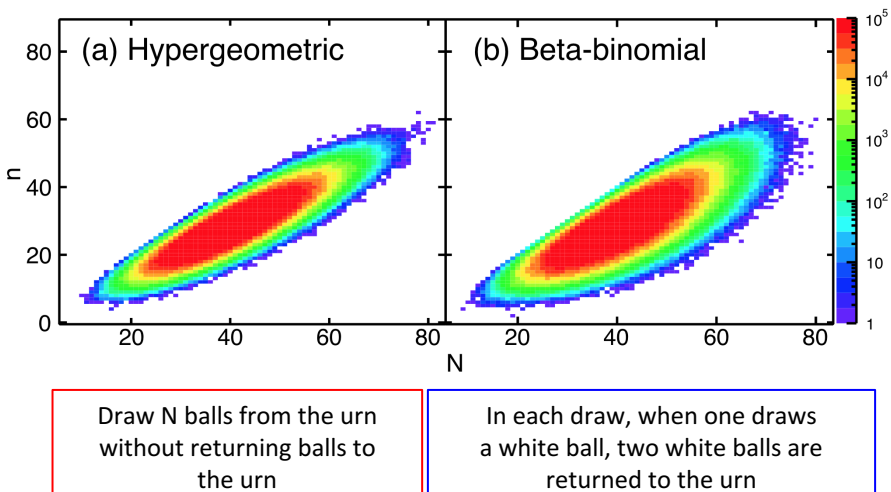
[T. Nonaka, M. Kitazawa, S. Esumi, Nucl.Instrum.Meth. A906 \(2018\) 10-17](#)

[T. Nonaka, M. Kitazawa, S. Esumi, Phys. Rev. C 95, 064912 \(2017\)](#)

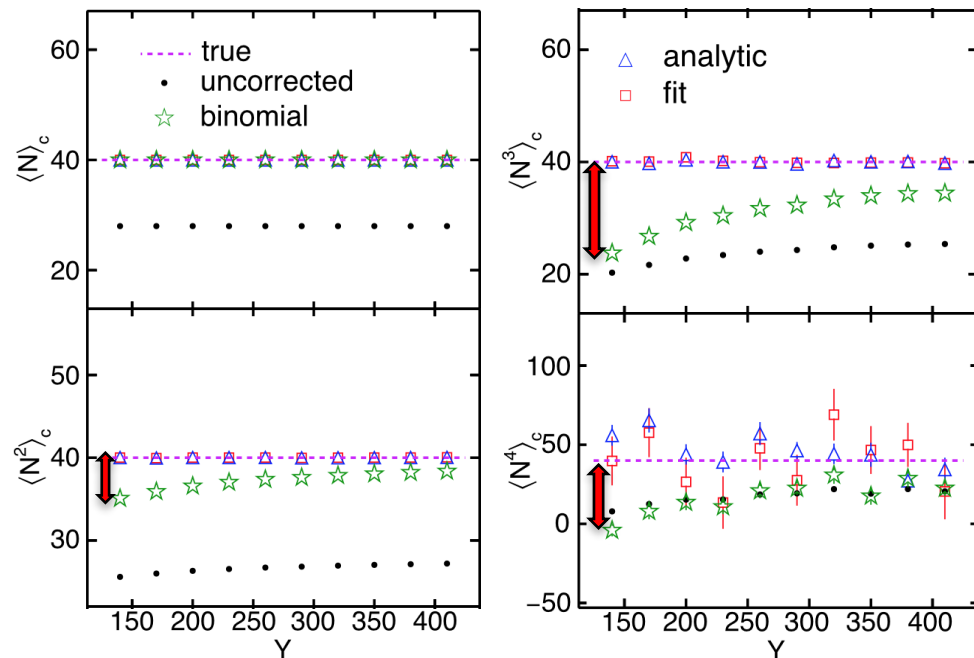
[Adam Bzdak, Volker Koch, Phys. Rev. C86, 044904 \(2012\)](#)

Efficiency correction

What if efficiency loss is not binomial?



Hypergeometric



T. Nonaka, M. Kitazawa, S. Esumi, Nucl.Instrum.Meth. A906 (2018) 10-17

T. Nonaka, M. Kitazawa, S. Esumi, Phys. Rev. C 95, 064912 (2017)

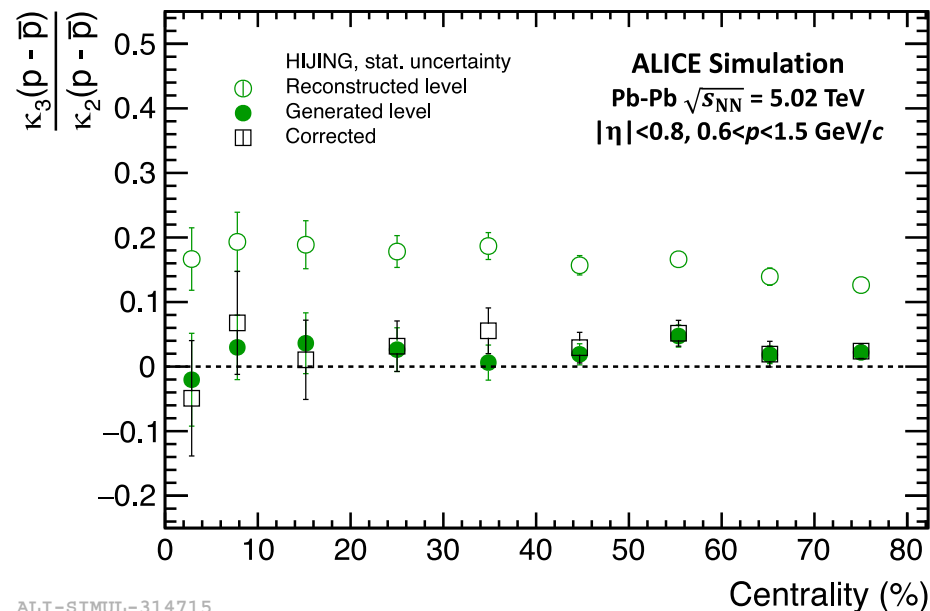
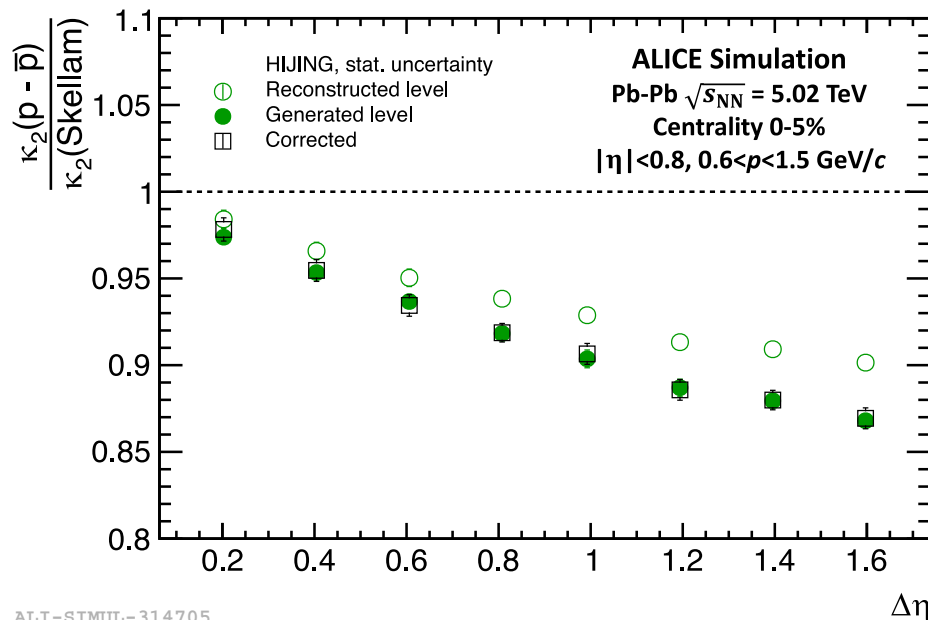
Adam Bzdak, Volker Koch, Phys. Rev. C86, 044904 (2012)

Efficiency correction

Efficiency correction with binomial assumption:

T. Nonaka, M. Kitazawa, S. Esumi, Phys. Rev. C 95, 064912 (2017)

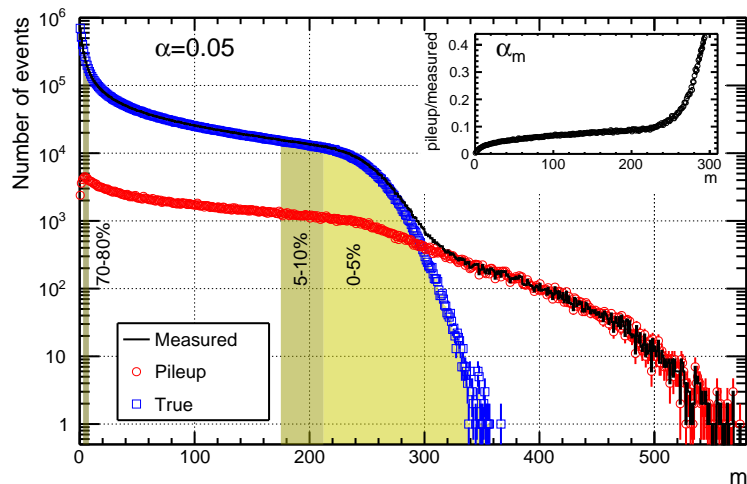
Adam Bzdak, Volker Koch, Phys. Rev. C 86, 044904 (2012)



- Monte-Carlo (MC) closure test is successful even though there is slight deviation from binomial detector response
- **Realistic MC description and track selection criteria are crucial**

Event pileup

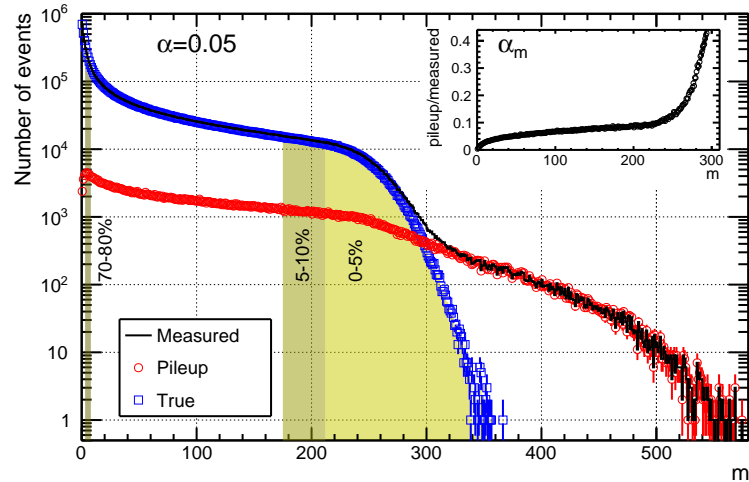
- **Event pileup:** When two collision events occur within a small space and time interval, they are identified as a single event
- Significant impact on the higher order cumulants
- Event selection criteria is crucial



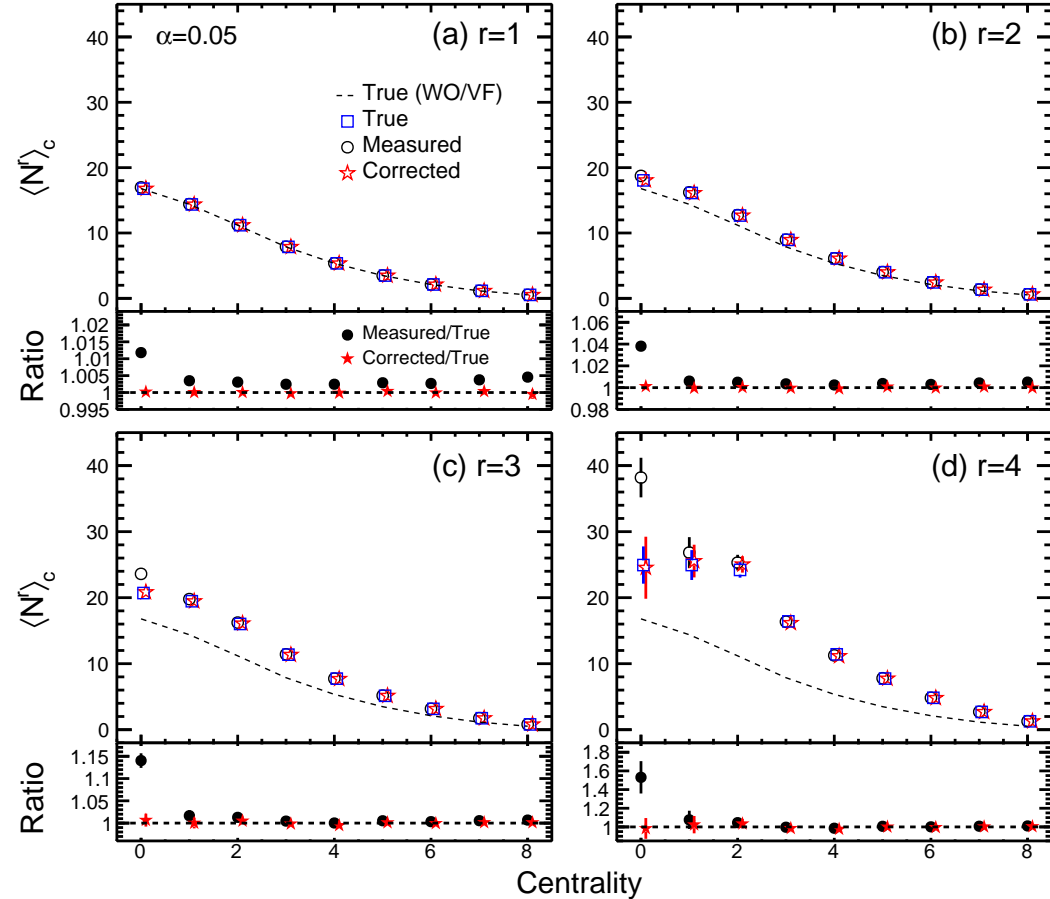
T. Nonaka, M. Kitazawa, S. Esumi, Nucl.Instrum.Meth. A984 (2020)

Event pileup

- **Event pileup:** When two collision events occur within a small space and time interval, they are identified as a single event
- Significant impact on the higher order cumulants
- Event selection criteria is crucial



T. Nonaka, M. Kitazawa, S. Esumi, Nucl.Instrum.Meth. A984 (2020)

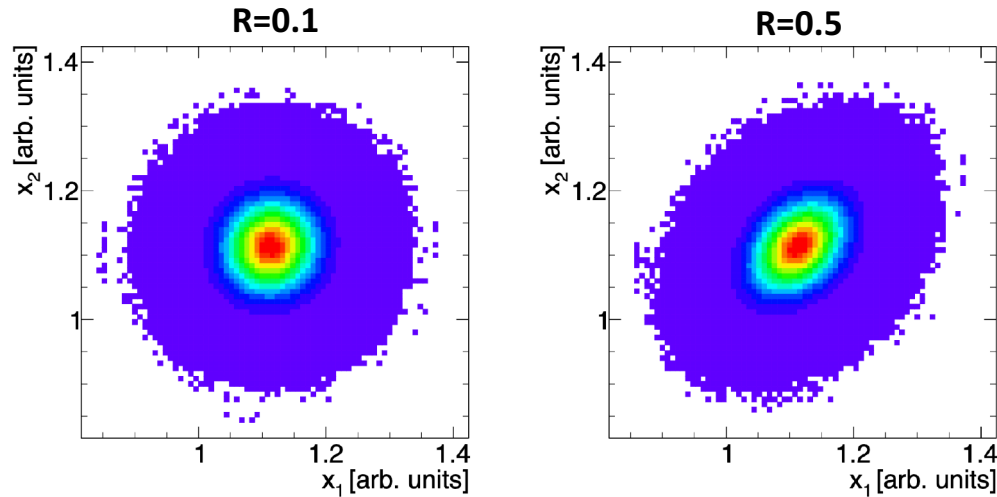


Particle-Set Identification method

Identity Method

+
solution to “correlations between dE/dx measurements of different particles”

$$R = \frac{\langle (x_1 - \langle x_1 \rangle)(x_2 - \langle x_2 \rangle) \rangle}{\sigma_{x_1} \sigma_{x_2}}$$



Two-particle distribution $p(x_1, x_2)$ for pions ($x \rightarrow dE/dx$)

Particle-Set Identification method

Run 1 & Run 2

- **1 nb⁻¹** of Pb–Pb collisions
- **≈1 kHz** Pb–Pb minimum bias readout rate
- **MWPC** based TPC
- ITS with **6 layers**
- **Offline reconstruction**

LS2 Upgrade



Run 3 & Run 4 (beyond 2021)

- **13 nb⁻¹** of Pb–Pb collisions
- **≈50 kHz** Pb–Pb minimum bias readout rate
- **GEM** based TPC
- ITS with **7 layers**
- **Online reconstruction**

Particle-Set Identification method

Run 1 & Run 2

- 1 nb^{-1} of Pb–Pb collisions
- $\approx 1 \text{ kHz}$ Pb–Pb minimum bias readout rate
- MWPC based TPC
- ITS with 6 layers
- Offline reconstruction

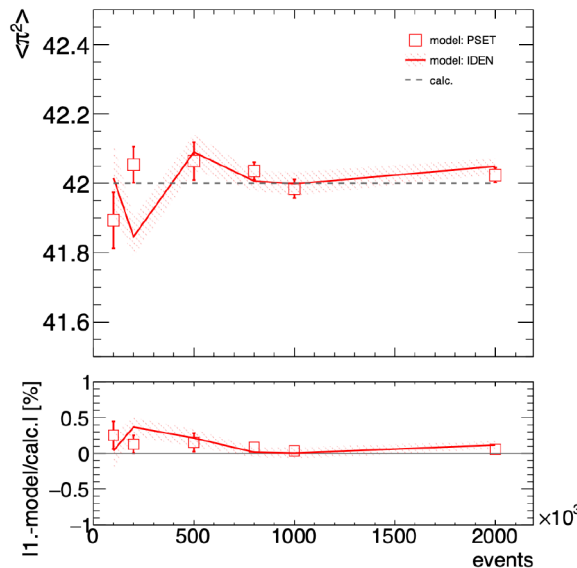
LS2 Upgrade



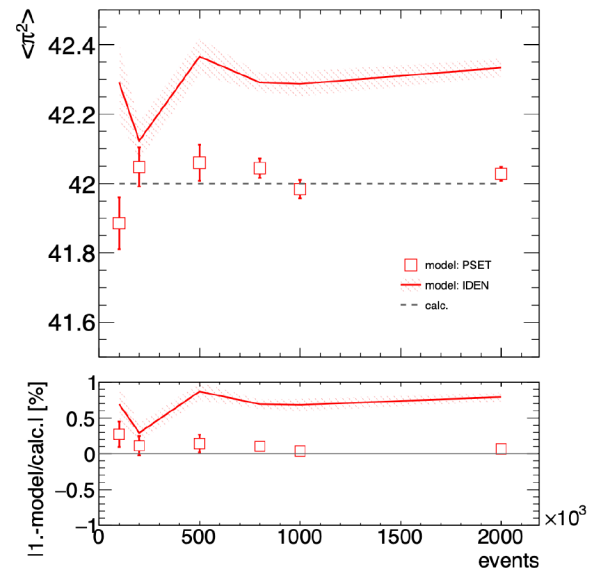
Run 3 & Run 4 (beyond 2021)

- 13 nb^{-1} of Pb–Pb collisions
- $\approx 50 \text{ kHz}$ Pb–Pb minimum bias readout rate
- GEM based TPC
- ITS with 7 layers
- Online reconstruction

R=0.1



R=0.5



- So far **Identity Method** performs well with **Run 1/2** data
- **PSET** could be an option for **Run 3/4**

Summary: Current status

- **Physics interpretation:** Volume fluctuations, resonance contributions, baryon number conservation, effect of hydrodynamic evolution, baryon stopping, deuteron formation ...

Summary: Current status

- **Physics interpretation:** Volume fluctuations, resonance contributions, baryon number conservation, effect of hydrodynamic evolution, baryon stopping, deuteron formation ...
- **Net-Q fluctuations:** Strongly dominated by resonance contributions
- **Net-p fluctuations:**
 - ✓ **1st order:** $T_{fo}^{ALICE} \sim T_{pc}^{LQCD}$
 - ✓ **2nd order:** Deviation from Skellam baseline is due to baryon number conservation
 - ALICE data suggests **long range correlations**
 - ✓ **3rd order:** Agrees with Skellam baseline “0” as a function of centrality and pseudorapidity
 - Achieved precision of **better than 5%** for the κ_3/κ_2 results is promising for the higher order cumulants
 - **Up to 3rd order ALICE data agree with the LQCD expectations**
 - **4th and higher order:** Ongoing analysis with Identity Method (or PSET method)
- **Net- Λ fluctuations:**
 - ✓ **2nd order:** Similar trend as for **net-p** → **Better precision** is needed to see the impact of strangeness conservation
 - **3rd and higher order:** Ongoing analysis with Identity method (or PSET method)

Summary: Current status

- **Physics interpretation:** Volume fluctuations, resonance contributions, baryon number conservation, effect of hydrodynamic evolution, baryon stopping, deuteron formation ...
- **Net-Q fluctuations:** Strongly dominated by resonance contributions
- **Net-p fluctuations:**
 - ✓ **1st order:** $T_{fo}^{ALICE} \sim T_{pc}^{LQCD}$
 - ✓ **2nd order:** Deviation from Skellam baseline is due to baryon number conservation
 - ALICE data suggests **long range correlations**
 - ✓ **3rd order:** Agrees with Skellam baseline “0” as a function of centrality and pseudorapidity
 - Achieved precision of **better than 5%** for the κ_3/κ_2 results is promising for the higher order cumulants
 - **Up to 3rd order ALICE data agree with the LQCD expectations**
 - **4th and higher order:** Ongoing analysis with Identity Method (or PSET method)
- **Net- Λ fluctuations:**
 - ✓ **2nd order:** Similar trend as for **net-p** → **Better precision** is needed to see the impact of strangeness conservation
 - **3rd and higher order:** Ongoing analysis with Identity method (or PSET method)

➤ Experimental Challenges:

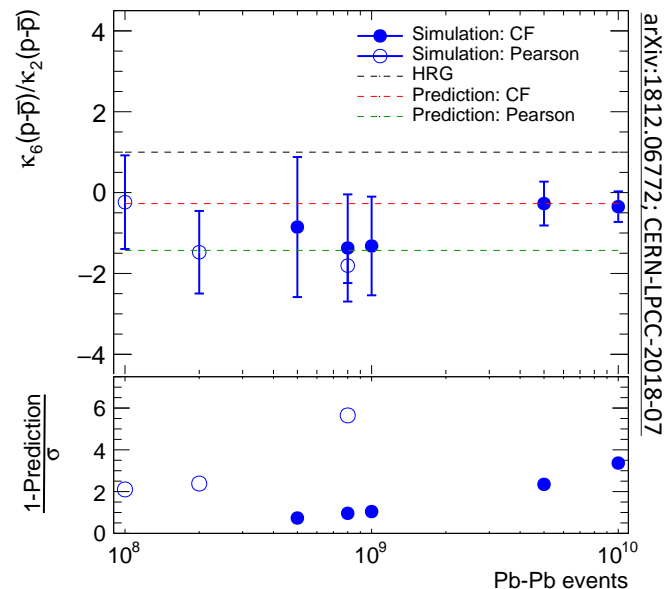
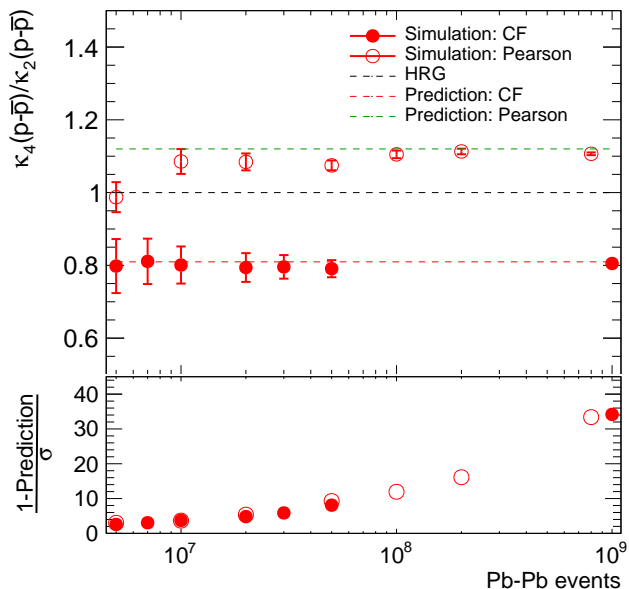
- ✓ Efficiency correction
- ✓ Event pileup
- ✓ **Correlations between dE/dx measurements of different particles**



- ☐ **Event/track selection**
- ☐ **Correction/analysis methods**
- ☐ **Realistic detector simulation**
- ☐ ...

Outlook: After ALICE upgrade

- **New ITS** (better vertexing) and **TPC** (continuous readout with GEM technology)
- Record minimum-bias Pb-Pb data at 50kHz → Order of magnitude more events

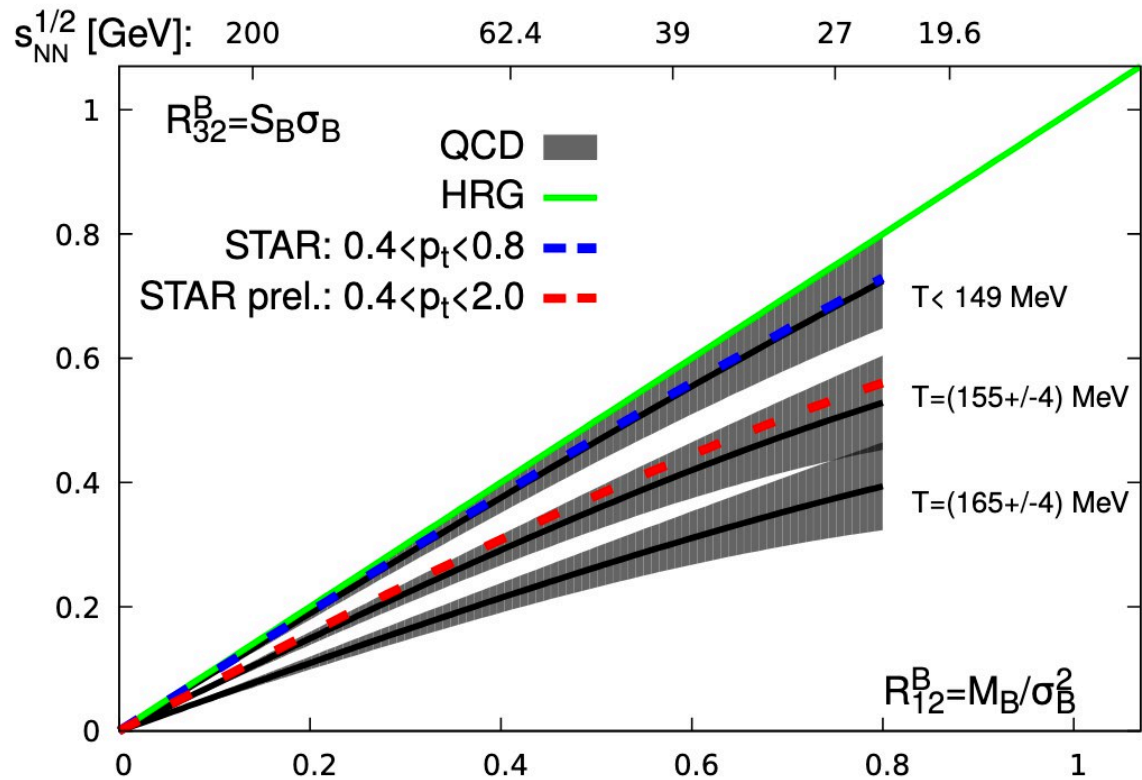


✓ **6th order and maybe beyond**

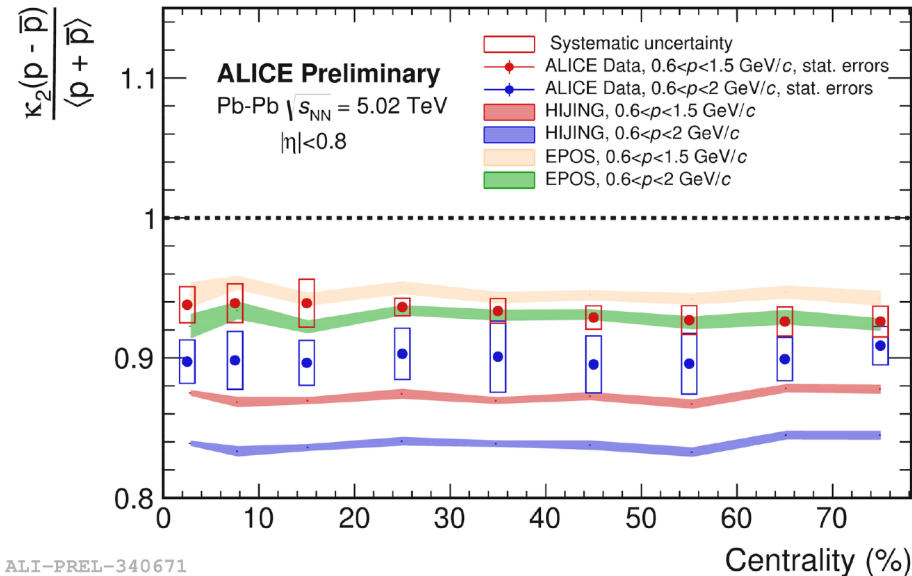
Polyakov-loop extended Quark- Meson model (PQM), G. A. Almasi, B. Friman, and K. Redlich, Phys. Rev.D96(2017) no. 1, 014027

Pearson Curve Method, N. K. Behera, arXiv:1706.06558 [nucl-ex]

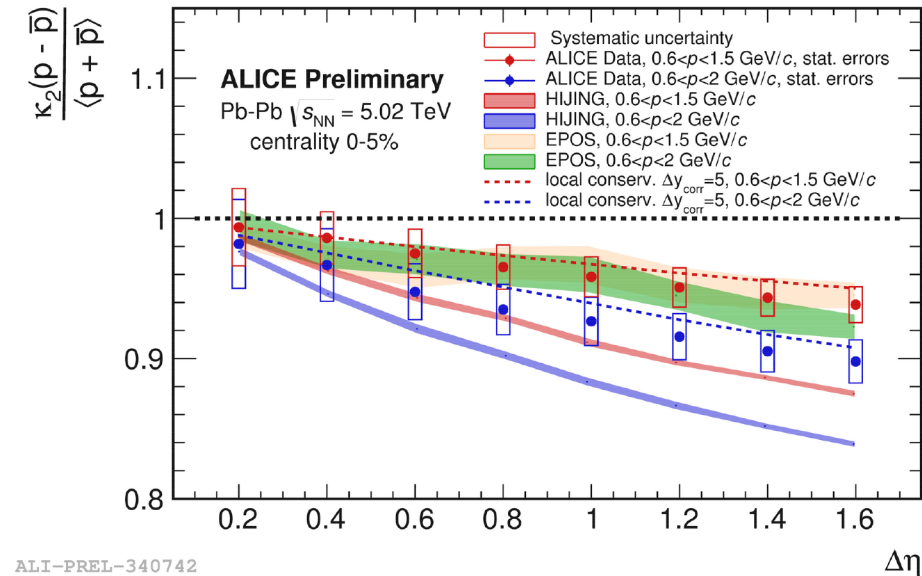
BACKUP



2nd order cumulants of net-p



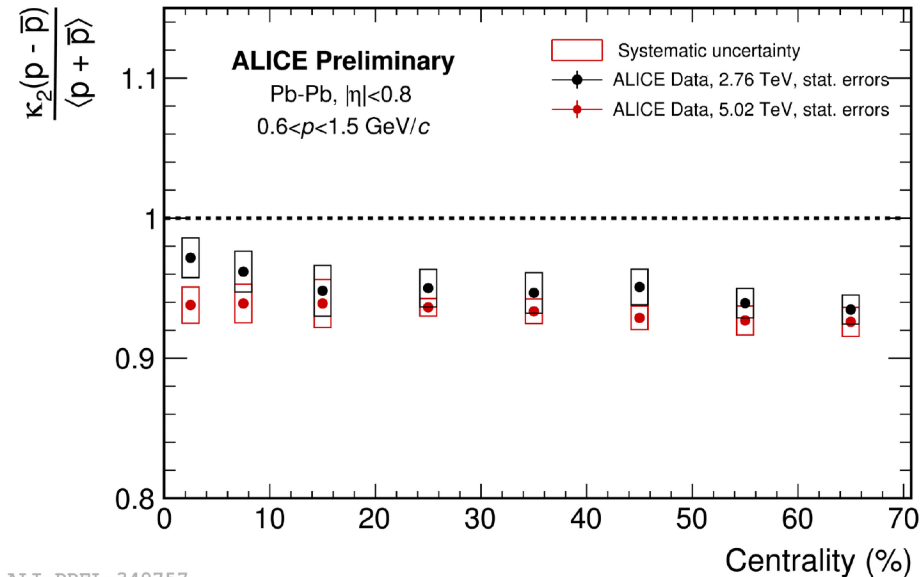
ALI-PREL-340671



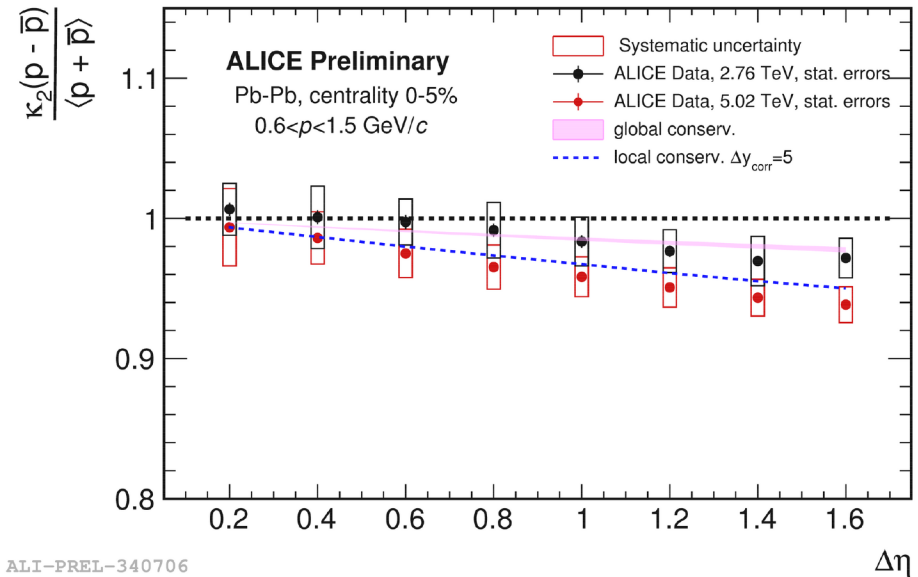
ALI-PREL-340742

- Consistent with the baryon number conservation picture
 - Increase in fraction of accepted $p, \bar{p} \rightarrow$ stronger constraint of fluctuations due to baryon number conservation
- EPOS & HIJING show this drop qualitatively

2nd order cumulants of net-p



ALI-PREL-340757

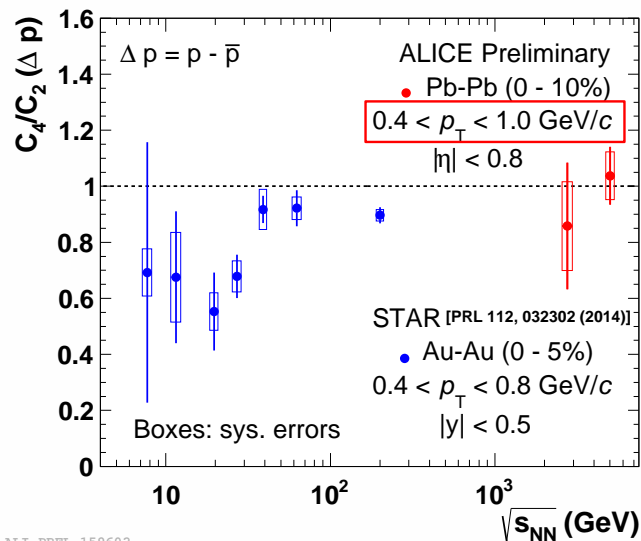


ALI-PREL-340706

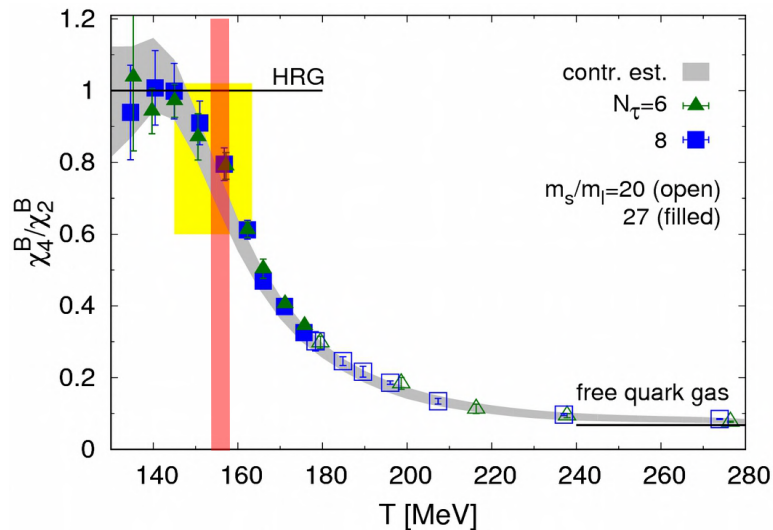
- Deviation from Skellam baseline is due to **baryon number conservation**
- ALICE data suggest **long range correlations**, $\Delta y = \pm 2.5$ unit or longer

C_3/C_2 and C_4/C_2 agree with Skellam at LHC energies?

- Small acceptance
- Low statistics
- Cut-based approach for PID

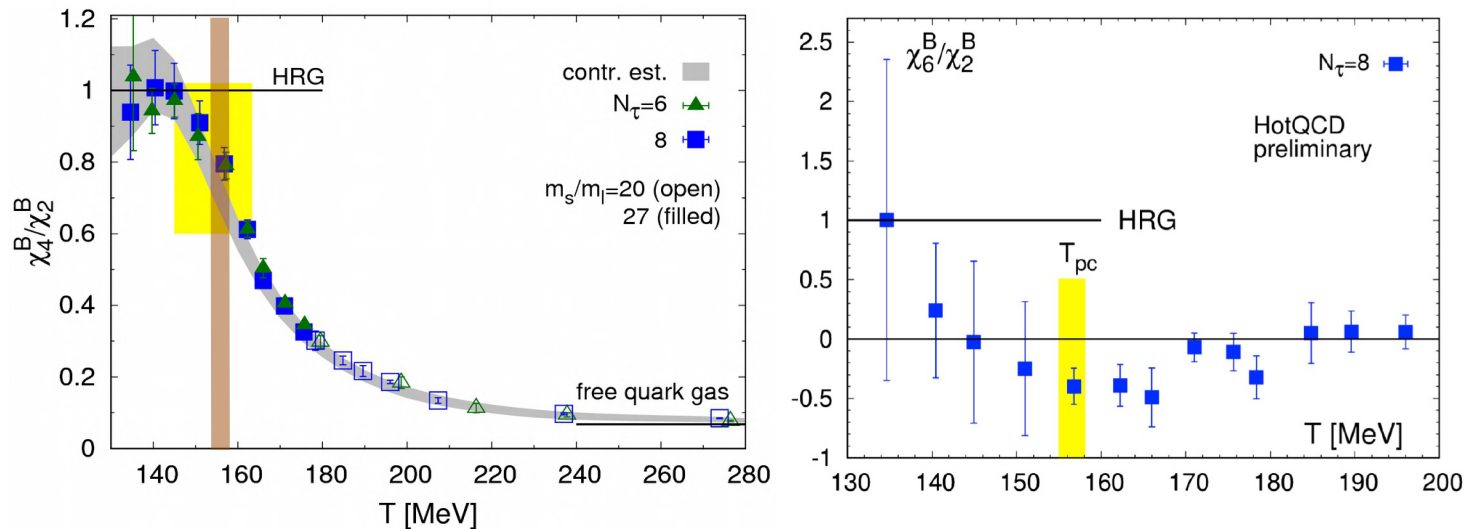


ALI-PREL-159602



Analysis within a larger kinematic acceptance using Identity Method is in progress

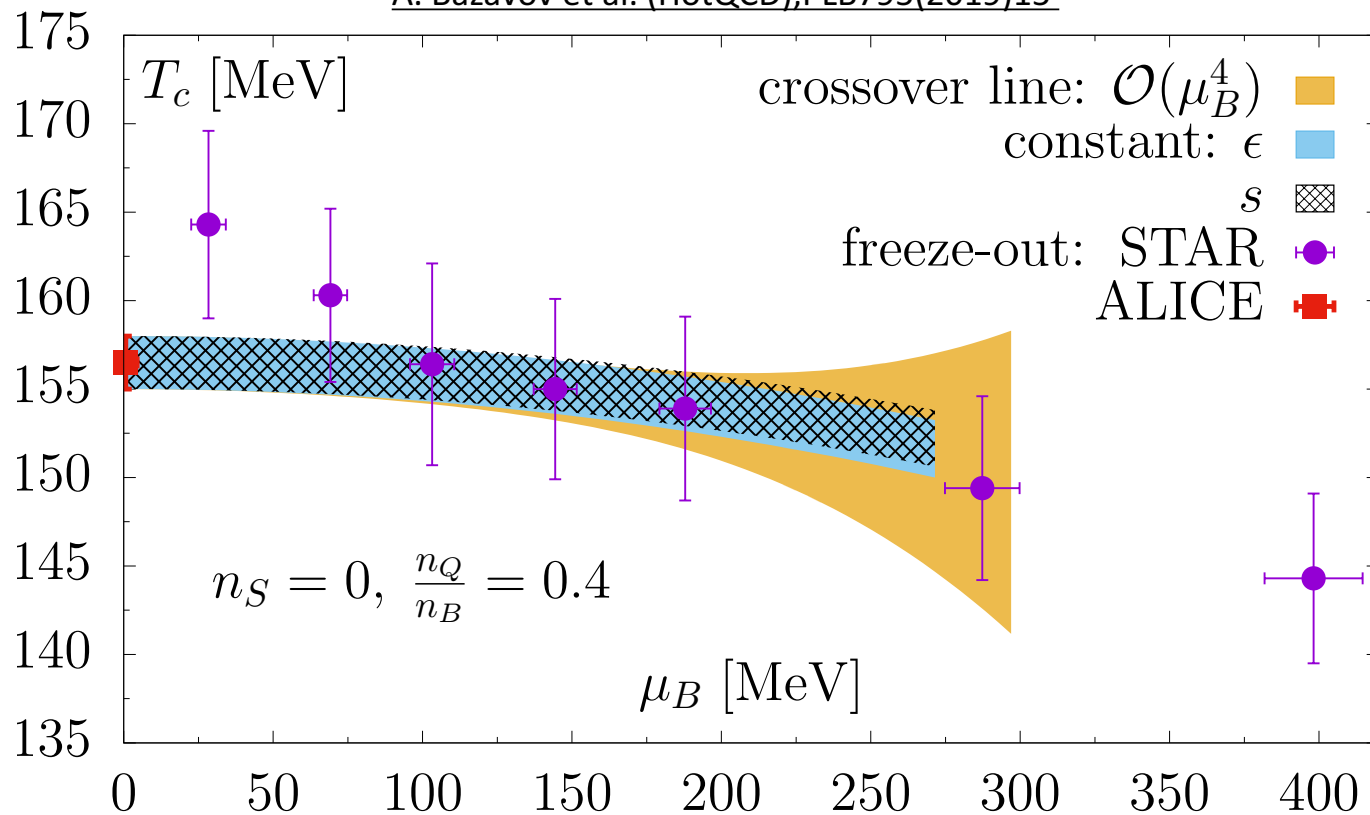
Holy grail: see critical behavior in 6th and higher order cumulants

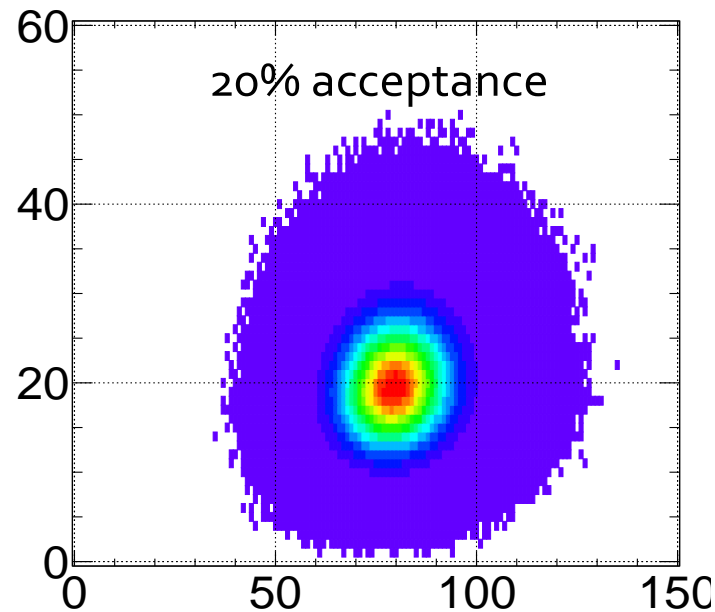
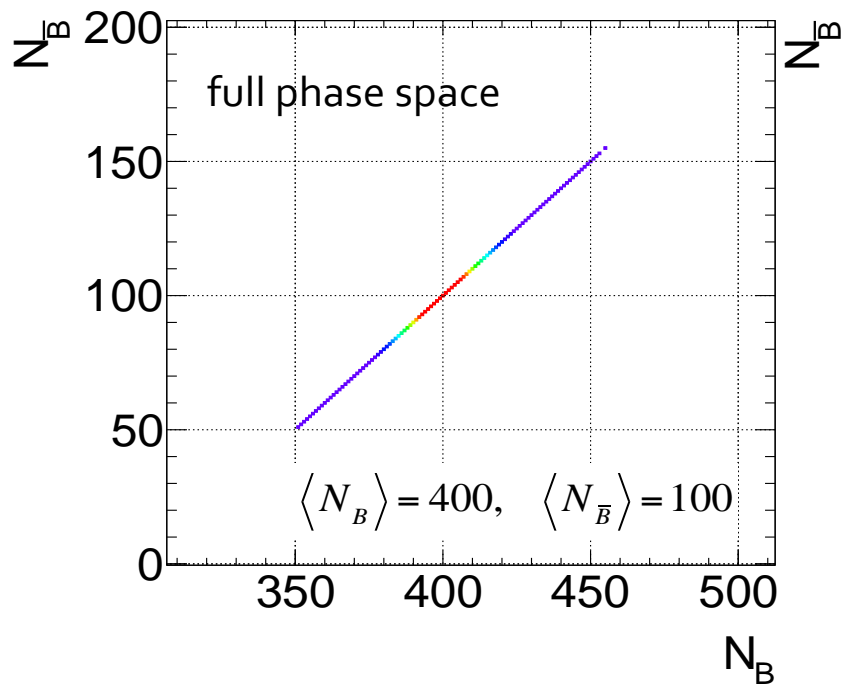


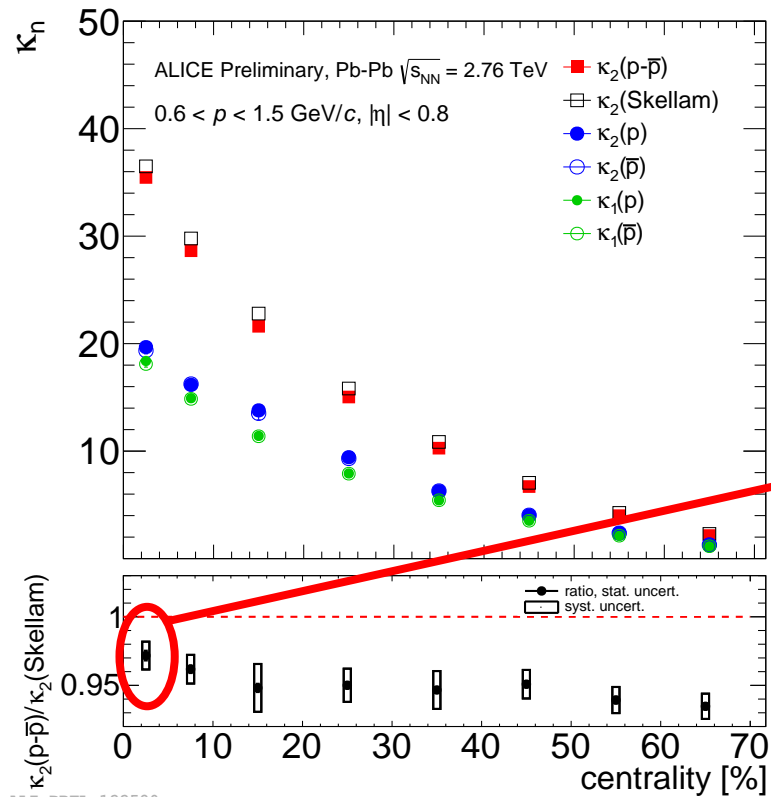
RUN 1: 2nd order (~13M min. bias events)

RUN 2: 4th order (~150M central events)

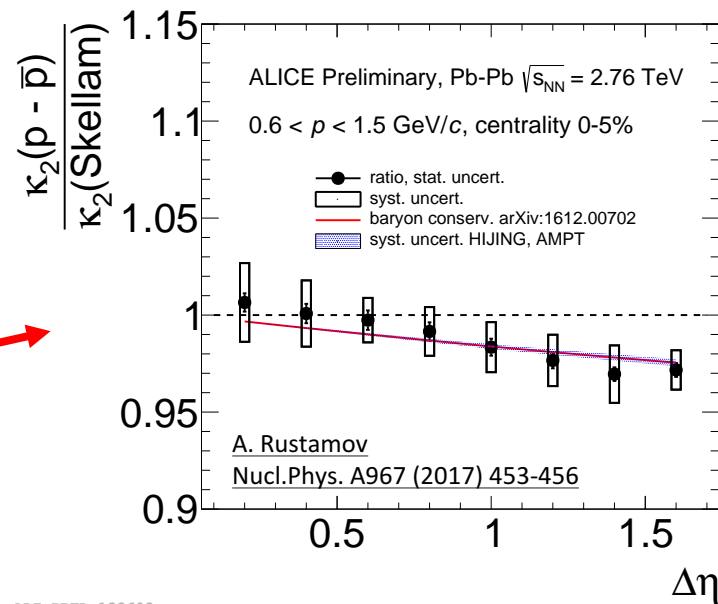
RUN 3: 6th and higher order? (>1000M central events)





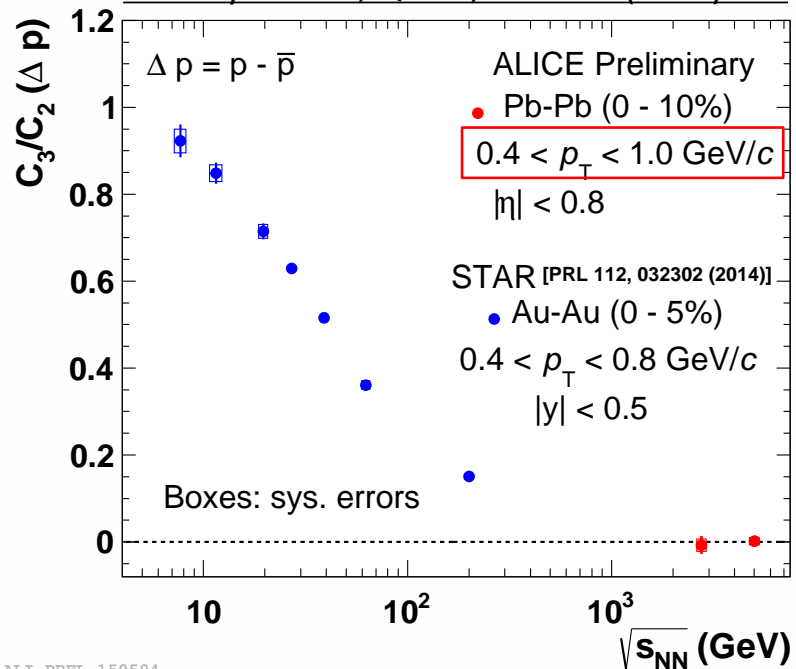


ALI-PREL-122590

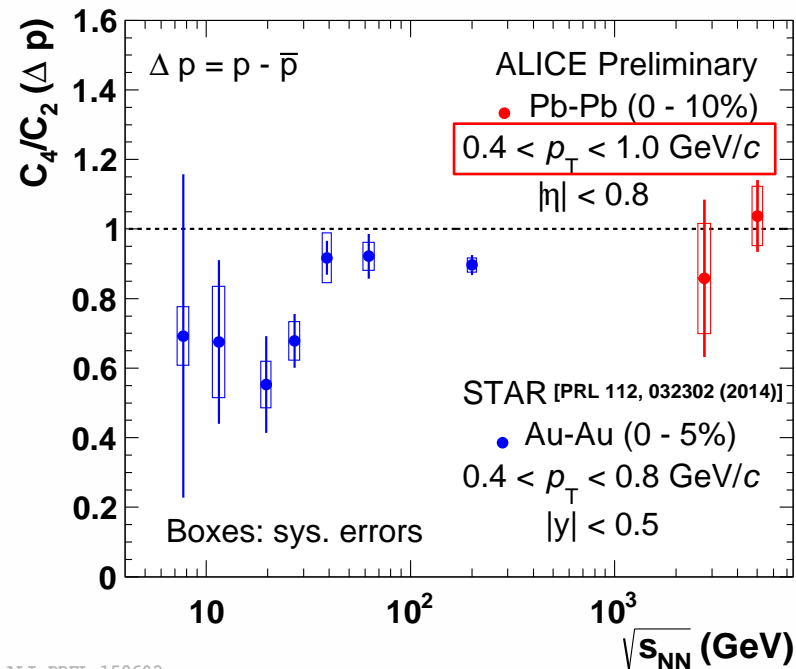


ALI-PREL-122602

Nirbhay Kubera, QM18, NPA 982 (2019) 851



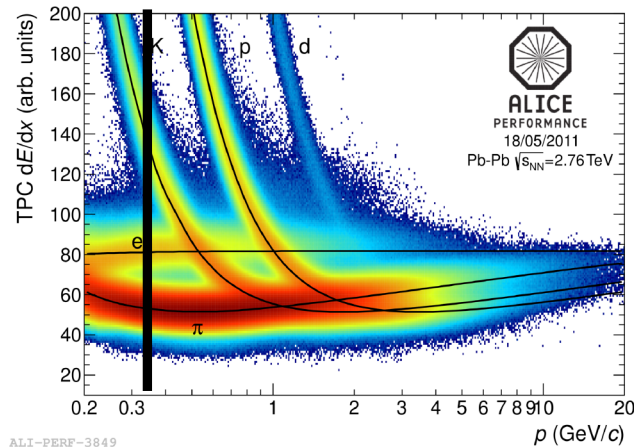
ALI-PREL-159594



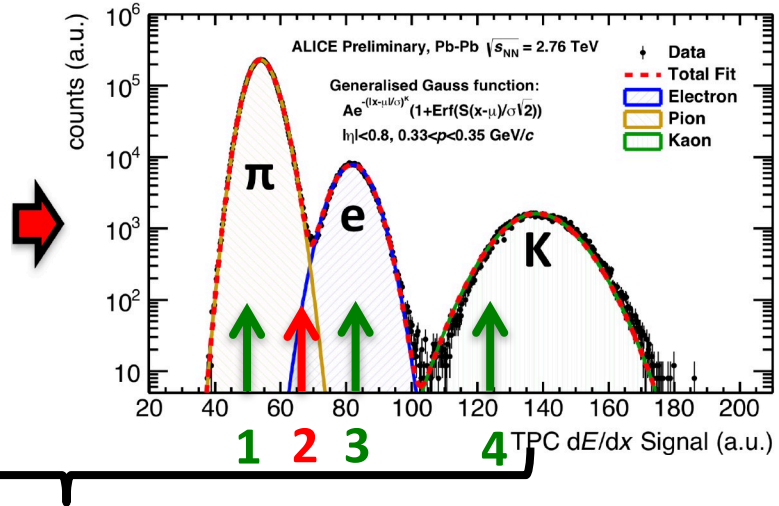
ALI-PREL-159602

Cut-based approach: count tracks of a given particle type

Identity method: count probabilities to be of a given particle type



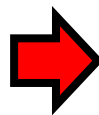
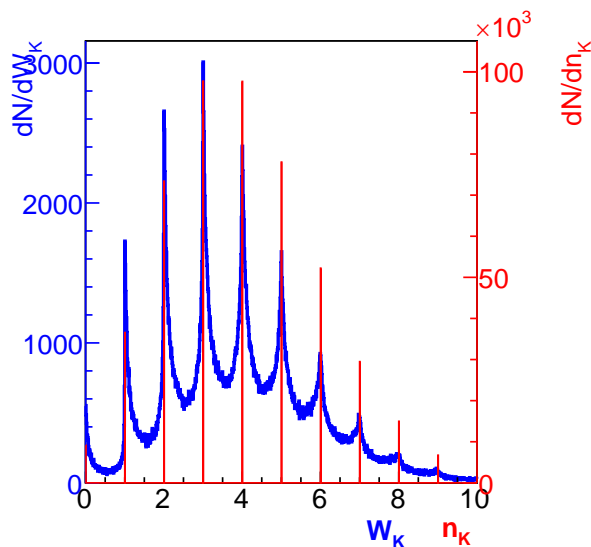
ALI-PERF-3849



$$\omega_{\pi}^{(1)} = 1, \quad \omega_{\pi}^{(2)} \cong 0.6, \quad \omega_{\pi}^{(3)} = 0, \quad \omega_{\pi}^{(4)} = 0 \Rightarrow W_{\pi} = 1.6 \neq N_{\pi}$$

A. Rustamov, M. Gazdzicki, M. I. Gorenstein, PRC 86, 044906 (2012), PRC 84, 024902 (2011)

A. Rustamov, M. Arslanodok, arXiv:1807.06370, NIM in print



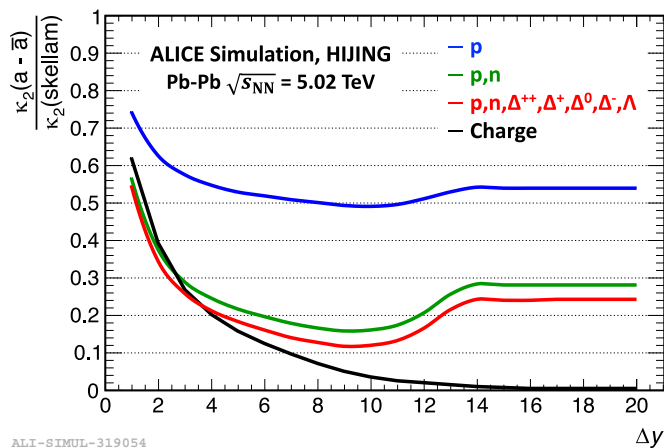
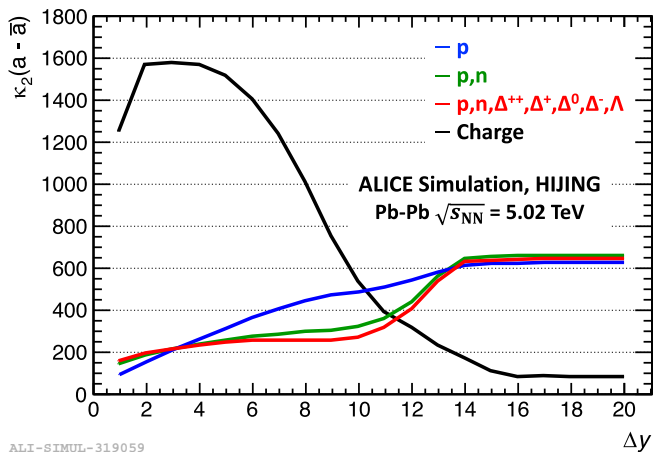
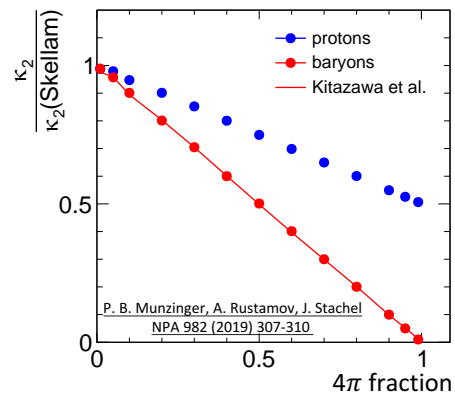
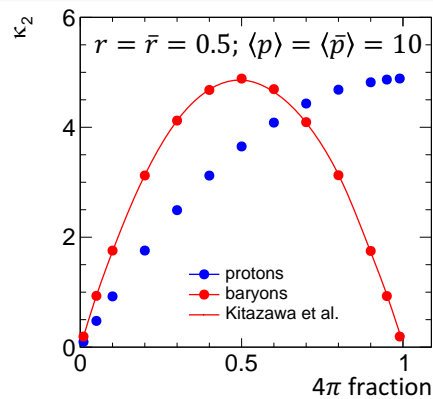
$$\langle N_j^n \rangle = A^{-1} \langle W_j^n \rangle$$

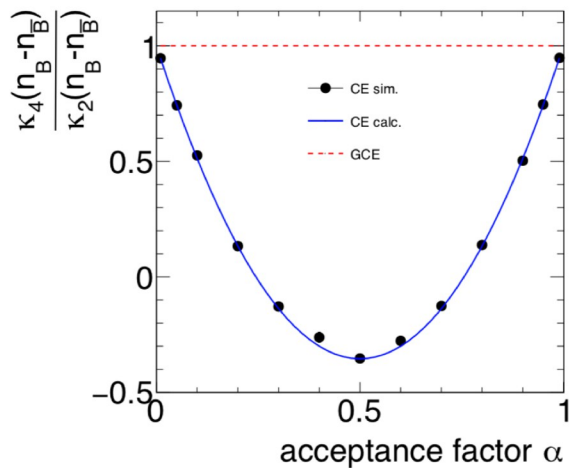
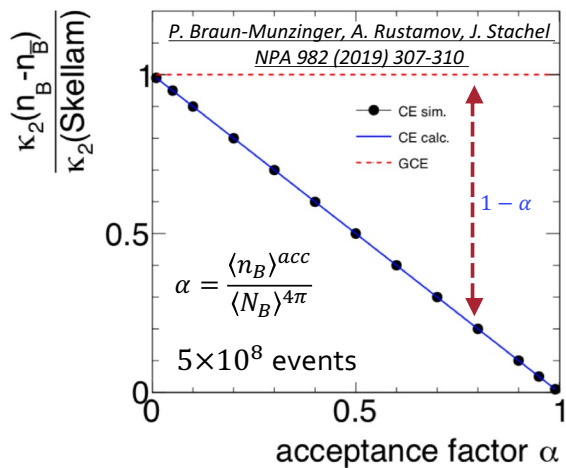
➤ **Cut-based approach**

- Uses additional detector information or reject a given phase space bin
- Challenge: efficiency correction and contamination

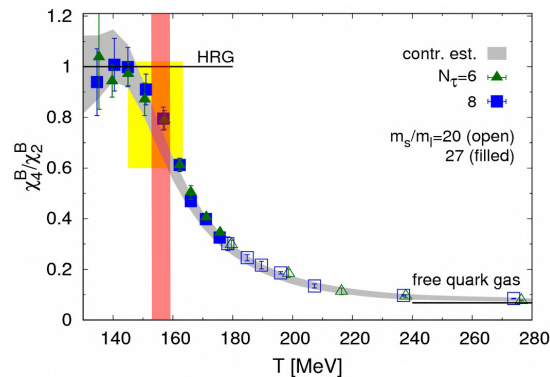
➤ **Identity Method**

- Gives folded multiplicity distribution
- Allows for larger efficiencies → smaller correction needed
- Ideal approach for low momentum ($p < 2 \text{ GeV}/c$)





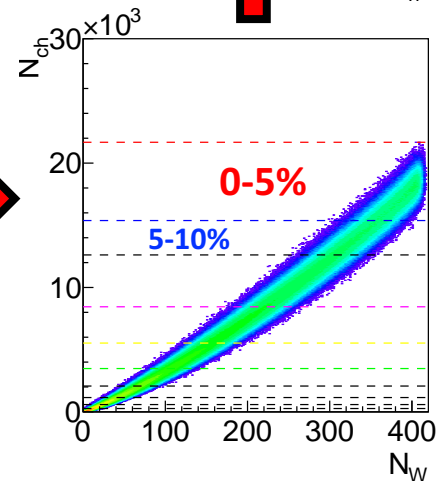
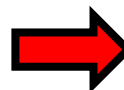
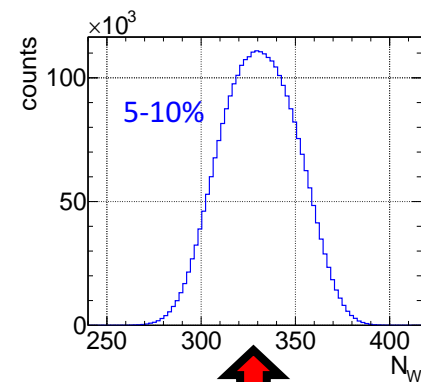
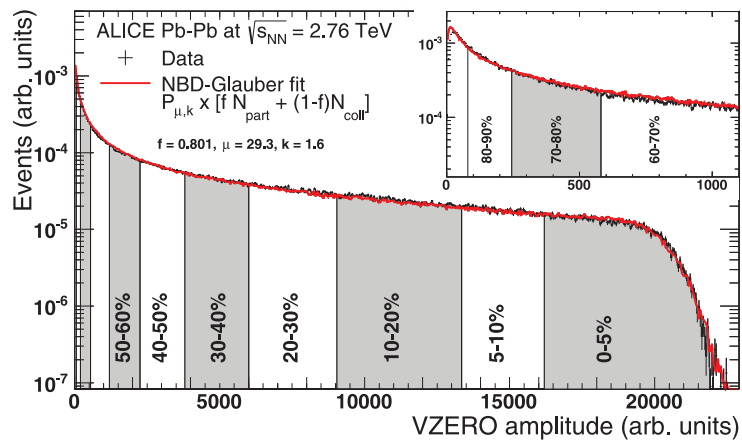
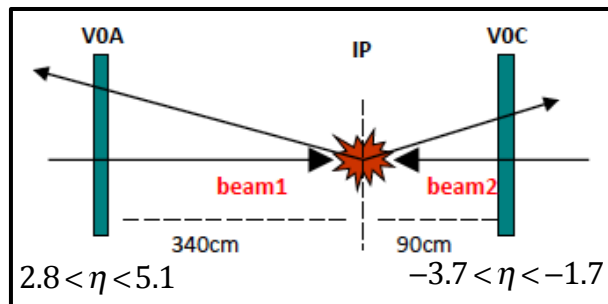
**Deviations from unity
are driven
by different mechanisms**



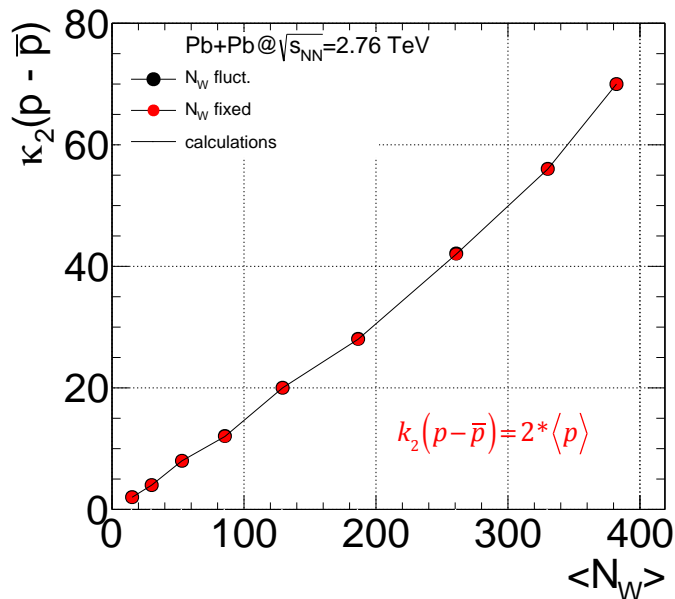
A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901

K. Redlich and L. Turko, Z. Phys. C5 (1980) 201

ALICE: Phys.Rev. C88 (2013) no.4, 044909



150*10⁶ Events

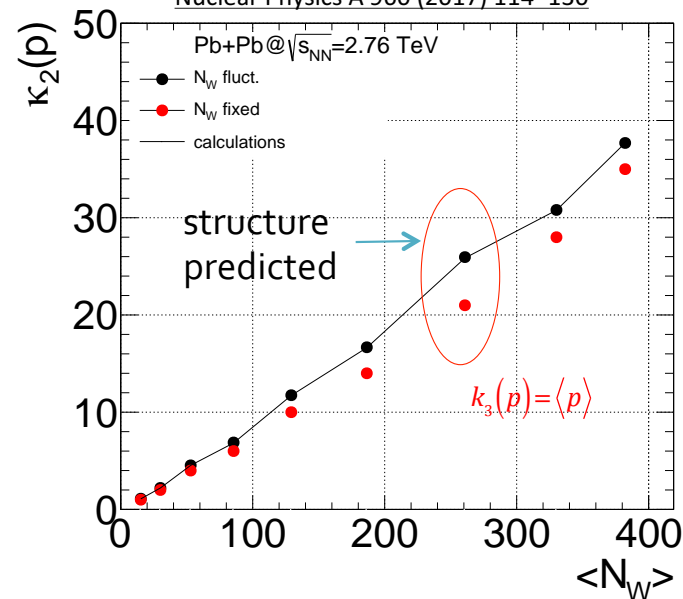


$$k_2(p - \bar{p}) = \langle N_w \rangle k_2(n - \bar{n}) + \langle n - \bar{n} \rangle^2 k_2(N_w)$$

↓
vanishes for ALICE

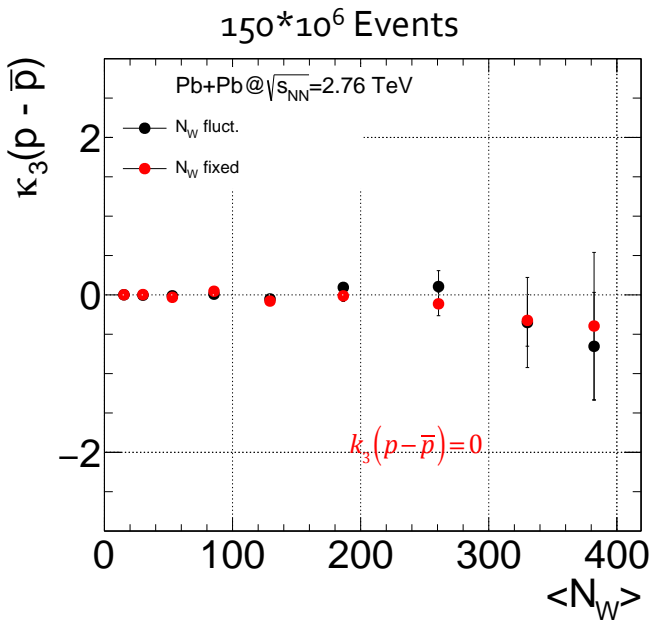
n, \bar{n} from single wounded nucleon

P. Braun-Munzinger, A. Rustamov, J. Stachel
Nuclear Physics A 960 (2017) 114–130



$$k_2(p) = \langle N_w \rangle k_2(n) + \langle n \rangle^2 k_2(N_w)$$

↓
does not vanish

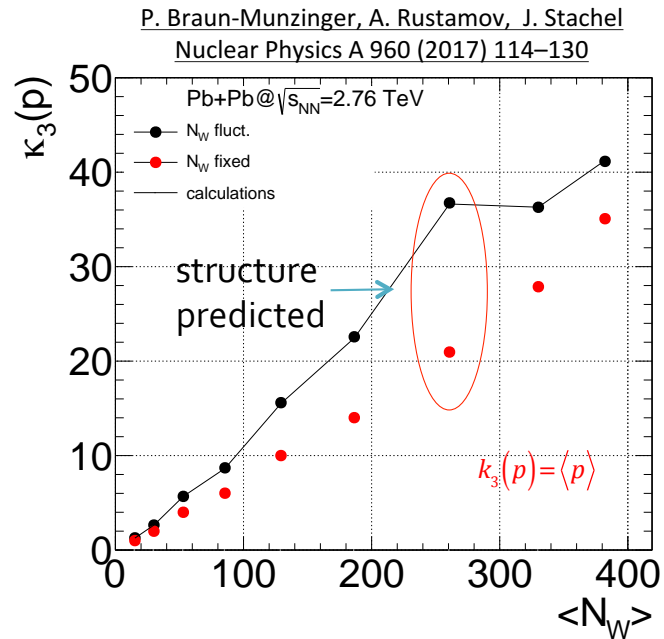


$$k_3(p - \bar{p}) = \langle N_w \rangle k_3(n - \bar{n}) + \langle n - \bar{n} \rangle (\dots)$$



vanishes for ALICE

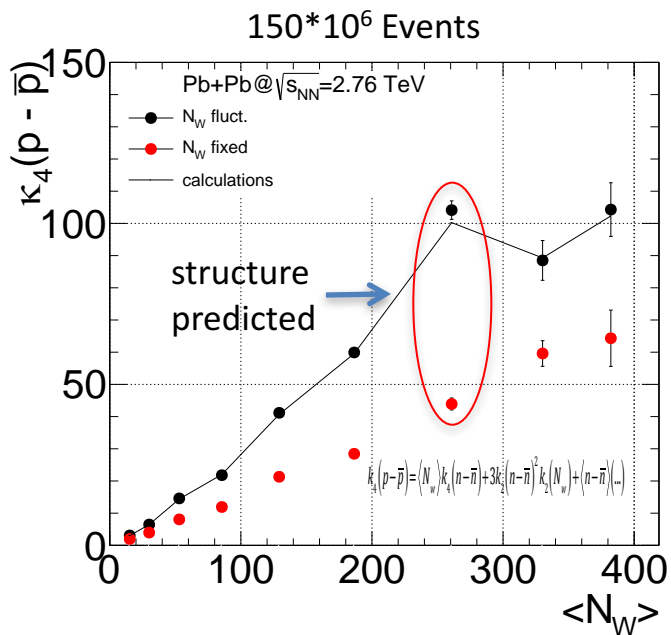
n, \bar{n} from single wounded nucleon



$$k_3(p) = \langle N_w \rangle k_3(n) + \langle n \rangle (\dots)$$

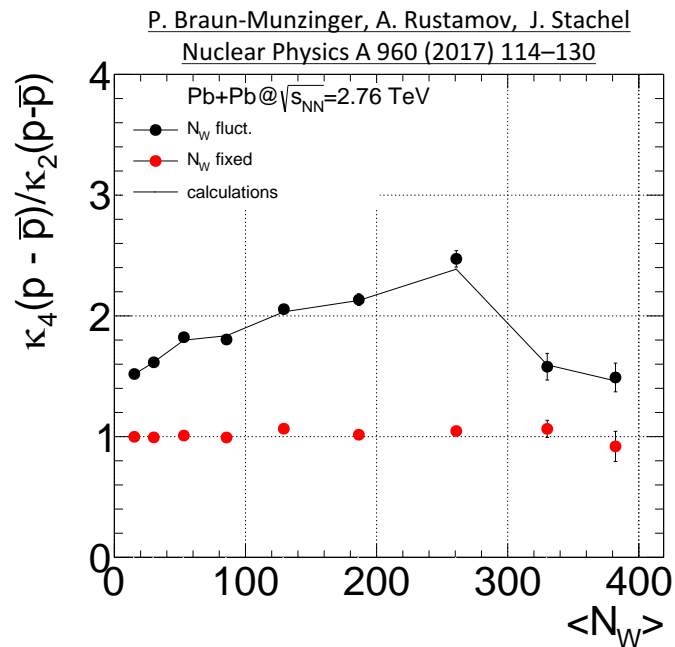


does not vanish

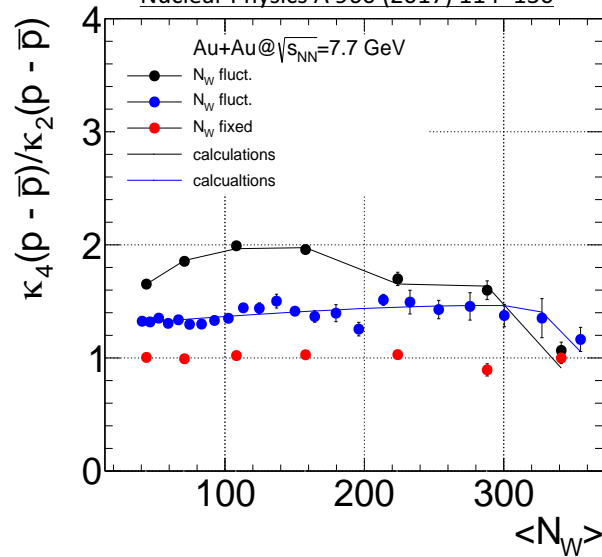
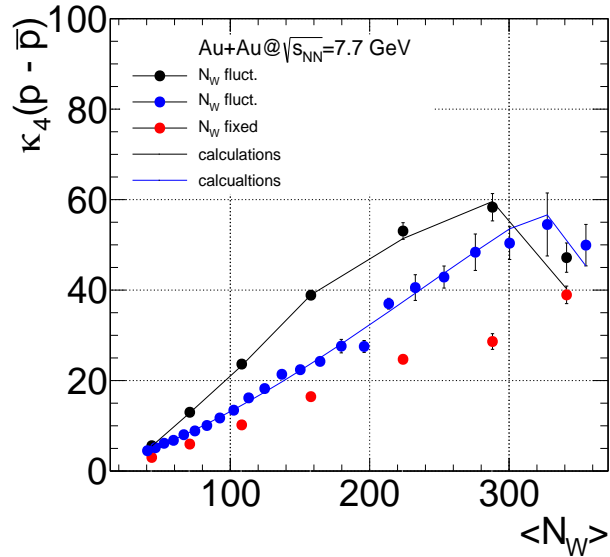


$$k_4(p-\bar{p}) = \langle N_w \rangle k_4(n-\bar{n}) + 3k_2(n-\bar{n})^2 k_2(N_w) + \langle n-\bar{n} \rangle (...)$$

$n, \bar{n} \rightarrow$ from single wounded nucleon

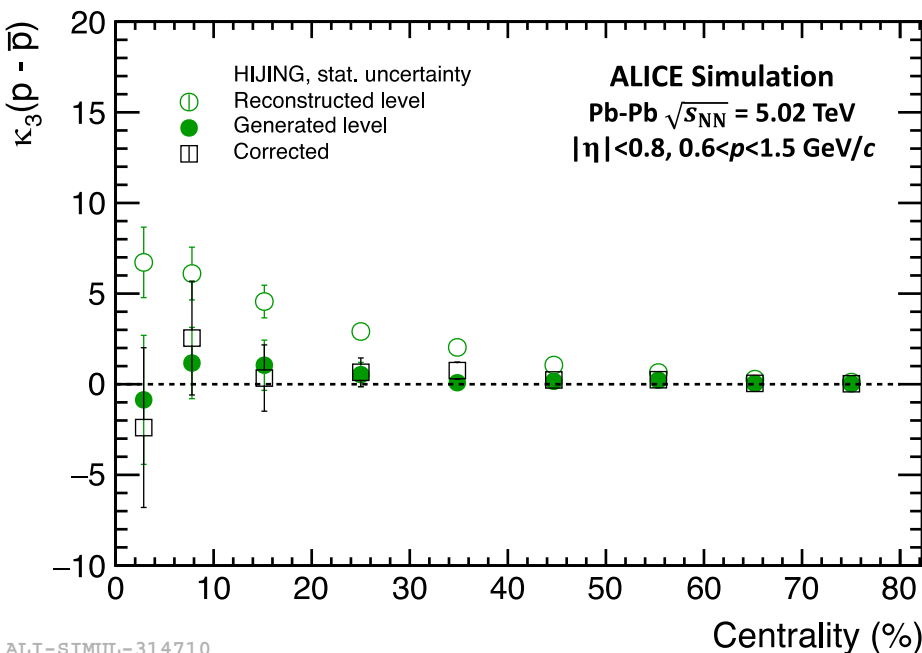


vanishes for ALICE

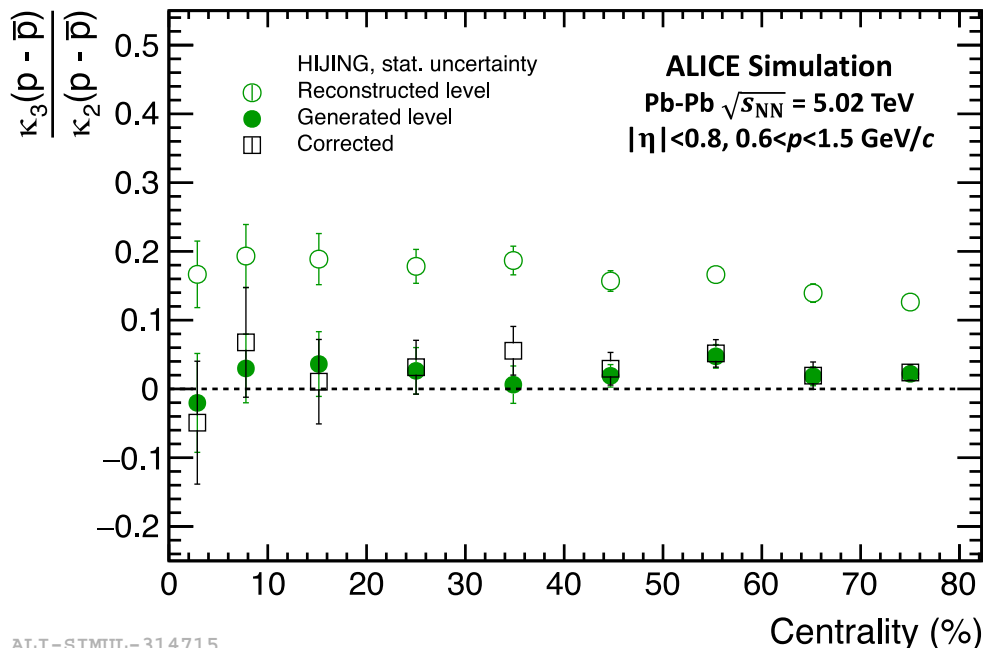


- Subdividing a given centrality bin into smaller ones and then merging them together **incoherently**.
- Incoherent addition of data from intervals with very small centrality bin width will **eliminate true dynamical fluctuations**.

Better publish uncorrected results



ALI-SIMUL-314710



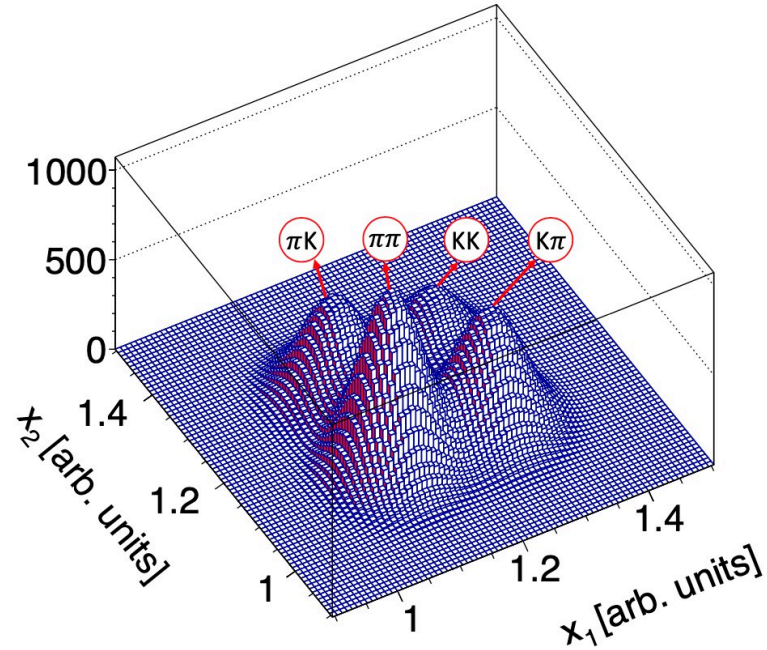
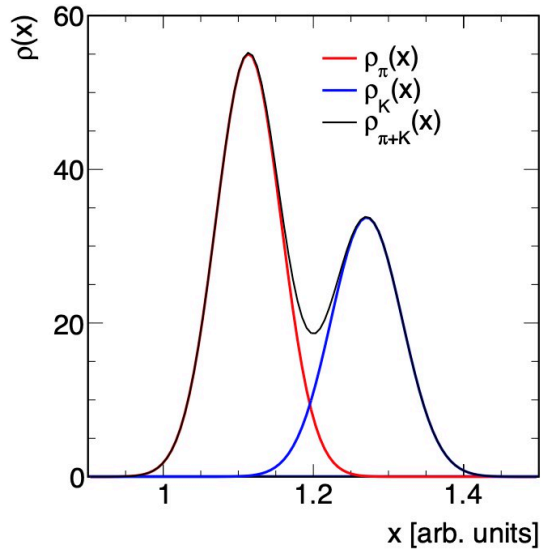
ALI-SIMUL-314715

Efficiency correction with binomial assumption:

T. Nonaka, M. Kitazawa, S. Esumi, Phys. Rev. C 95, 064912 (2017)

Adam Bzdak, Volker Koch, Phys. Rev. C 86, 044904 (2012)

Particle-Set Identification method



$$\rho(x_1, x_2)$$

Particle-Set Identification method

Probability density function of the bi-variate normal distribution

$$f(x_1, x_2) = \frac{1}{2\pi\sqrt{|\Sigma|}} e^{-0.5(\mathbf{x} - \langle \mathbf{x} \rangle)^T \Sigma^{-1} (\mathbf{x} - \langle \mathbf{x} \rangle)}$$

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \langle \mathbf{x} \rangle = \begin{pmatrix} \langle x_1 \rangle \\ \langle x_2 \rangle \end{pmatrix}, \Sigma = \begin{pmatrix} \sigma_{x_1}^2 & R\sigma_{x_1}\sigma_{x_2} \\ R\sigma_{x_1}\sigma_{x_2} & \sigma_{x_2}^2 \end{pmatrix}$$

$$R = \frac{\langle (x_1 - \langle x_1 \rangle)(x_2 - \langle x_2 \rangle) \rangle}{\sigma_{x_1}\sigma_{x_2}}$$

Correlations between x_1 and x_2 are introduced only if they belong to the same particle, otherwise they are generated independently, i.e., R is set to 0 in this case

Particle-Set Identification method

$$\begin{aligned}
 \rho(x_1, x_2) &\equiv \rho_{\pi\pi}(x_1, x_2) + \rho_{KK}(x_1, x_2) + \rho_{\pi K}(x_1, x_2) + \rho_{K\pi}(x_1, x_2) \\
 &\equiv \langle N_{\pi\pi} \rangle \cdot f_{\pi\pi}(x_1, x_2) + \langle N_{KK} \rangle \cdot f_{KK}(x_1, x_2) + \langle N_{\pi K} \rangle \cdot f_{\pi K}(x_1, x_2) + \langle N_{K\pi} \rangle \cdot f_{K\pi}(x_1, x_2) \\
 &\equiv \langle N^{(2)} \rangle (r_{\pi\pi} \cdot f_{\pi\pi}(x_1, x_2) + r_{KK} \cdot f_{KK}(x_1, x_2) + r_{\pi K} \cdot f_{\pi K}(x_1, x_2) + r_{K\pi} \cdot f_{K\pi}(x_1, x_2)),
 \end{aligned}$$

$$F(x_1, x_2 | r_{\pi\pi}, r_{KK}) = r_{\pi\pi} \cdot f_{\pi\pi}(x_1, x_2) + r_{KK} \cdot f_{KK}(x_1, x_2) + r_{\pi K} \cdot f_{\pi K}(x_1, x_2) + r_{K\pi} \cdot f_{K\pi}(x_1, x_2)$$

$$l(\mathcal{X}^{(2)} | r_{\pi\pi}, r_{KK}) = - \sum_{j=1}^{N^{(2)}} \ln(F((x_1, x_2)_j | r_{\pi\pi}, r_{KK}))$$

Maximum Likelihood minimization

$$r_{\pi\pi} + r_{KK} + 2 \cdot r_{\pi K} = 1$$

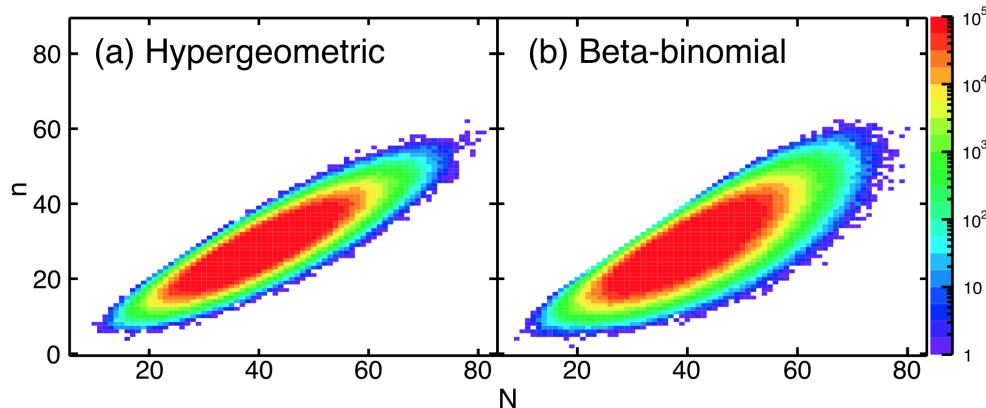
Calculated from 1D dE/dx distribution

$$\langle N_{\pi\pi} \rangle = \frac{1}{2} \langle N_{\pi} (N_{\pi} - 1) \rangle = \frac{\langle N_{\pi}^2 \rangle - \langle N_{\pi} \rangle}{2}$$

Calculated from 2D dE/dx distribution

Efficiency correction

What if efficiency loss is not binomial?



Draw N balls from the urn
without returning balls to
the urn

In each draw, when one draws
a white ball, two white balls are
returned to the urn

T. Nonaka, M. Kitazawa, S. Esumi, Nucl.Instrum.Meth. A906 (2018) 10-17

T. Nonaka, M. Kitazawa, S. Esumi, Phys. Rev. C 95, 064912 (2017)

Adam Bzdak, Volker Koch, Phys. Rev. C86, 044904 (2012)

