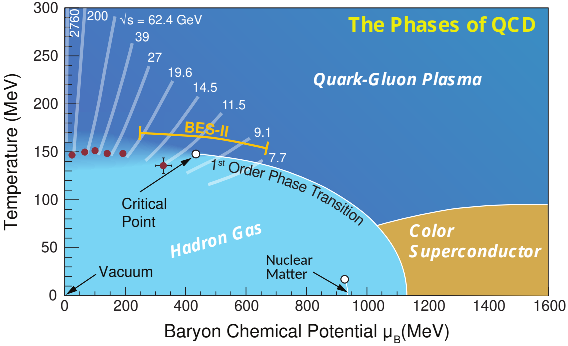
Search for the QCD critical point:
The remaining tasks

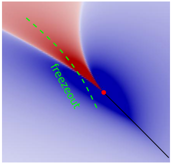
M. Stephanov

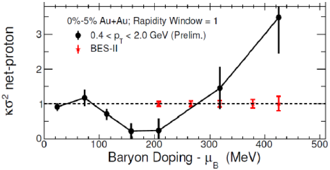
Critical point: intriguing hints

Where on the QCD phase boundary is the CP?



Equilibrium κ_4
vs T and μ_B:





“intriguing hint” (2015 LRPNS)

Motivation for phase II of BES at RHIC and BEST topical collaboration.

BEST Framework

- Lattice EOS + CP $\to$ parametric EOS
- Initial conditions and transition to viscous hydrodynamics.
- Dynamics of critical fluctuations. Freezeout of fluctuations.
- "Particlization" and hadronic phase evolution.
- Comparison with experiment. Bayesian analysis.

Theory/experiment gap: predictions assume equilibrium, but in heavy-ion collisions

non-equilibrium physics is essential near the critical point.

Challenge: develop hydrodynamics *with fluctuations* capable of describing *non-equilibrium* effects on critical-point signatures.

Dynamics of (critical) fluctuations

Stochastic

Random hydro variables: $\breve{\psi}$

$$\partial_t \breve{\psi} = -\nabla \cdot \Big(\text{Flux}[\breve{\psi}] + \text{ Noise} \Big)$$

$+$ fewer variables and eqs.

$-$ cutoff dependence

Deterministic

$\psi \equiv \langle \breve{\psi} \rangle$, $G \equiv \langle \breve{\psi}\breve{\psi} \rangle$, etc.

$$\partial_t \psi = -\nabla \cdot \text{Flux}[\psi, G];$$

$$\partial_t G = \mathsf{L}[G; \psi].$$

$-$ more variables and eqs.

$+$ cutoff independence after renormalization

Hydro+

Yin, MS, 1712.10305

- Hydro+ extends Hydro with new *non-hydrodynamic* d.o.f..
- At the CP, the *slowest* (i.e., most out of equilibrium) new d.o.f. is the 2-pt function $\langle \delta m \delta m \rangle$ of the slowest hydro variable:

$$\phi_{\boldsymbol{Q}}(\boldsymbol{x}) = \int_{\Delta \boldsymbol{x}} \langle \delta m\,(\boldsymbol{x}_+)\ \delta m\,(\boldsymbol{x}_-) \rangle\ e^{i\boldsymbol{Q}\cdot\Delta\boldsymbol{x}}$$

where $\boldsymbol{x} = (\boldsymbol{x}_+ + \boldsymbol{x}_-)/2$ and $\Delta\boldsymbol{x} = \boldsymbol{x}_+ - \boldsymbol{x}_-$.

Relaxation of fluctuations towards equilibrium

- As usual, equilibration maximizes entropy $S = \sum_i p_i \log(1/p_i)$:

$$s_{(+)}(\epsilon, n, \phi_{\boldsymbol{Q}}) = s(\epsilon, n) + \frac{1}{2}\int_{\boldsymbol{Q}} \left(\log\frac{\phi_{\boldsymbol{Q}}}{\bar{\phi}_{\boldsymbol{Q}}} - \frac{\phi_{\boldsymbol{Q}}}{\bar{\phi}_{\boldsymbol{Q}}} + 1 \right)$$

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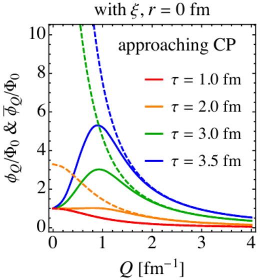
- The equation for $\phi_{\boldsymbol{Q}}$ is a relaxation equation with rate

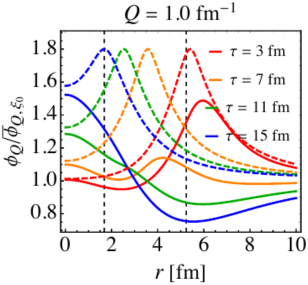
$$\Gamma(\boldsymbol{Q}) \approx 2DQ^2 \quad \text{for} \quad Q \ll \xi^{-1}, \quad D \sim 1/\xi.$$

- Impact on fluctuation observables: “memory” effects

(Berdnikov-Rajagopal, Mukherjee-Venugopalan-Yin, . . .)

Implementation of Hydro+ and lessons





Rajagopal et al, 1908.08539
Du et al, 2004.02719

- Conservation laws
- Memory and lag
- Advection
- Feedback is small

non-Gaussian fluctuations are sensitive signatures of the critical point

Nonlinearity and multiplicative noise

An et al 2009.10742

- Now nonlinearity and multiplicative noise matter even more:

$$\partial_t\breve\psi = -\nabla\cdot\Big(\text{Flux}[\breve\psi] + \text{ Noise}\Big)\,, \qquad \langle\text{Noise Noise}\rangle \sim 2Q[\breve\psi]\,.$$

- Define the problem in terms of *physical properties*

$$\text{Onsager matrix } M \text{ and entropy } S \equiv \log P_{\rm eq},$$

to avoid ambiguities associated with multiplicative noise.

- Equivalent to an *infinite* hierarchy of coupled equations for cumulants $G_n^{\rm c} \equiv \langle\delta\breve\psi\ldots\delta\breve\psi\rangle^{\rm c}$

Perturbation theory

- *Small* fluctuations are Gaussian

- Introduce expansion parameter: ε.

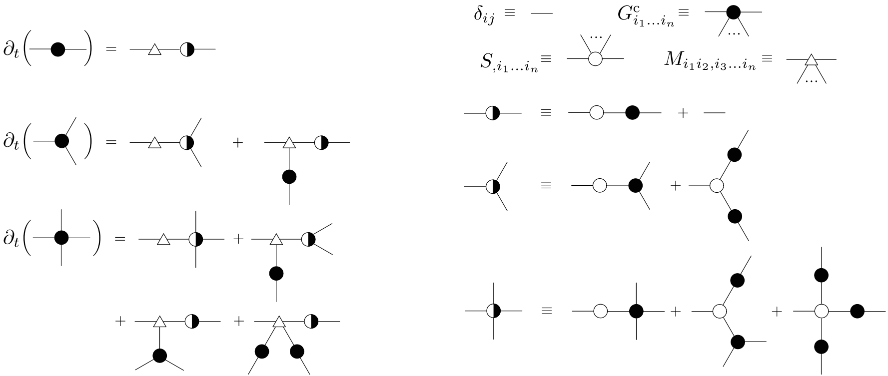
  Power counting: $S'' \sim \varepsilon^{-1}$, so that $\delta\breve{\psi} \sim \sqrt{\varepsilon}$.

  Then $G_n^{\mathsf{c}} \equiv \langle \delta\breve{\psi}_1 \ldots \delta\breve{\psi}_n \rangle^{\mathsf{c}} \sim \varepsilon^{n-1}$.

- In hydrodynamics the small parameter is $(q/\Lambda)^3$, i.e., fluctuation wavelength $1/q \gg$ size of hydro cell $1/\Lambda$ (UV cutoff).

Diagrammatic representation

Systematically truncate each equation at leading order in ε:



- Tree diagrams at leading order in ε.
- In higher-orders, loops describe feedback of fluctuations (e.g., long-time tails).

Generalizing Wigner transform

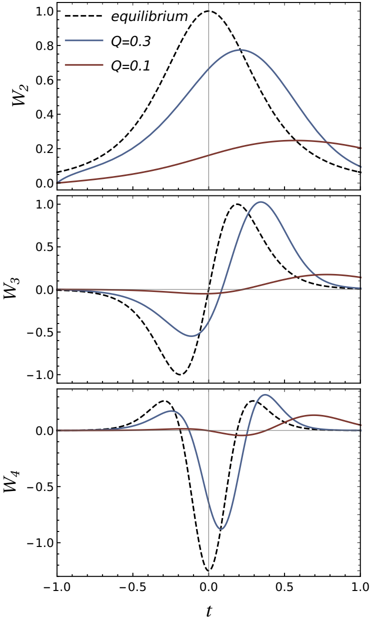
- $$W_n(\boldsymbol{x}, \boldsymbol{q}_1, \ldots, \boldsymbol{q}_n) \equiv \int d\boldsymbol{y}_1^3 \ldots \int d\boldsymbol{y}_n^3 \, G_n\left(\boldsymbol{x}+\boldsymbol{y}_1, \ldots, \boldsymbol{x}+\boldsymbol{y}_n\right) \delta^{(3)}\left(\frac{\boldsymbol{y}_1+\ldots+\boldsymbol{y}_n}{n}\right) e^{-i(\boldsymbol{q}_1\cdot\boldsymbol{y}_1+\ldots+\boldsymbol{q}_n\cdot\boldsymbol{y}_n)};$$

  $$G_n\left(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n\right) = \int \frac{d\boldsymbol{q}_1^3}{(2\pi)^3} \ldots \int \frac{d\boldsymbol{q}_n^3}{(2\pi)^3} \, W_n(\boldsymbol{x}, \boldsymbol{q}_1, \ldots, \boldsymbol{q}_n)(2\pi)^3 \delta^{(3)}\left(\boldsymbol{q}_1+\ldots+\boldsymbol{q}_n\right) e^{i(\boldsymbol{q}_1\cdot\boldsymbol{x}_1+\ldots+\boldsymbol{q}_n\cdot\boldsymbol{x}_n)}.$$

  - Properties similar to the usual ($n=2$) Wigner transform.
  - Takes advantage of the scale separation: long-scale x-dependence and short-scale $\boldsymbol{y}_n$-dependence.

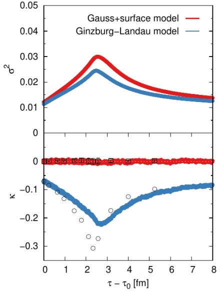
Example: expansion through a critical region

- Two main features:
  - Lag, or "memory".
  - Smaller Q – slower evolution. Conservation laws.
- The magnitude of the observed critical point signatures depends on the scale of fluctuations probed.



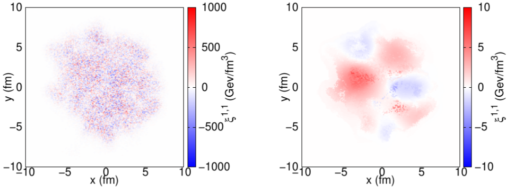
Stochastic approach

Nahrgang et al



Gaussian noise generates non-Gaussian fluctuations via non-linearities in EOS.
Baryon number fluctuations show expected memory effects in higher-order cumulants

Singh et al



Noise filter. Renormalization of kinetic coefficients (filter/cutoff dependence)

Experiments measure not hydro variables, but particle multiplicities

Freezing out (critical) hydrodynamic fluctuations

Pradeep et al, in preparation

- Cooper-Frye deals with with 1-particle observables.
  We need 2-particle (and n-particle) *correlations*.
- Critical contribution to fluctuations of $f(x,p)$:

$$\delta f = \frac{\partial f}{\partial \sigma}\delta\sigma, \quad \text{via} \quad \delta m = g\delta\sigma.$$

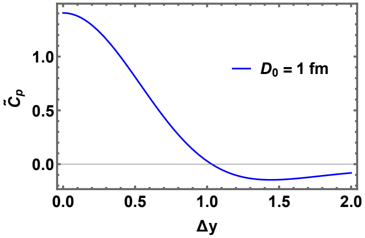
$$\langle \delta\sigma\delta\sigma \rangle \sim \text{ F.T. } \phi_{\boldsymbol{Q}}$$

- Critical contribution to observables

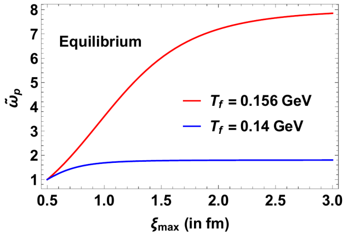
$$\langle \delta N^2 \rangle = \int\int \langle \delta f \delta f \rangle \sim \int\int \text{ F.T. } \phi_{\boldsymbol{Q}}$$

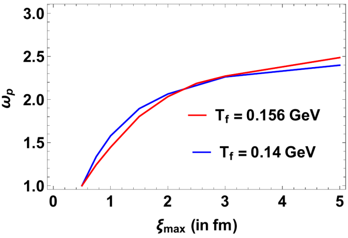
Freeze out in Hydro+: model calculation and lessons

- Effect of conservation laws on particle (anti)correlations at freezeout



- Dependence of particle fluctuations on proximity to C.P. (in terms of $\xi_{\rm max}$). Sensitivity to $T_{\rm freezeout}$ is weaker due to noneq. effects:





Work in progress and remaining tasks

- *Non-Gaussian* fluctuations in *full* relativistic hydrodynamics.

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- First-order transition in *fluctuating* hydrodynamics.

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- Compare with experiment.