Nuclear Science Computing Center at CCNU

Charge fluctuations in strong magnetic fields: new opportunities

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HTD, S.-T. Li, Q. Shi, X.-D. Wang, arXiv: 2104.06843

online workshop on the physics of RHIC Beam Energy Scan and Beyond
16-17 Aug, 2021

Strong magnetic fields

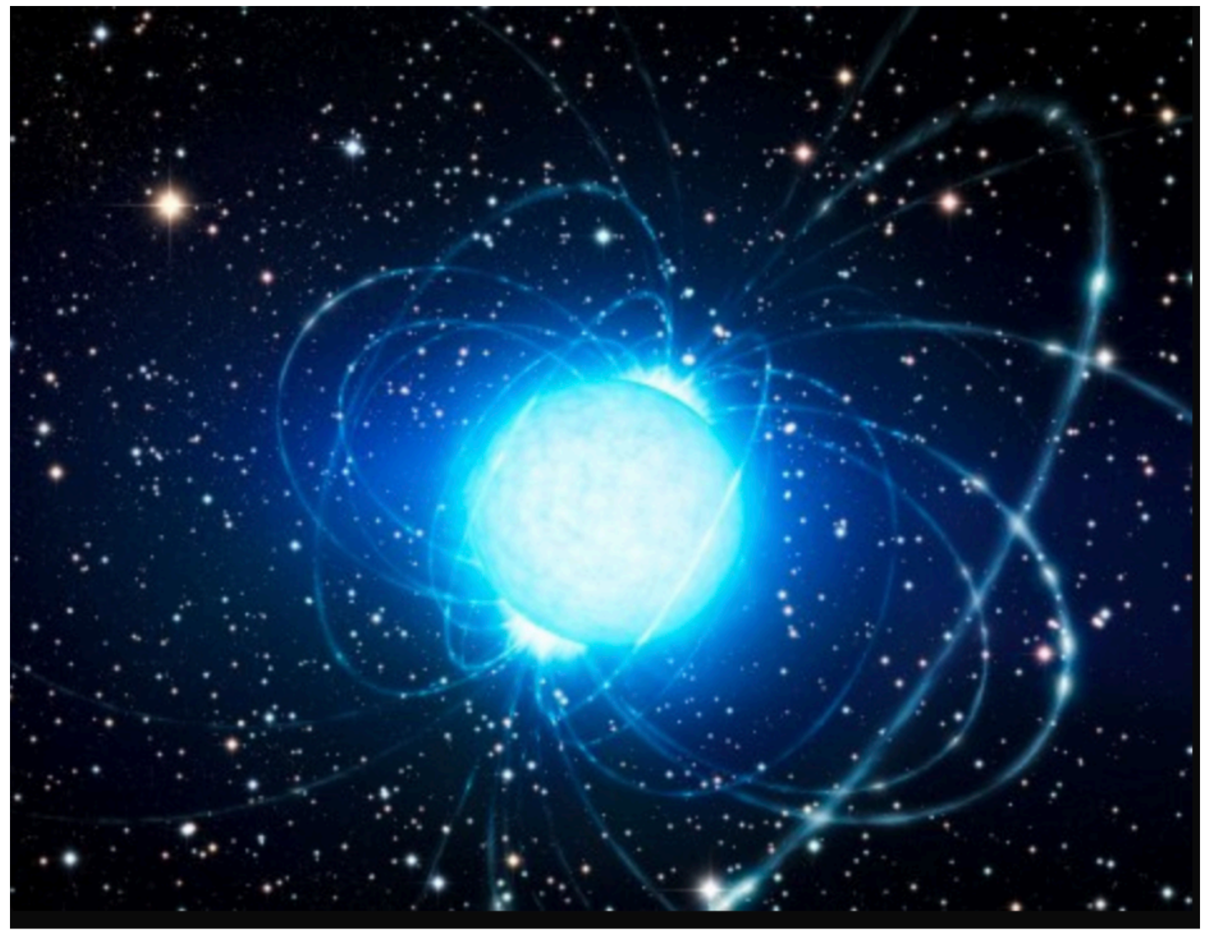
Earth

0.6 Gauss

A common, hand-held magnet

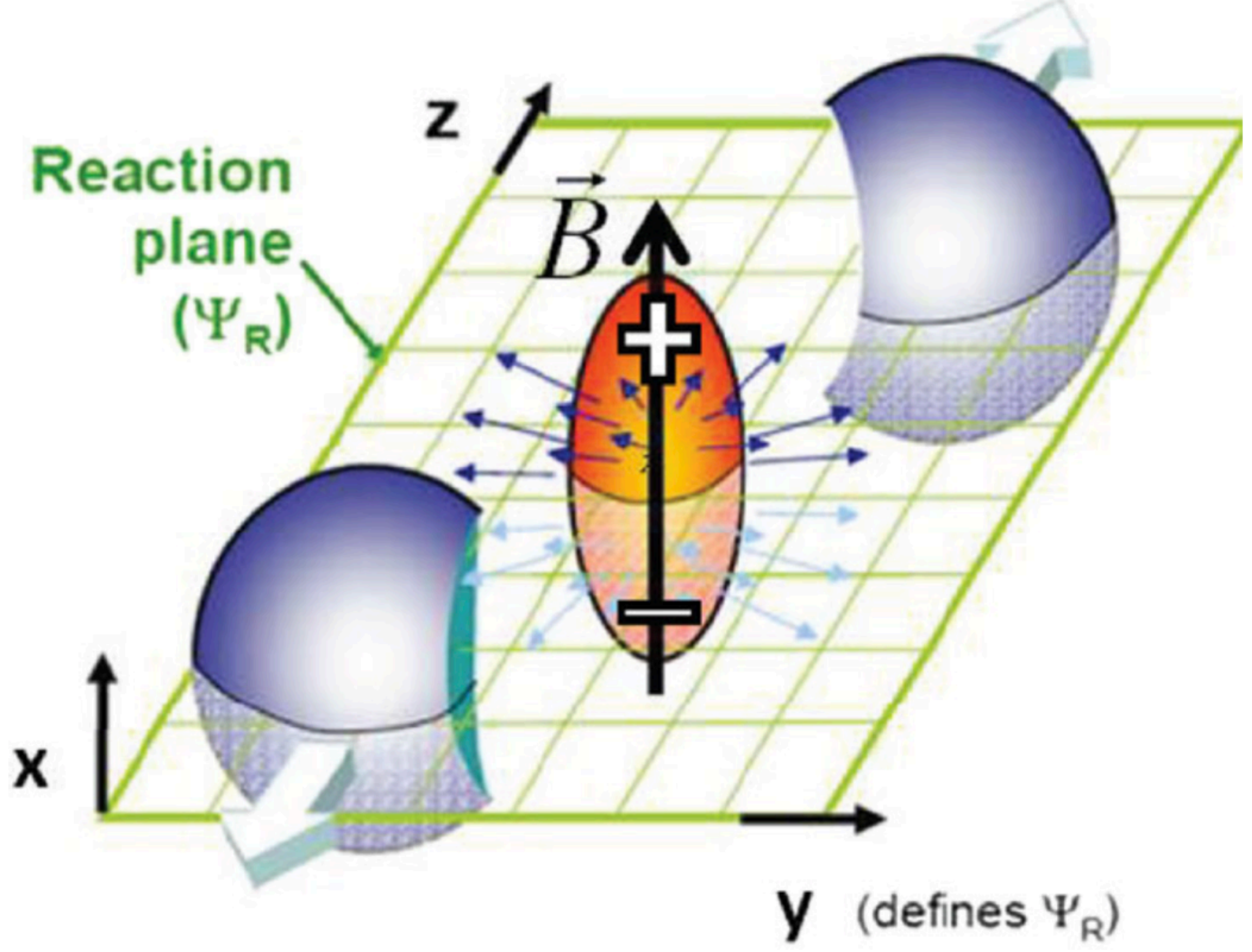
100 Gauss

Magnetar



10^{15} Gauss

Heavy-Ion collision

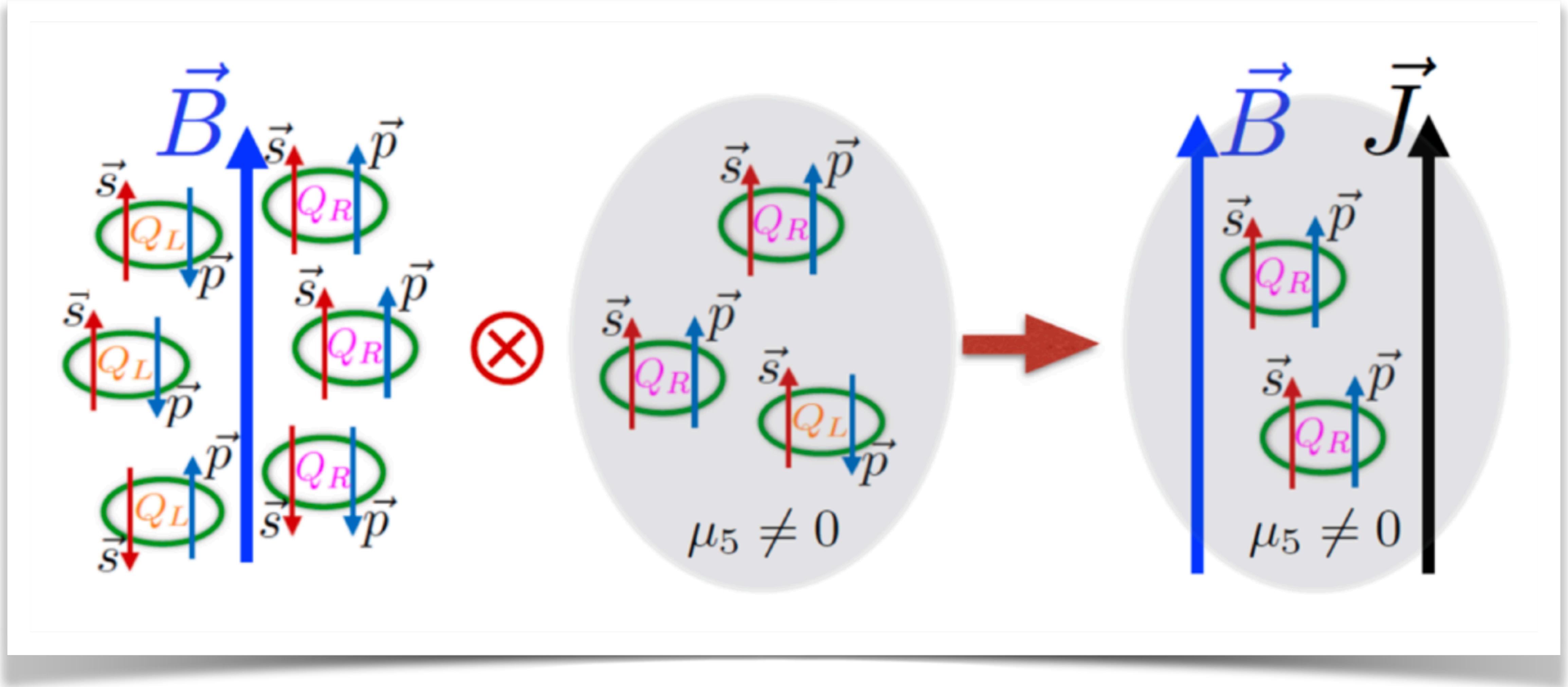


$10^{17\text{-}18}$ Gauss

$$\Lambda_{QCD}^2 \sim 10^4 \text{ MeV}^2 \sim 10^{17} \text{ Gauss}$$

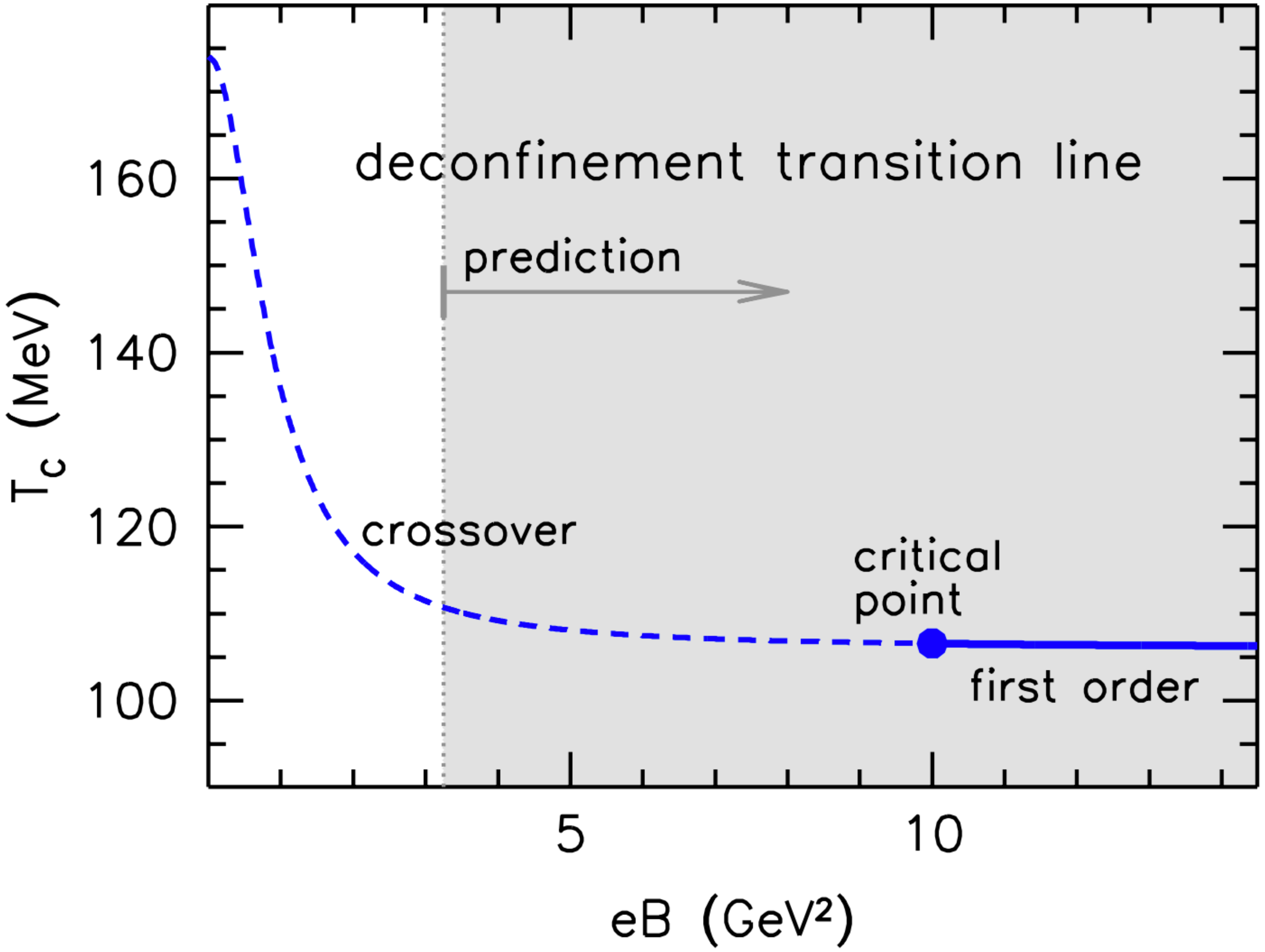
$$1 \text{ Gauss} = 1.95 \times 10^{-14} \text{ MeV}^2$$

eB induced effects:
Chiral magnetic effects, QCD critical end point…



See recent reviews e.g.
D.E. Kharzeev and J. Liao, Nature Rev. Phys. 3(2021)55

T-*eB* plane



G. Endrodi, JHEP 1507(2015) 173

Lattice QCD in strong magnetic fields

No sign problem

- B pointing to the z direction & Gauge link multiplied by a U(1) factor

$$u_x(n_x,n_y,n_z,n_\tau) = \begin{cases} \exp[-iqa^2BN_xn_y] & (n_x = N_x - 1) \\ 1 & (\text{otherwise}) \end{cases}$$

$$u_y(n_x,n_y,n_z,n_\tau) = \exp[iqa^2Bn_x],$$

$$u_z(n_x,n_y,n_z,n_\tau) = u_t(n_x,n_y,n_z,n_\tau) = 1.$$

- Quantization of the magnetic field

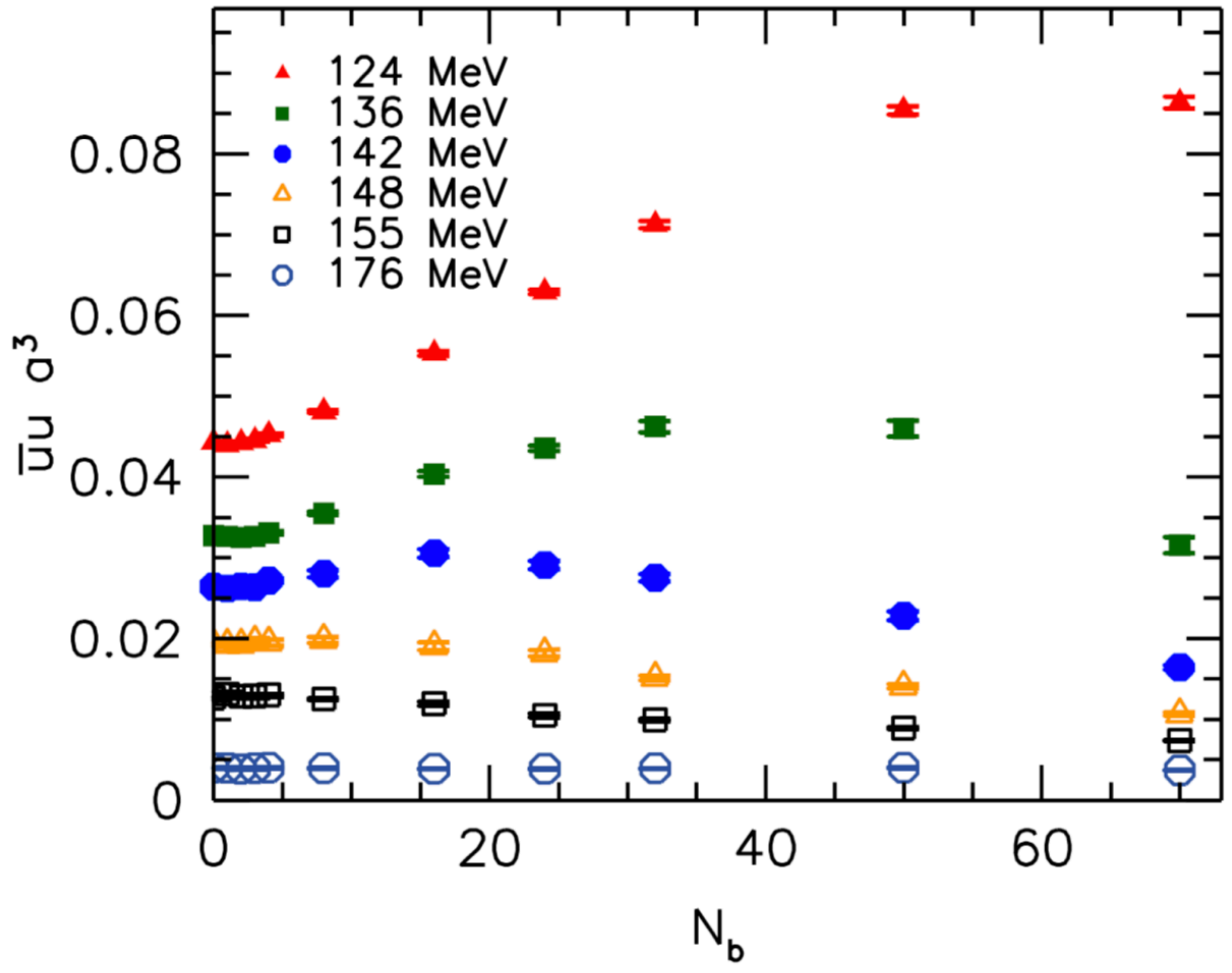
$$qB = \frac{2\pi N_b}{N_xN_y}a^{-2} \xrightarrow{q_u=2/3e,\ q_d=-1/3e,\ q_s=-1/3e} eB = \frac{6\pi N_b}{N_xN_y}a^{-2}$$

Inverse magnetic catalyses and Tpc

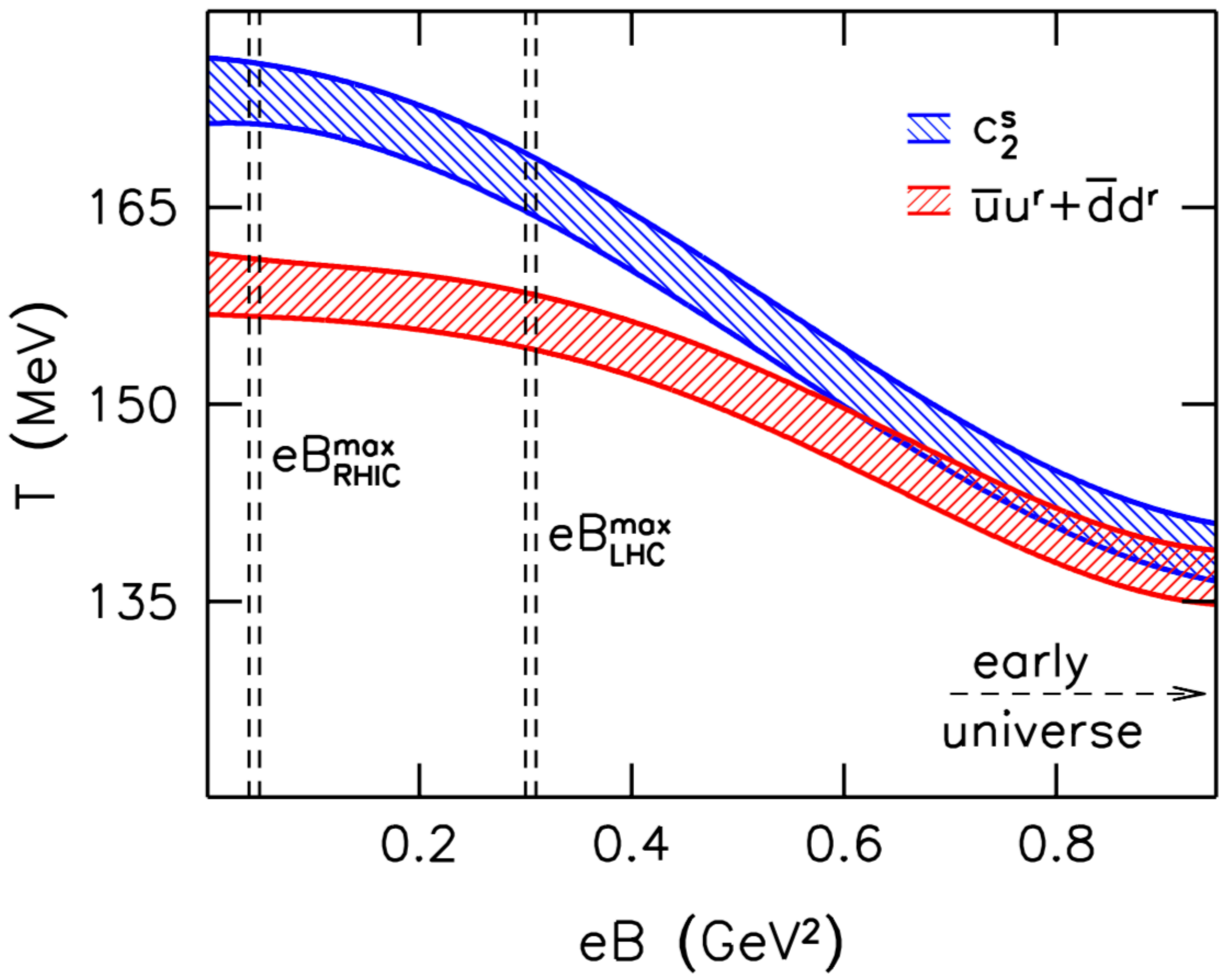
Continuum extrapolated lattice QCD results with physical pion mass

Bali et al., JHEP02(2012)044

Inverse magnetic catalysis



$eB\uparrow\ T_{pc}\downarrow$



See recent reviews e.g.
Gaoqing Cao, arXiv:2103.00456
Andersen et al., Rev. Mod. Phys. 88(2016)02001

Reduction of T_{pc} associated with inverse magnetic catalysis?

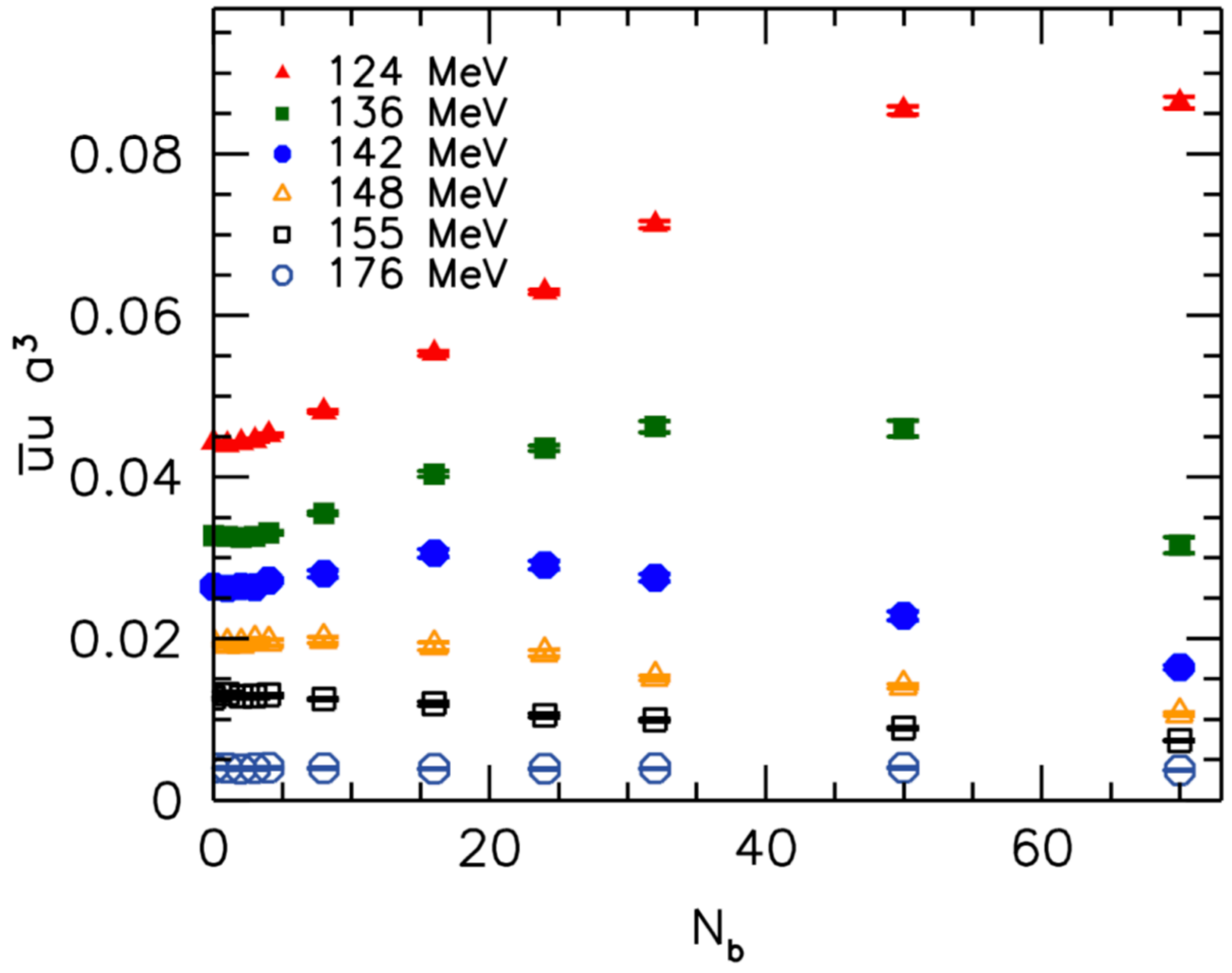
Role of hadrons?

Inverse magnetic catalyses and Tpc

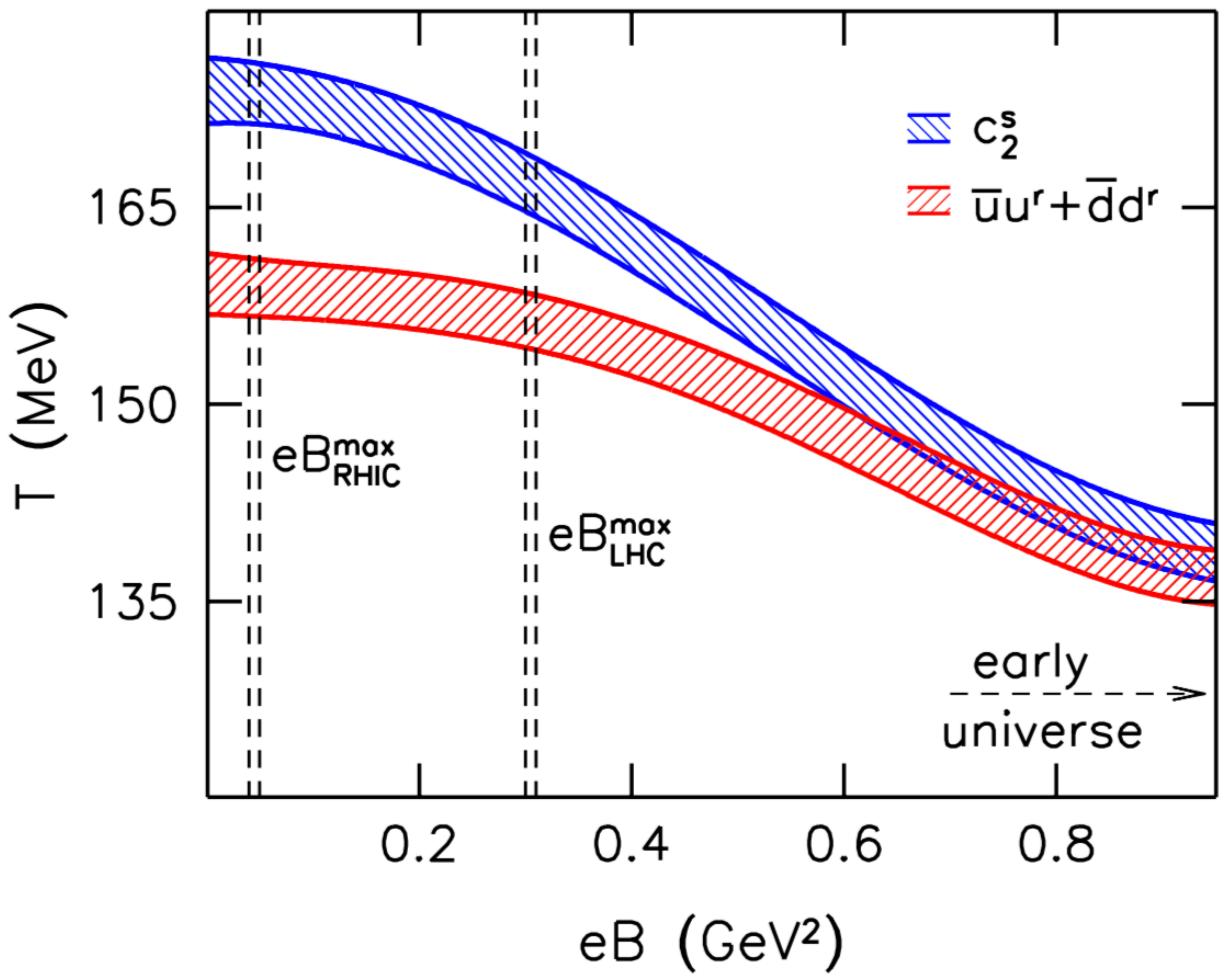
Continuum extrapolated lattice QCD results with physical pion mass

Bali et al., JHEP02(2012)044

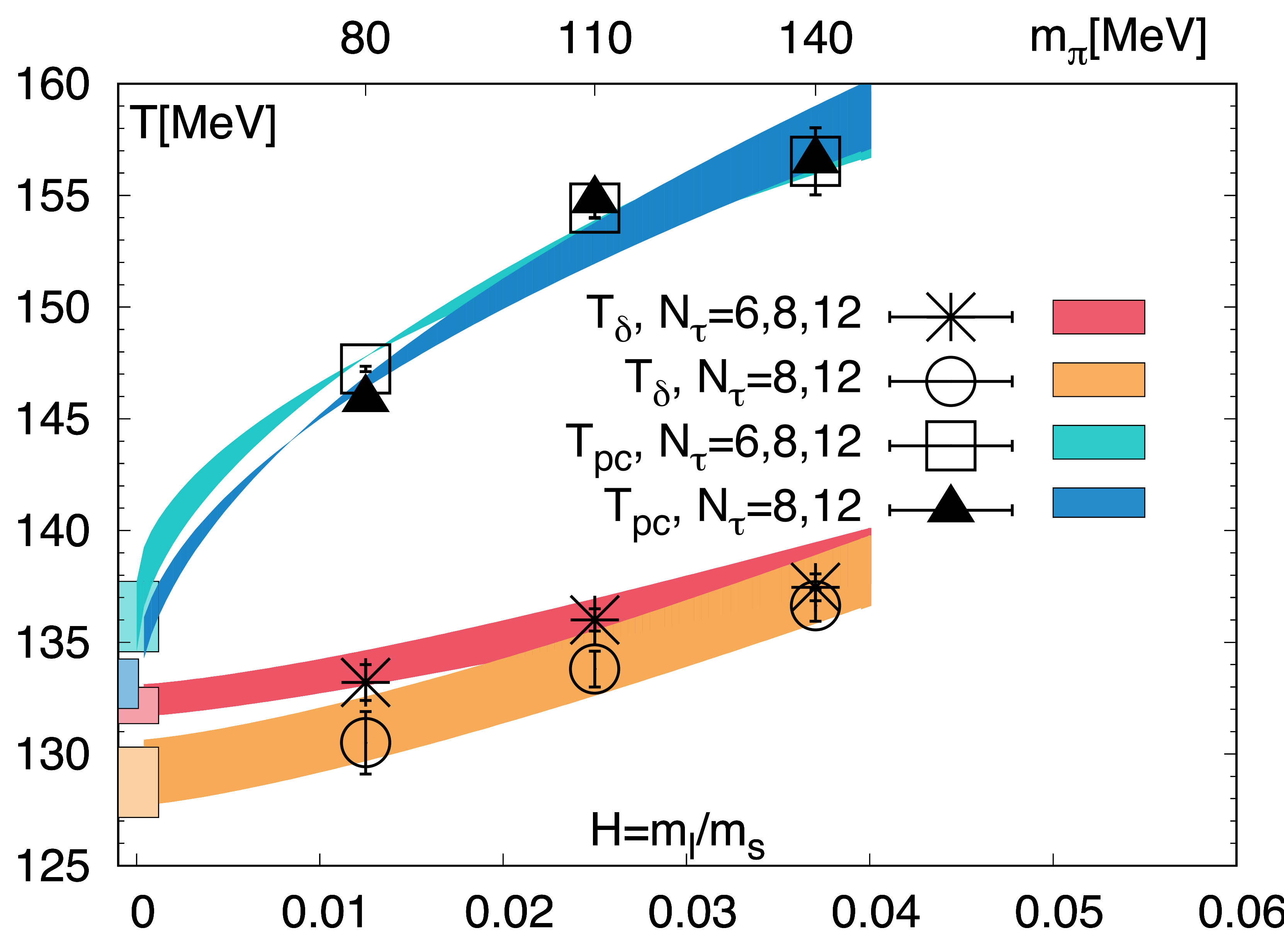
Inverse magnetic catalysis



$eB\uparrow\ T_{pc}\downarrow$



eB=0, Nf=2+1 QCD



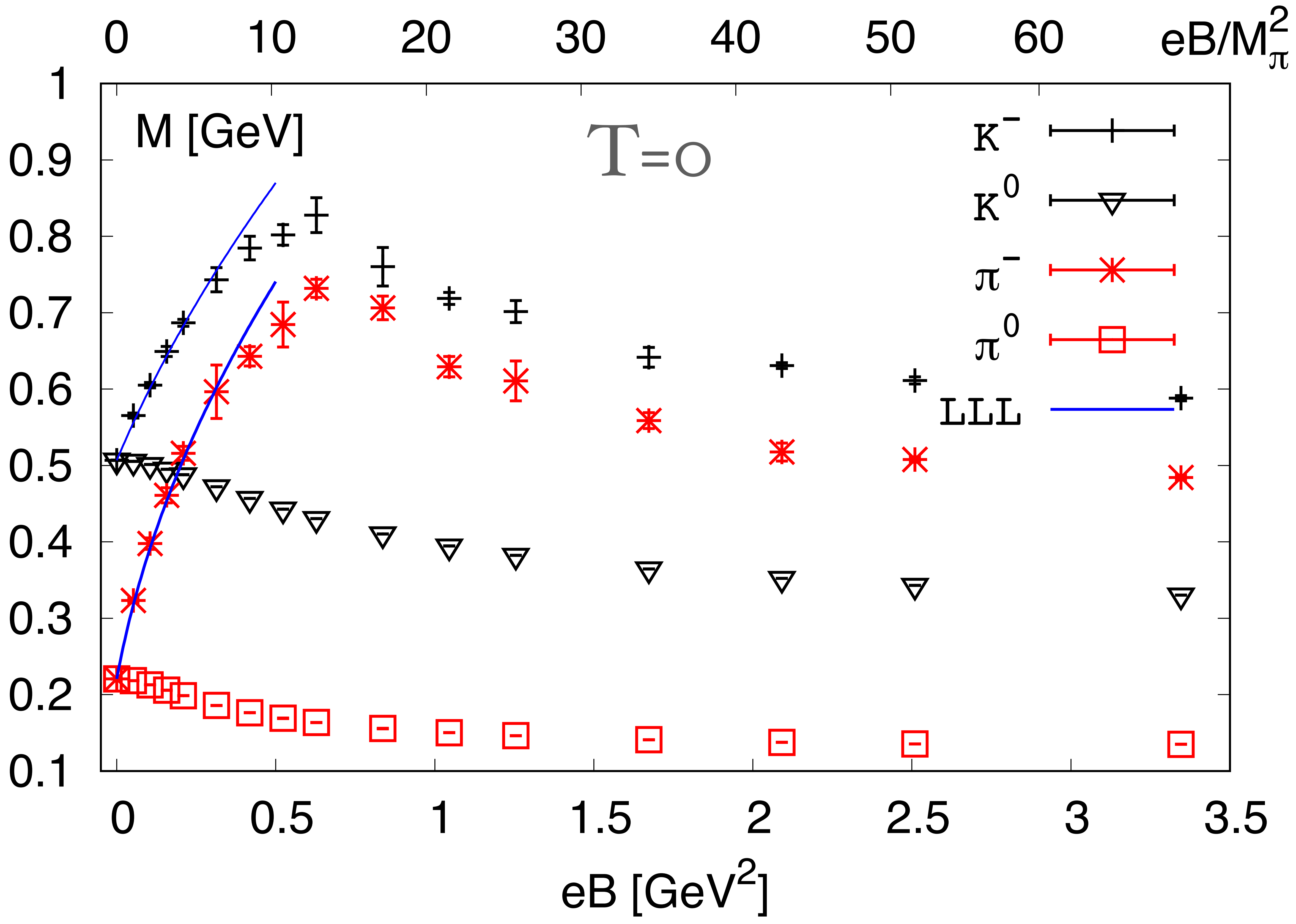
HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys. Rev. Lett. 123 (2019) 062002
HTD, arXiv:2002.11957

Reduction of T_{pc} associated with inverse magnetic catalysis?

Role of hadrons?

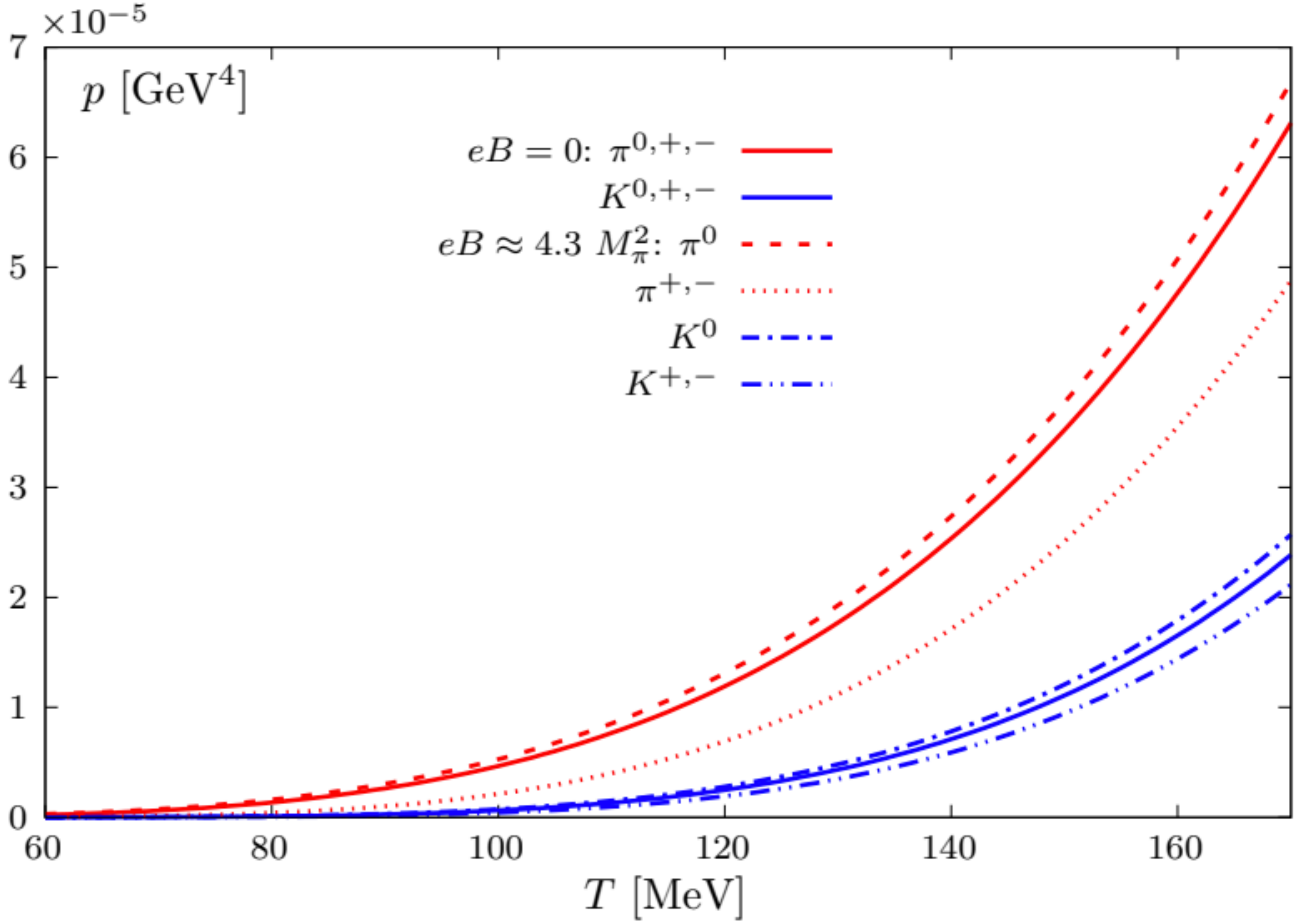
Masses of $\pi^{0,\pm}$ and $K^{0,\pm}$ and pressure

$N_{f=2+1}$ QCD, $M_\pi(eB=0) \approx 220$ MeV,
$32^3 \times 96$ lattices with $a^{-1} \approx 1.7$ GeV and HISQ action



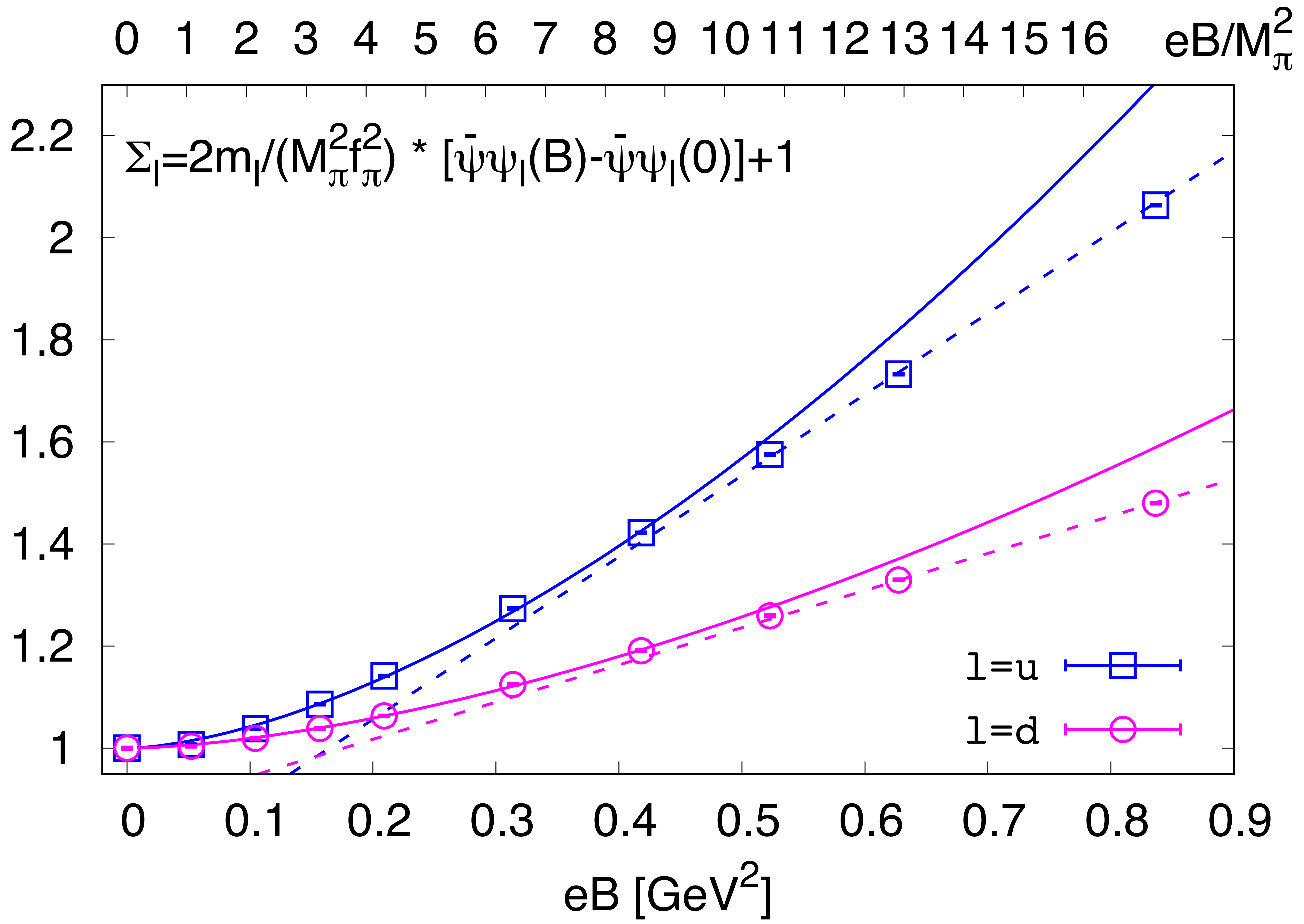
HTD, S.-T. Li, A. Tomiya, X.-D. Wang, Y. Zhang, PRD 126 (2021) 082001

Pressure in Hadron resonance gas model



HTD, S.-T. Li, Q. Shi, A. Tomiya, X.-D. Wang, Y. Zhang, arXiv: 2011.04870

Isospin symmetry breaking at $eB\neq0$ manifested in chiral condensates

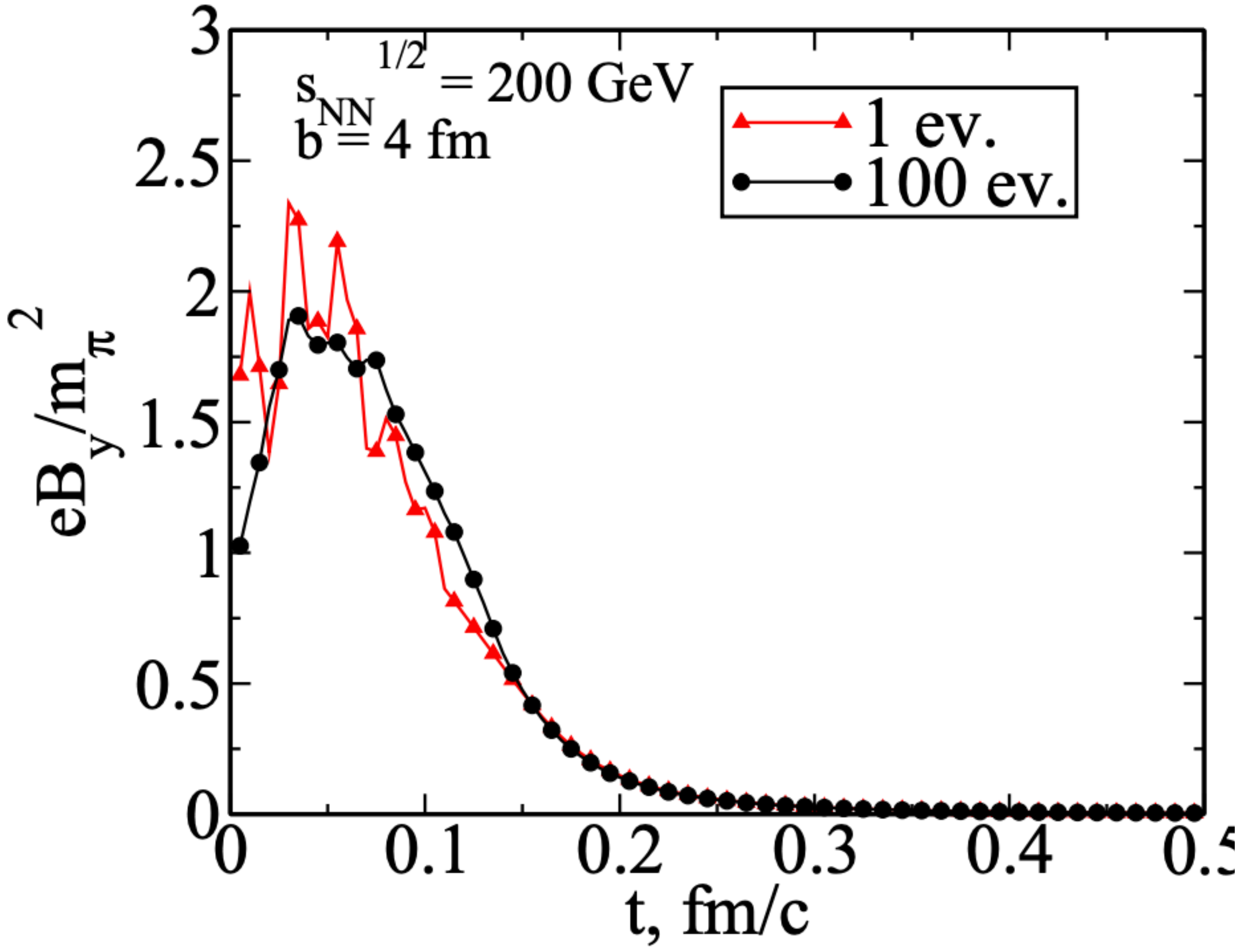


Not accessible in HIC experiments

HTD, S.-T. Li, A. Tomiya, X.-D. Wang and Y. Zhang, PRD 126 (2021) 082001

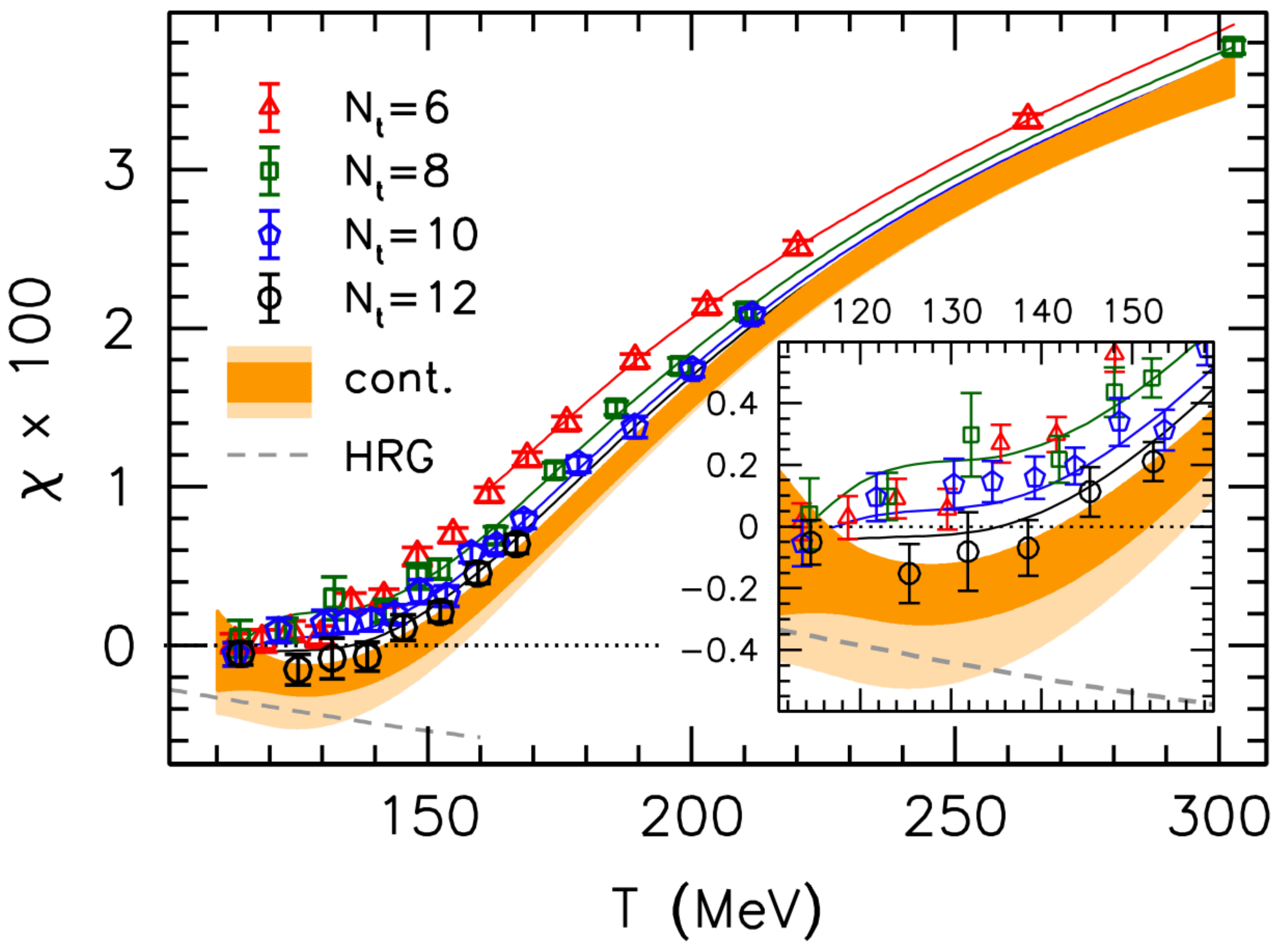
See also in e.g. Bali et al., Phys.Rev.D86(2012)071502

Magnetic field created in the early stage of HIC



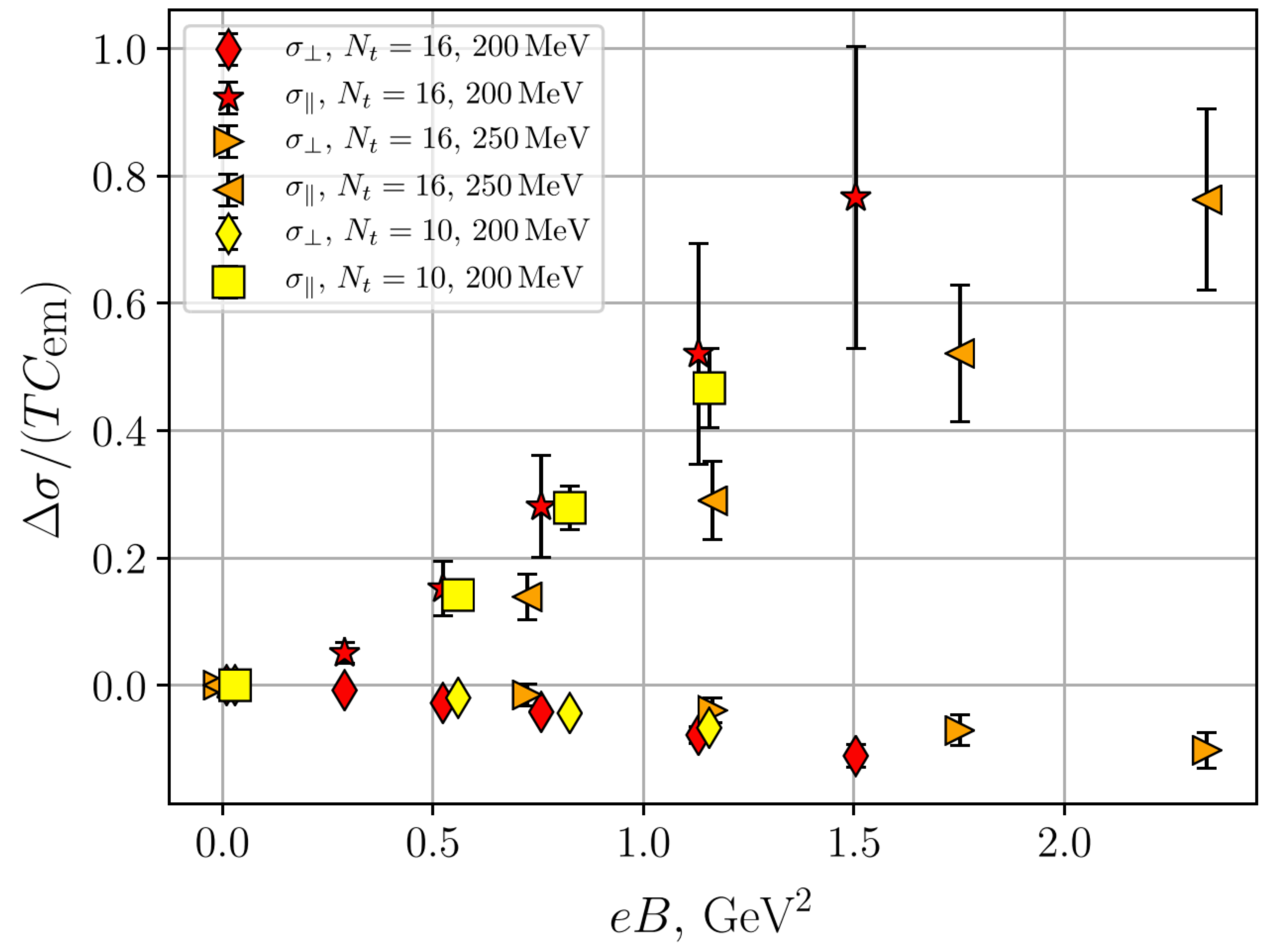
Skokov, Illarionov and V.Toneev, IJMPA 24 (2009) 5925

Magnetic susceptibility



Bali, Endrodi, Piemonte, JHEP 07 (2020) 183

Difference to electromagnetic conductivity at eB=0



Astrakhantsev et al., PRD 102 (2020) 054516

t=0: RHIC: $eB \sim m_\pi^2$

LHC: $eB \sim 15 m_\pi^2$

T> 155 MeV: Paramagnetic

T<155 MeV: Diamagnetic

Parallel: ↑

Transverse: ↓

Fluctuations of net baryon number, electric charge and strangeness

- Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln \mathcal{Z}(T,V,\hat\mu_u,\hat\mu_d,\hat\mu_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!}\left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Taylor expansion coefficients at μ=0 are computable in LQCD

$$\hat\chi_{ijk}^{uds} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_u/T\right)^i \partial\left(\mu_d/T\right)^j \partial\left(\mu_s/T\right)^k}\right|_{\mu_{u,d,s}=0}$$

$$\hat\chi_{ijk}^{\mathrm{BQS}} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial\left(\mu_\mathrm{B}/T\right)^i \partial\left(\mu_\mathrm{Q}/T\right)^j \partial\left(\mu_\mathrm{S}/T\right)^k}\right|_{\mu_{\mathrm{B,Q,S}}=0}$$

$$\mu_u = \frac{1}{3}\mu_\mathrm{B} + \frac{2}{3}\mu_\mathrm{Q}\,, \quad \mu_d = \frac{1}{3}\mu_\mathrm{B} - \frac{1}{3}\mu_\mathrm{Q}\,, \quad \mu_s = \frac{1}{3}\mu_\mathrm{B} - \frac{1}{3}\mu_\mathrm{Q} - \mu_\mathrm{S}\,.$$

- At eB=/=0 a lot more need to be explored

HRG: G. Kadam et al., JPG 47 (2020) 125106, Ferreira et al., PRD 98(2018)034003, Fukushima and Hidaka, PRL117 (2016)102301
Bhattacharyya et al., EPL115(2016)62003

PNJL: W.-J. Fu, Phys. Rev. D 88 (2013) 014009

High T: Ideal gas limit

- At eB=0: $\varepsilon^2 = m^2 + |\vec{p}|^2$ Kapusta & Gale, Finite-temperature field theory: Principles and applications

$$\frac{p}{T^4} = \frac{8\pi^2}{45} + \frac{7\pi^2}{20} + \sum_{f=u,d,s}\left[\frac{1}{2}\hat{\mu}_f^2 + \frac{1}{4\pi^2}\hat{\mu}_f^4\right]$$

- At eB=/=0: $\varepsilon_l^2 = p_z^2 + m^2 + 2qB(l + 1/2 - s_z)$ HTD, S.-T. Li, Q. Shi and X.-D. Wang, 2104.06843

$$\frac{p}{T^4} = \frac{8\pi^2}{45} + \sum_{f=u,d,s}\frac{3|q_f|B}{\pi^2 T^2}\left[\frac{\pi^2}{12} + \frac{\hat{\mu}_f^2}{4} + 2\frac{\sqrt{2|q_f|B}}{T}\sum_{l=1}^{\infty}\sqrt{l}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\cosh\left(k\hat{\mu}_f\right) \times \mathrm{K}_1\left(\frac{k\sqrt{2|q_f|Bl}}{T}\right)\right]$$

K_1: first-order modified Bessel function of the second kind

High T: Ideal gas limit

$$\frac{\chi_2^{\rm B}}{eB} = \frac{4}{9\pi^2}\left(\frac{1}{2} + \hat{b}\sum_{l=1}^{\infty}\sqrt{l}\sum_{k=1}^{\infty}(-1)^{k+1}k\times\left[\sqrt{2}\,{\rm K}_1\left(k\,\hat{b}\sqrt{2l}\right) + {\rm K}_1\left(k\,\hat{b}\sqrt{l}\right)\right]\right)$$

$$\frac{\chi_{11}^{\rm BQ}}{eB} = \frac{4}{9\pi^2}\left(\frac{1}{4} + \hat{b}\sum_{l=1}^{\infty}\sqrt{l}\sum_{k=1}^{\infty}(-1)^{k+1}k\times\left[2\sqrt{2}\,{\rm K}_1\left(k\,\hat{b}\sqrt{2l}\right) - {\rm K}_1\left(k\,\hat{b}\sqrt{l}\right)\right]\right)$$

$$\hat{b} = \sqrt{2eB/3}/T$$

$$\sqrt{eB}/T \to \infty$$

Quantity	Value
χ_2^u/eB	$1/\pi^2$
$\chi_2^{d/s/S}/eB$	$1/(2\pi^2)$
$\chi_{11}^{ud}/eB = \chi_{11}^{us}/eB = \chi_{11}^{ds}/eB{=}0$	0
$\chi_2^{\rm B}/eB$	$2/(9\pi^2)$
$\chi_2^{\rm Q}/eB$	$5/(9\pi^2)$
$\chi_{11}^{\rm BQ}/eB$	$1/(9\pi^2)$
$\chi_{11}^{\rm QS}/eB = -\chi_{11}^{\rm BS}/eB = \chi_2^{\rm S}/3eB$	$1/(6\pi^2)$

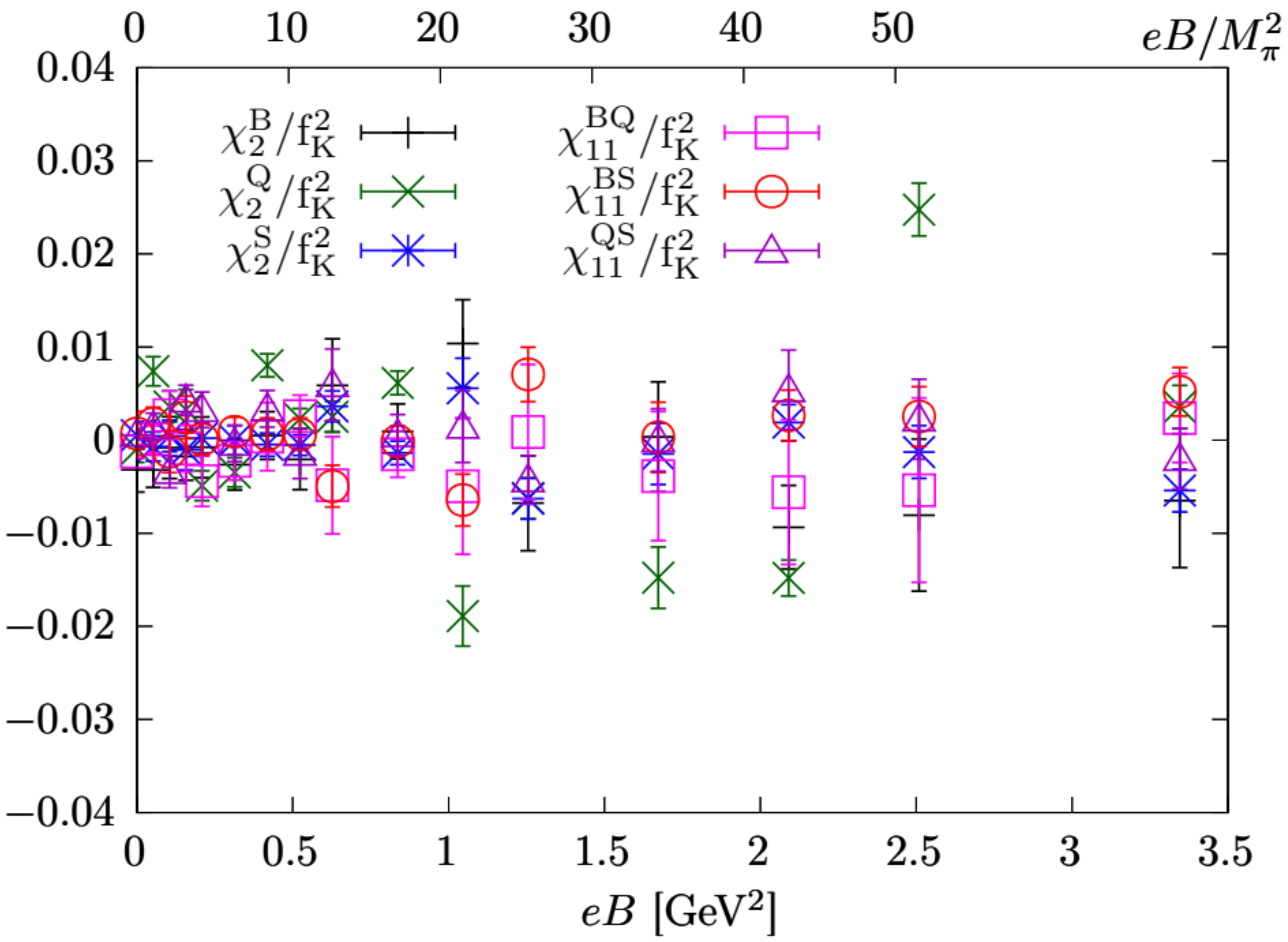
$$eB = 0$$

$$\chi_2^{\rm B} = \chi_{11}^{\rm QS} = -\chi_{11}^{\rm BS} = \chi_2^{\rm Q}/2 = \chi_2^{\rm S}/3 = 1/3$$
$$\chi_{11}^{\rm BQ} = 0\,.$$

Holds at both eB=0 and eB=/=0 with T→∞

$$\chi_{11}^{\rm BS}/\chi_2^{\rm S} = -\chi_{11}^{\rm QS}/\chi_2^{\rm S} = -\frac{1}{3}$$

No evidence for a Superconducting phase at T=0

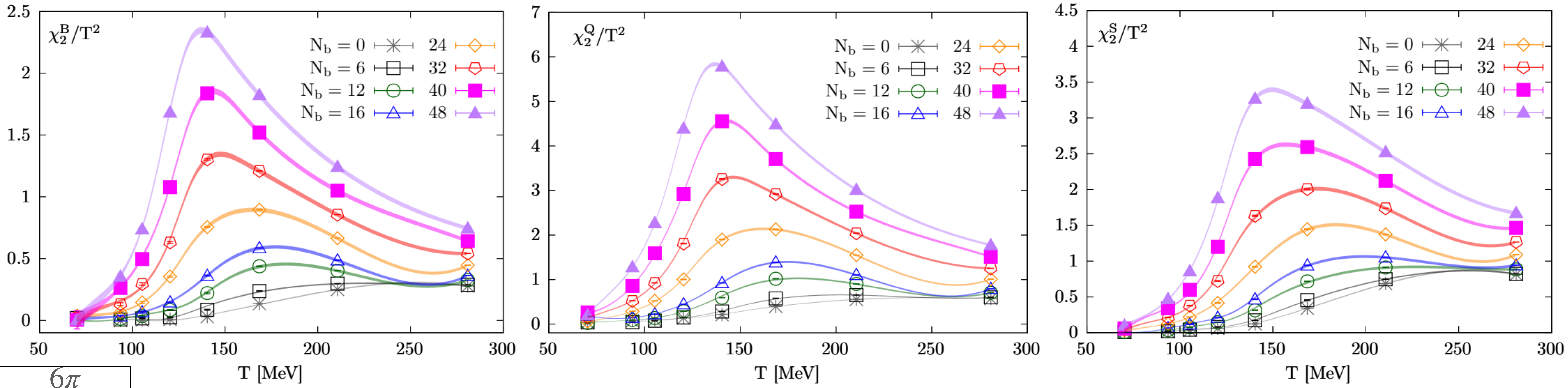


rho is a boson

$$f(E) = \frac{1}{e^{E/kT} - 1}$$

If the energy of rho becomes zero, electric charge fluctuation χ_2^Q shall be divergent

2nd order fluctuations of net baryon number, electric charge and strangeness



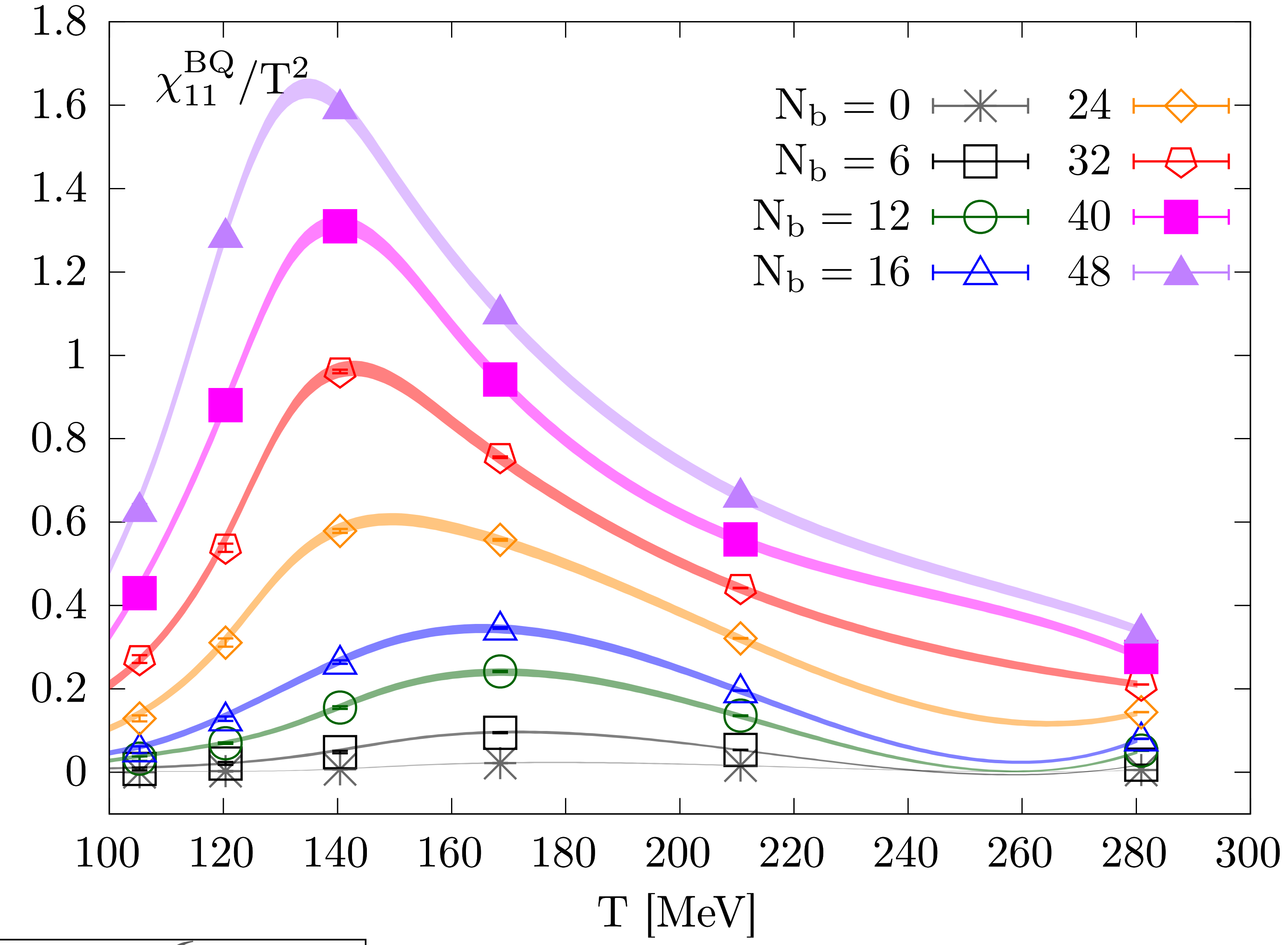
$$eB = \frac{6\pi}{N_x N_y} \mathrm{N_b}$$

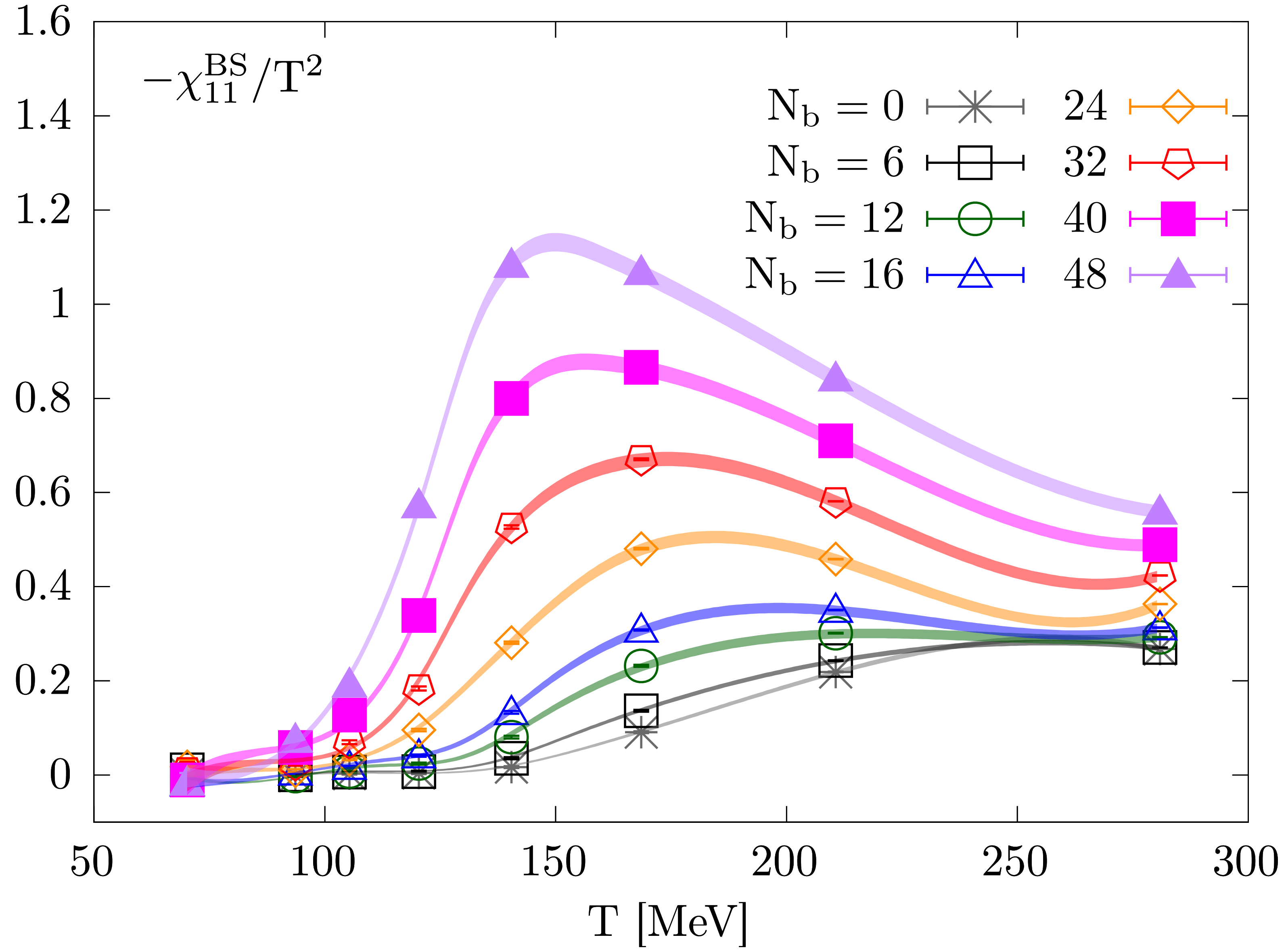
HTD, S.-T. Li, Q. Shi and X.-D. Wang, 2104.06843

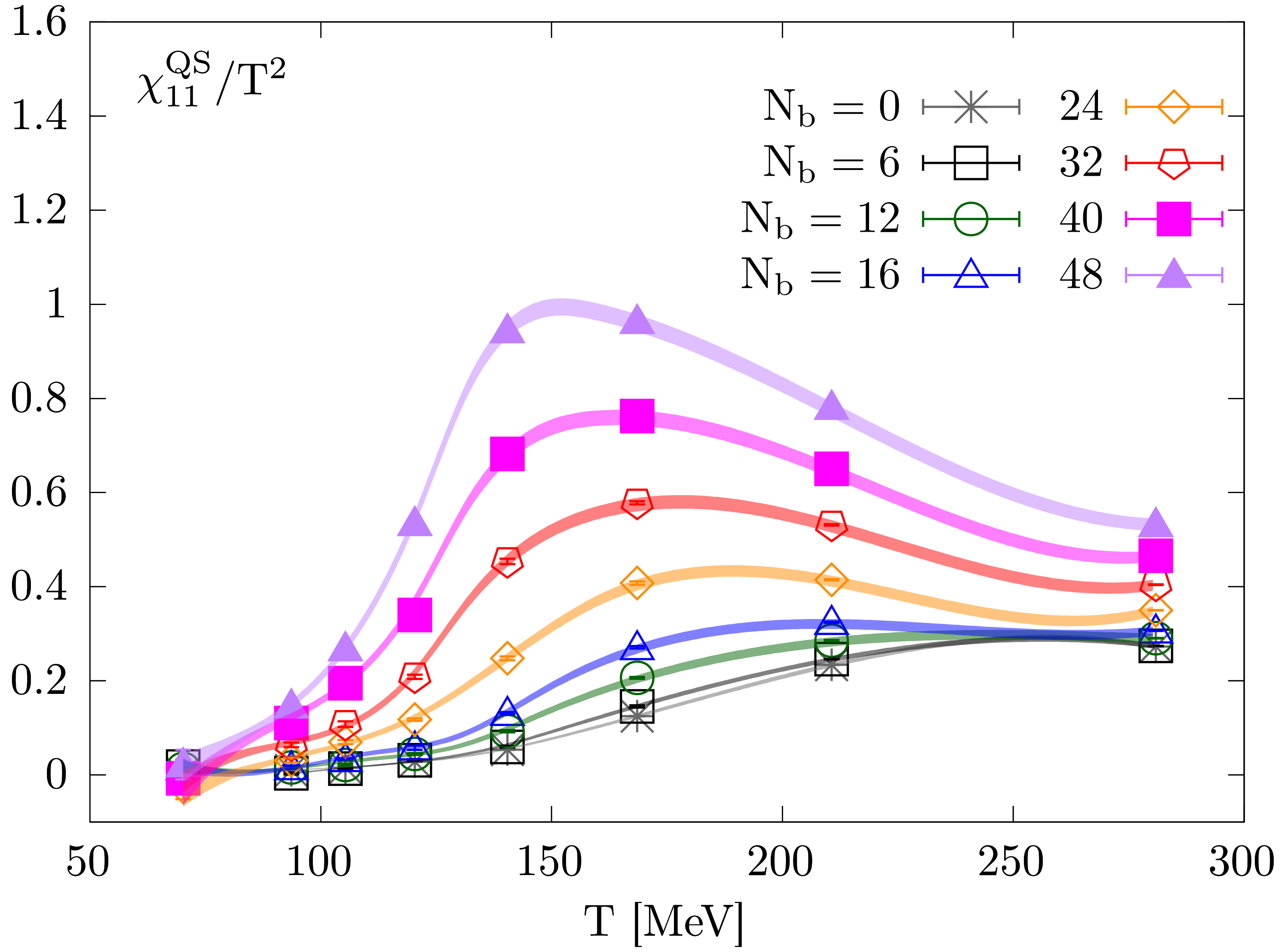
Peak locations shift to lower T in stronger eB

Consistent with the reduction of T_{pc} in a stronger magnetic field

2nd order correlations of net baryon number, electric charge and strangeness







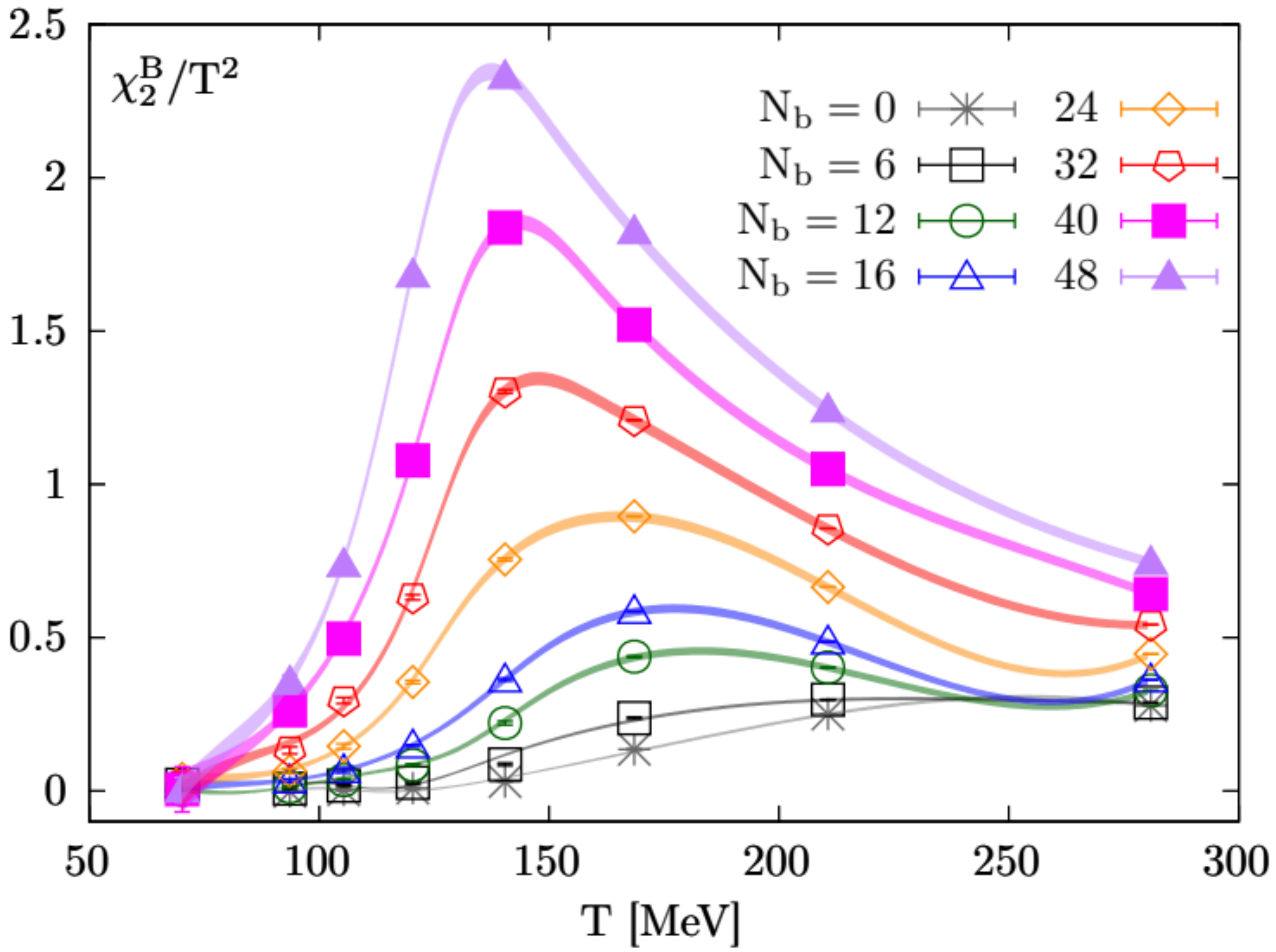
$$eB = \frac{6\pi}{N_x N_y} N_b$$

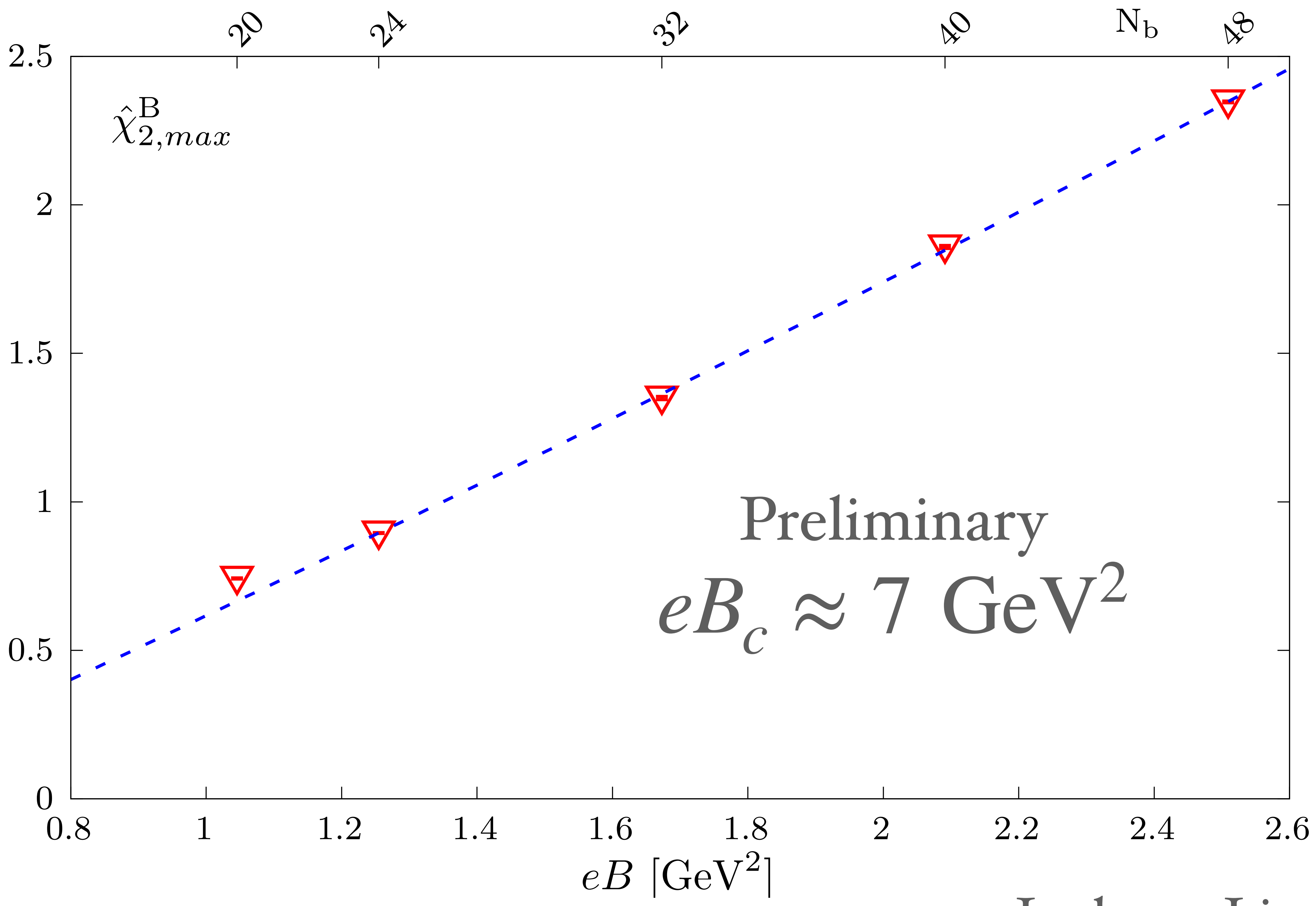
HTD, S.-T. Li, Q. Shi and X.-D. Wang, 2104.06843

Similar to the 2nd order fluctuations

Signal for a critical end point in the T-*eB* plane of QCD phase diagram?

A rough estimate of a CEP in T-eB plane





Junhong Liu, work in progress

At eB=0:

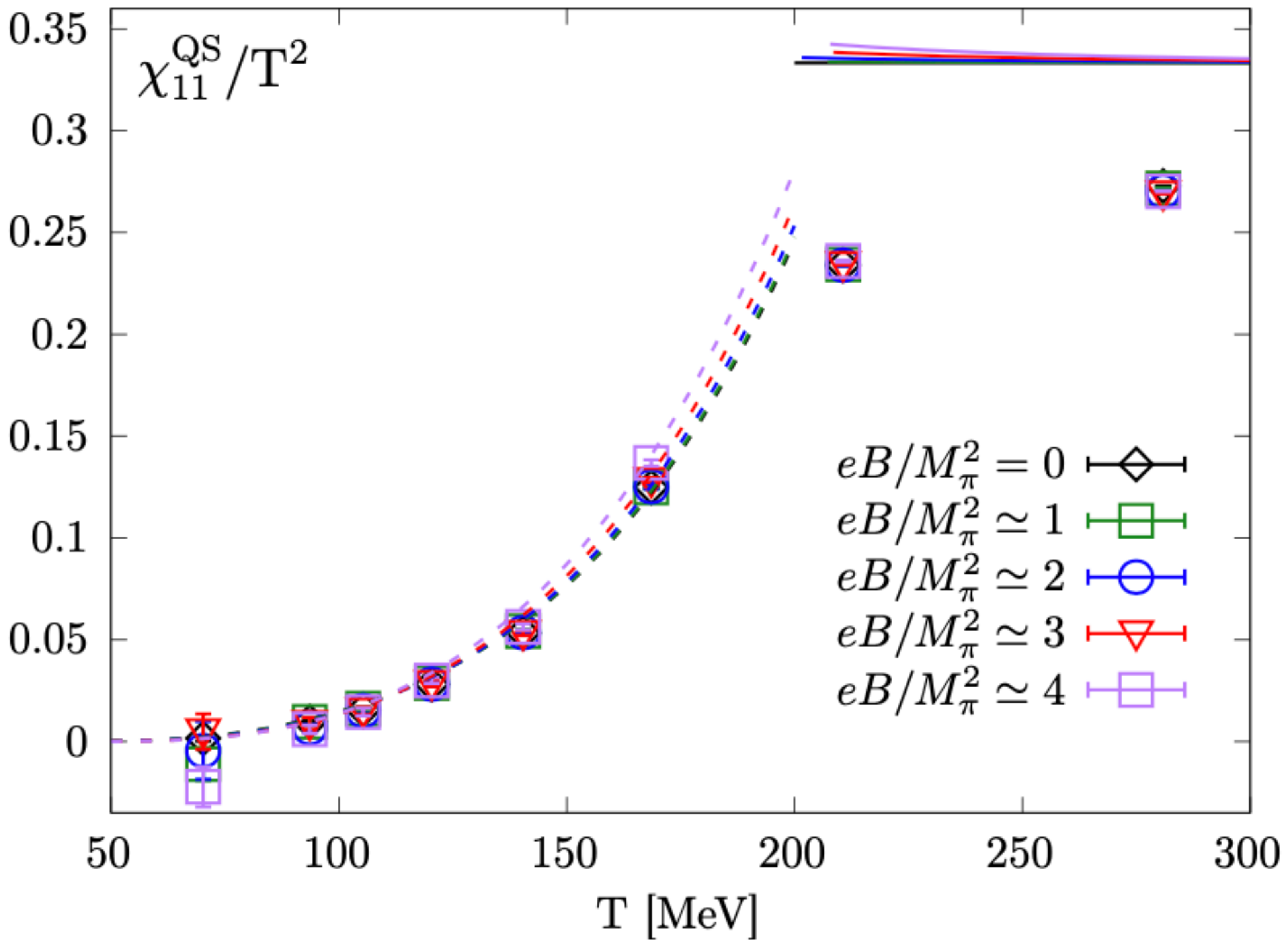
$$\chi_n^B \propto (-2\kappa_q)^{n/2}\, h^{(2-\alpha-n/2)/\beta\delta} f_f^{(n/2)}(z)$$

Friman et al., Eur. Phys. J. C 71(2011) 1694

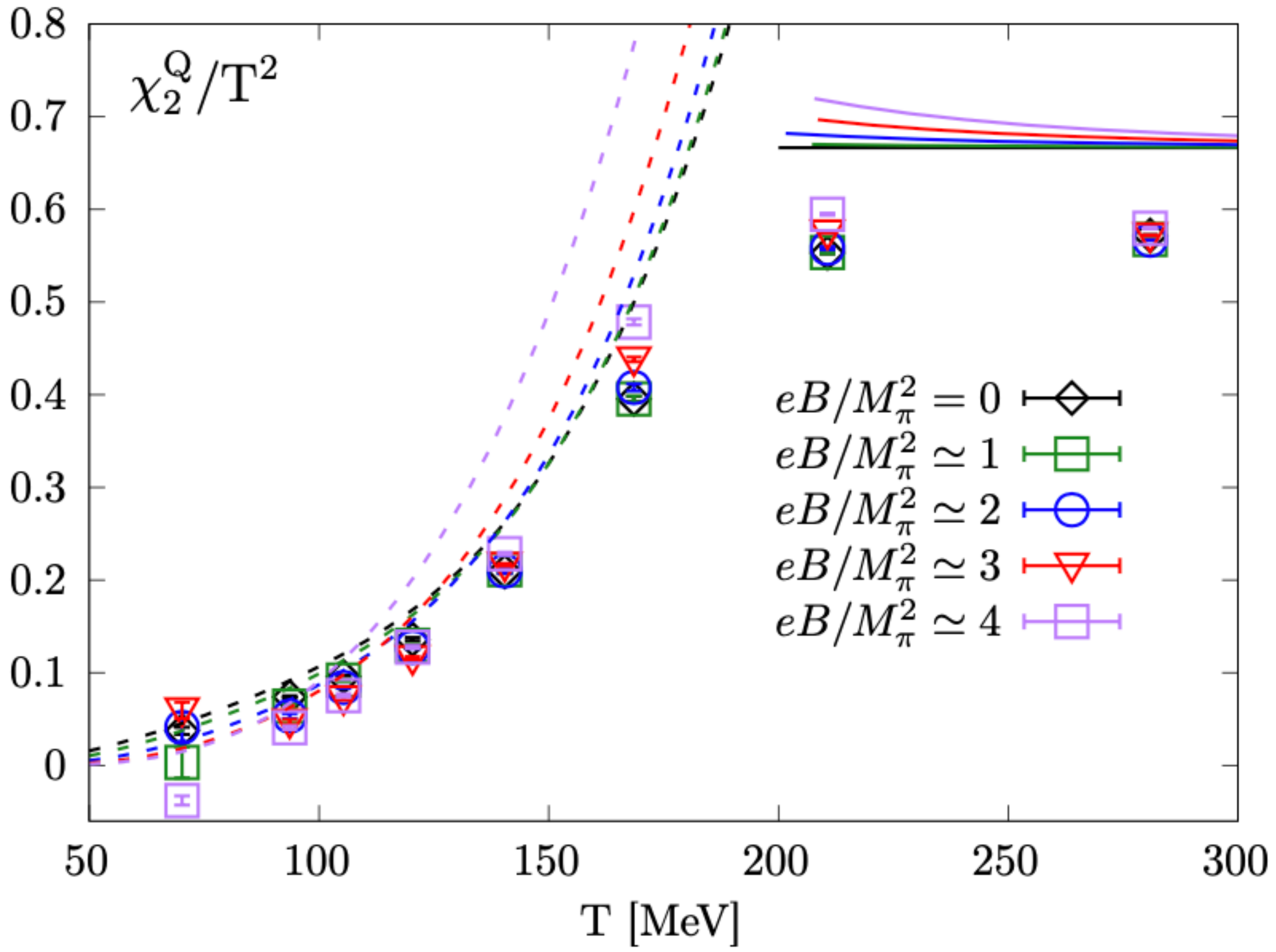
$$\chi^B_{2,max} = b\,(eB_c - eB)^{(1-\alpha)/\beta\delta} + d$$

	$\beta\delta$	α	$(1-\alpha)/\beta\delta$
Z(2)	1.5654	0.1088	0.5693
O(4)	1.8468	-0.2268	0.6643

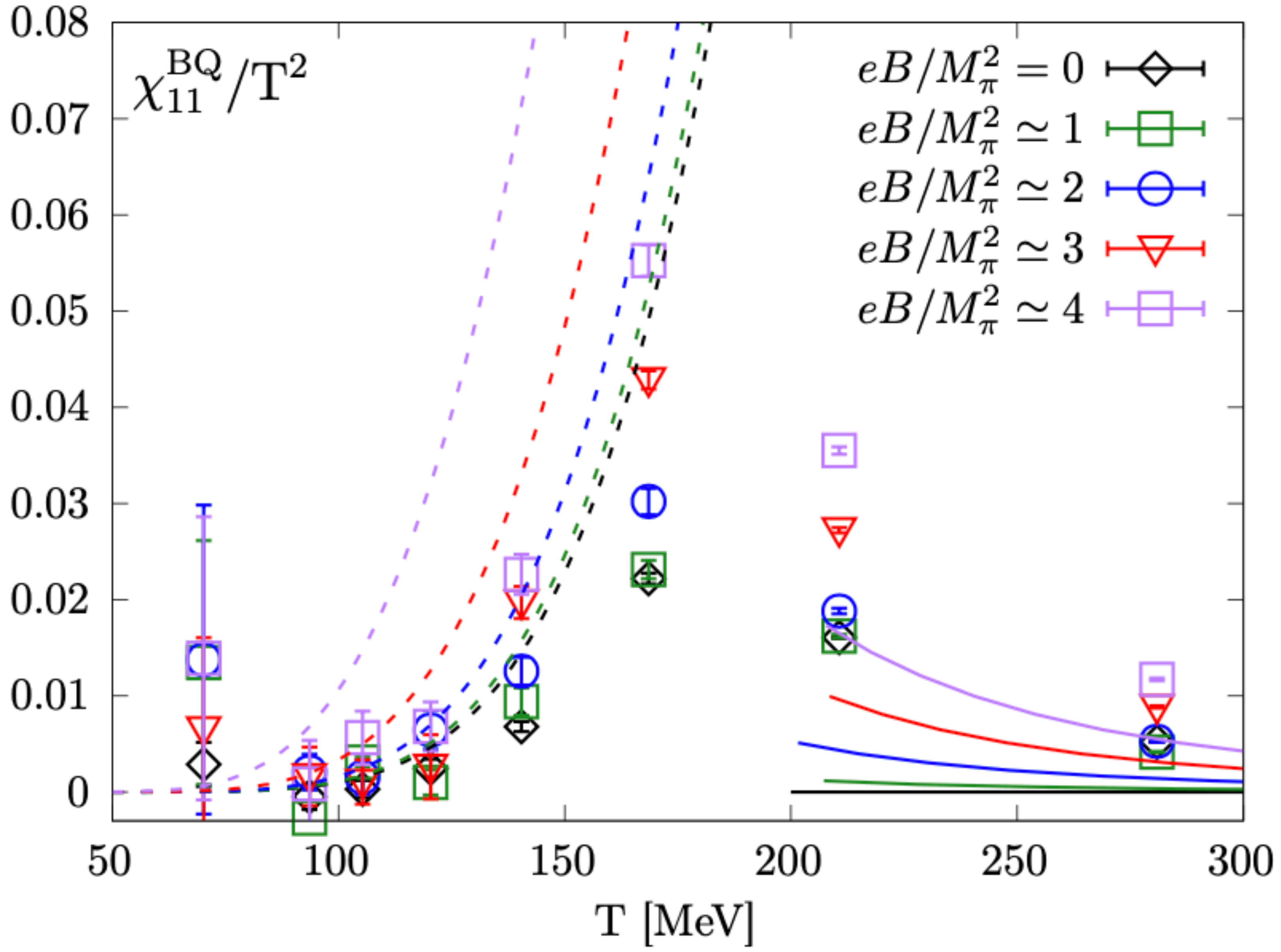
Comparison to HRG and idea gas limit



Low T: Kaons dominated

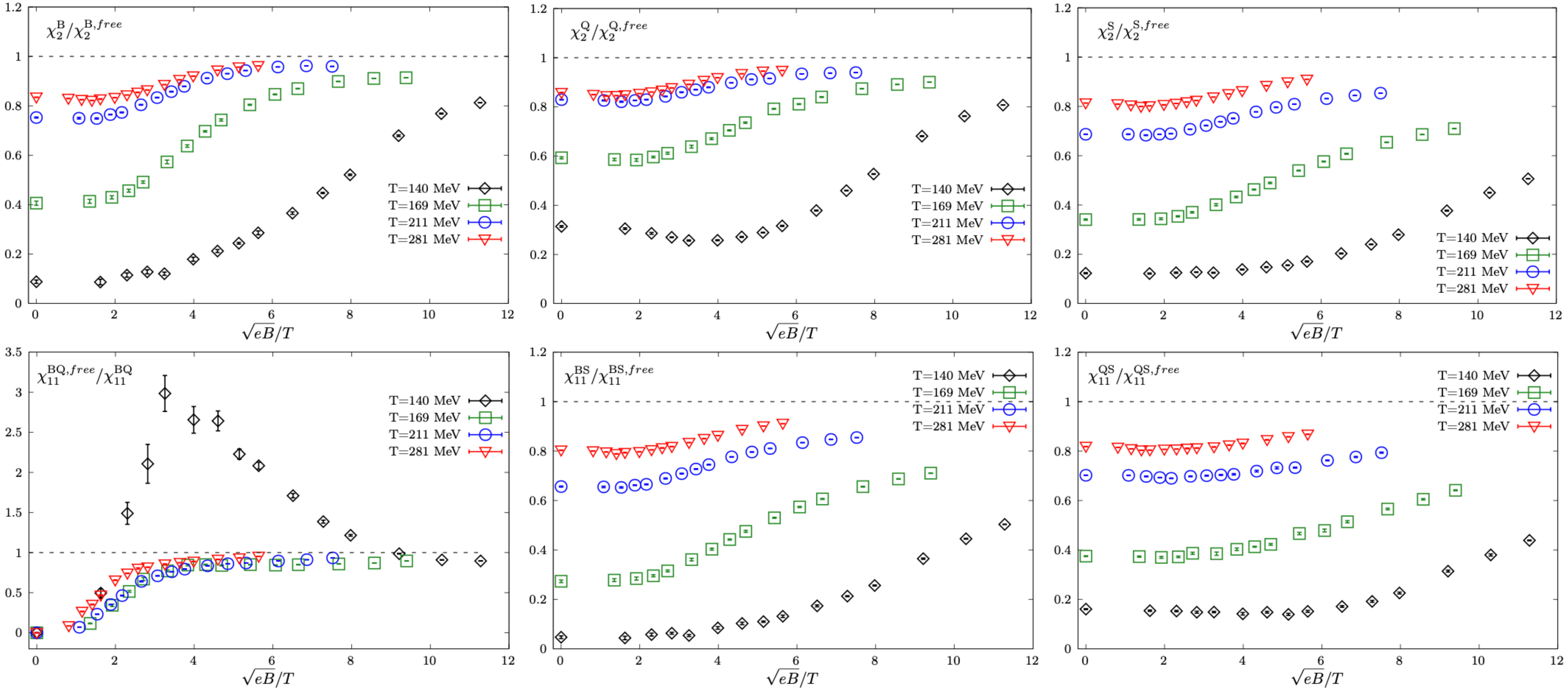


Pions dominated



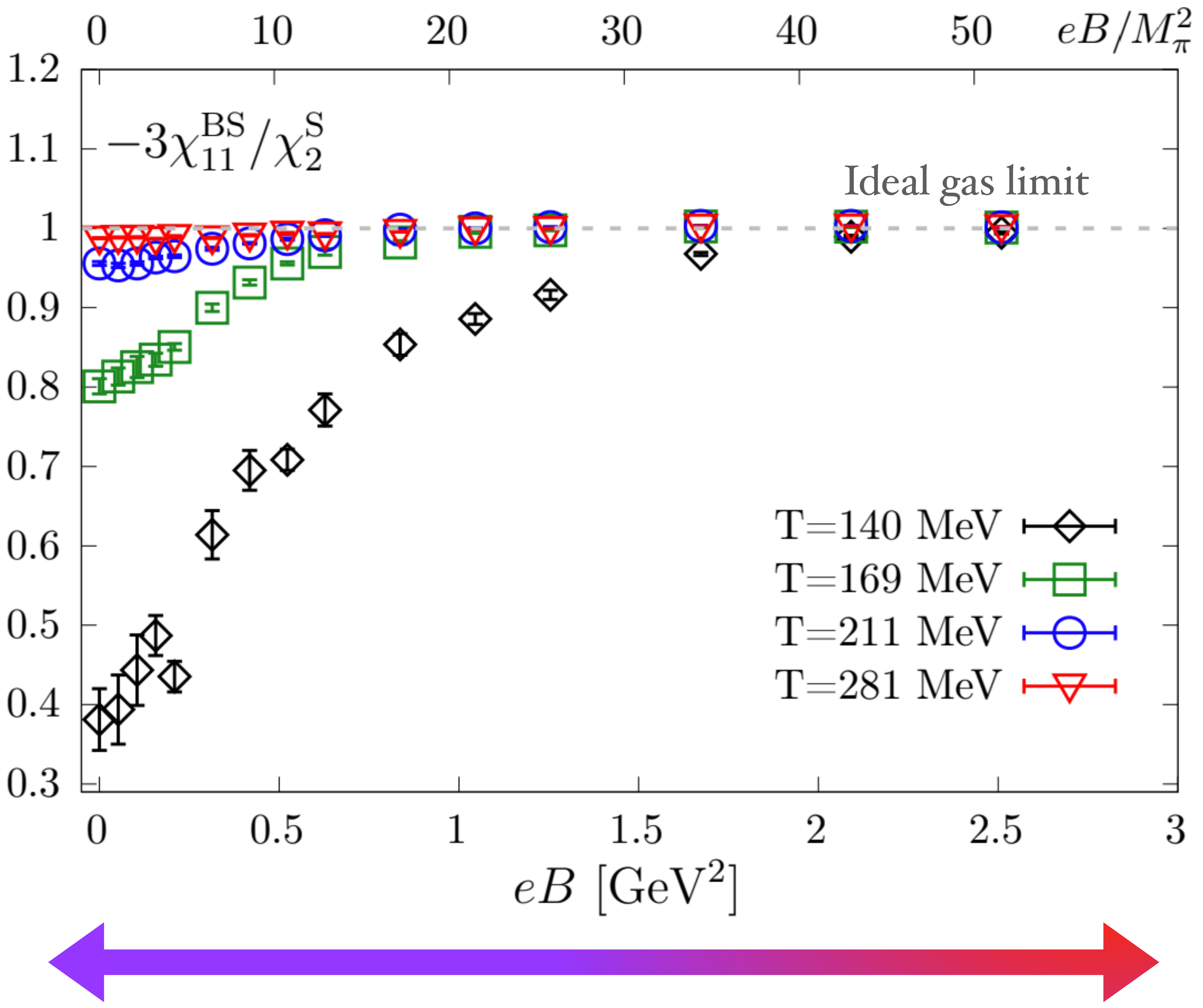
Baryons dominated

Comparison to the ideal gas limit



Baryon-strangeness correlations: a diagnostic of strongly interacting matter

Koch, Majumder and Randrup, Phys. Rev. Lett. 95(2005)182301



Central Collisions

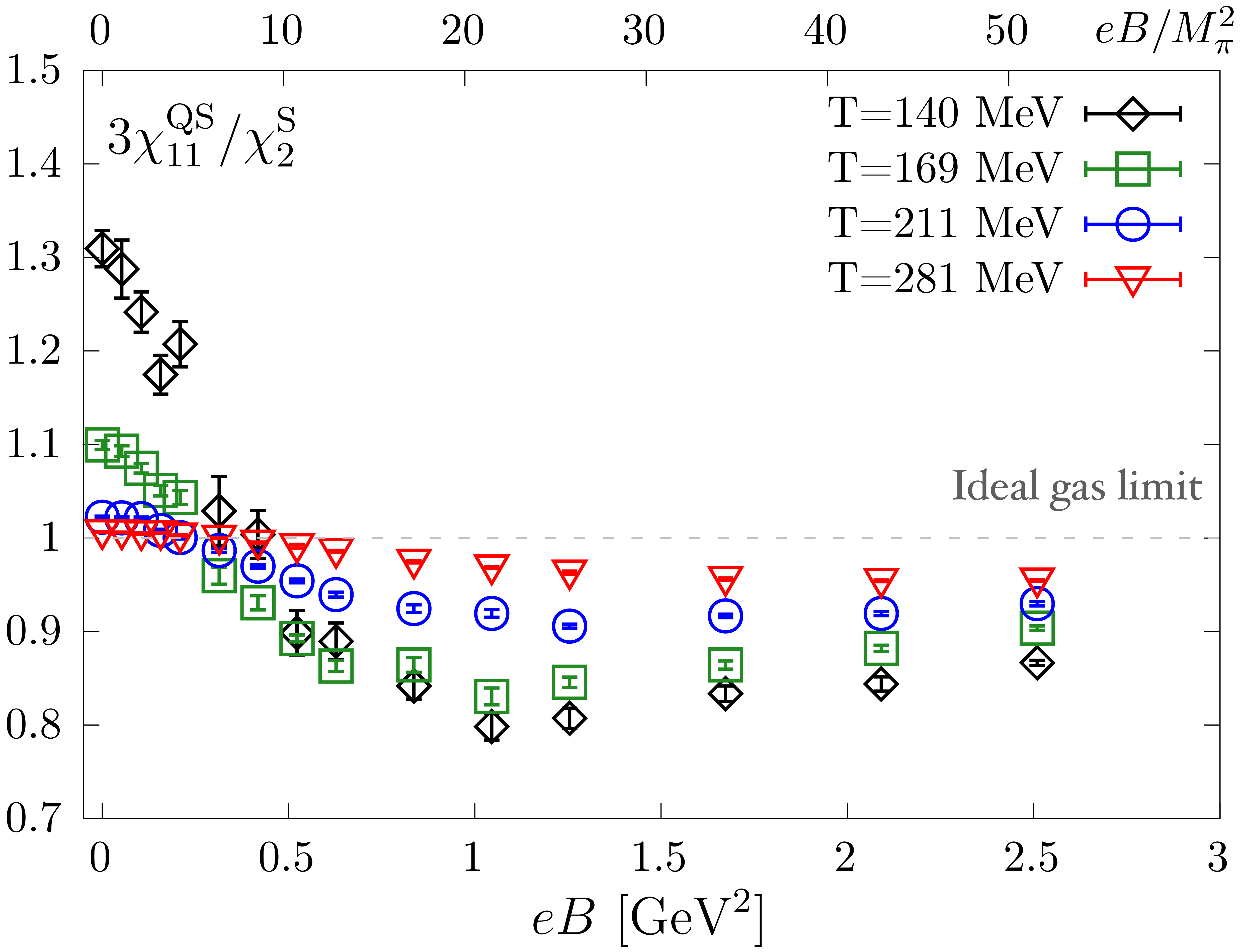
Peripheral Collisions

At both eB=0 and eB=/=0 with T→∞ :

$$-3\chi_{11}^{BS}/\chi_2^S = 3\chi_{11}^{QS}/\chi_2^S = 1$$

HTD, S.-T. Li, Q. Shi and X.-D. Wang, 2104.06843

Electric charge-strangeness correlations



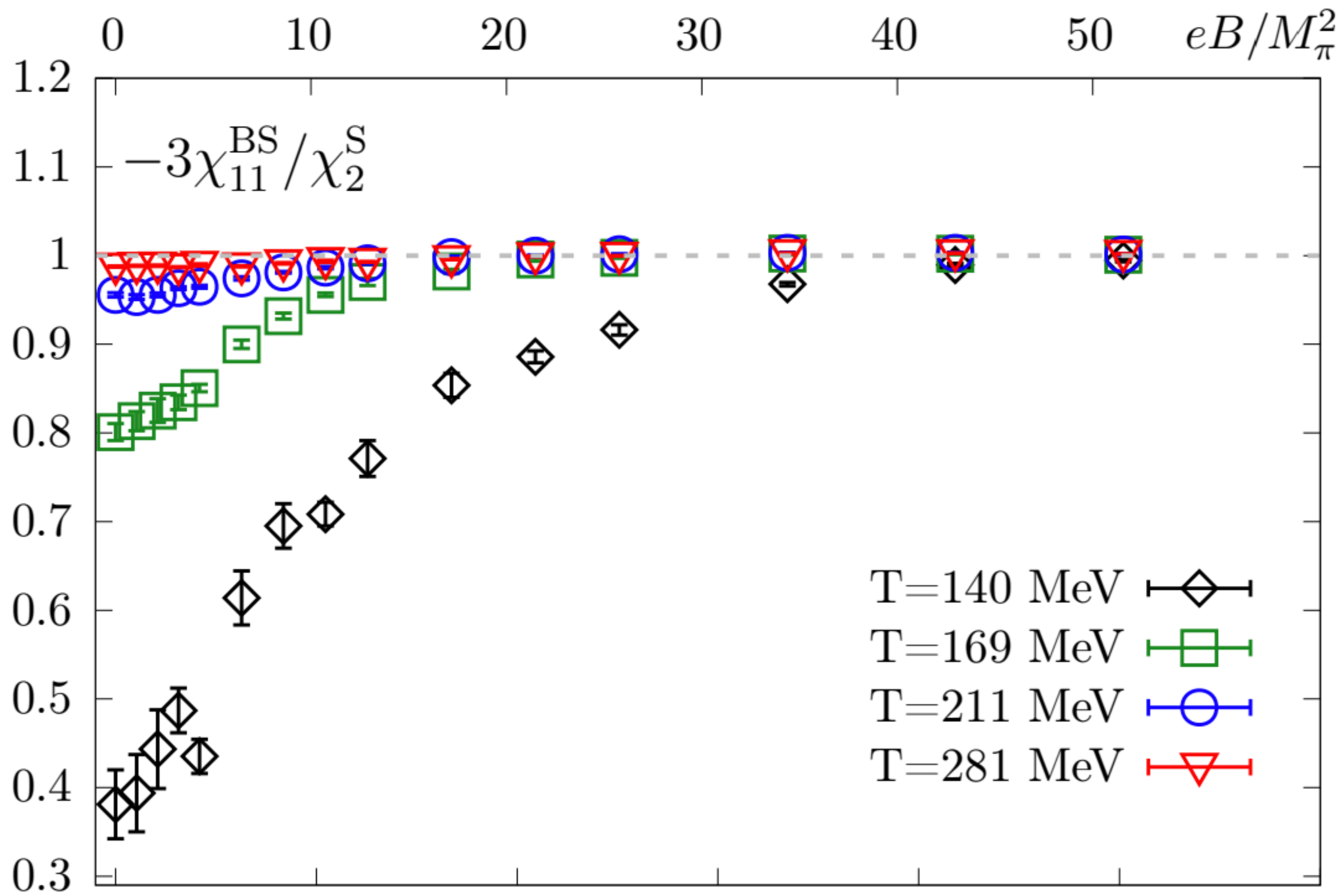
At both eB=0 and eB=/=0 with T→∞:

$$-3\chi_{11}^{BS}/\chi_2^S = 3\chi_{11}^{QS}/\chi_2^S = 1$$

HTD, S.-T. Li, Q. Shi and X.-D. Wang, 2104.06843

LatticeQCD meets experiment

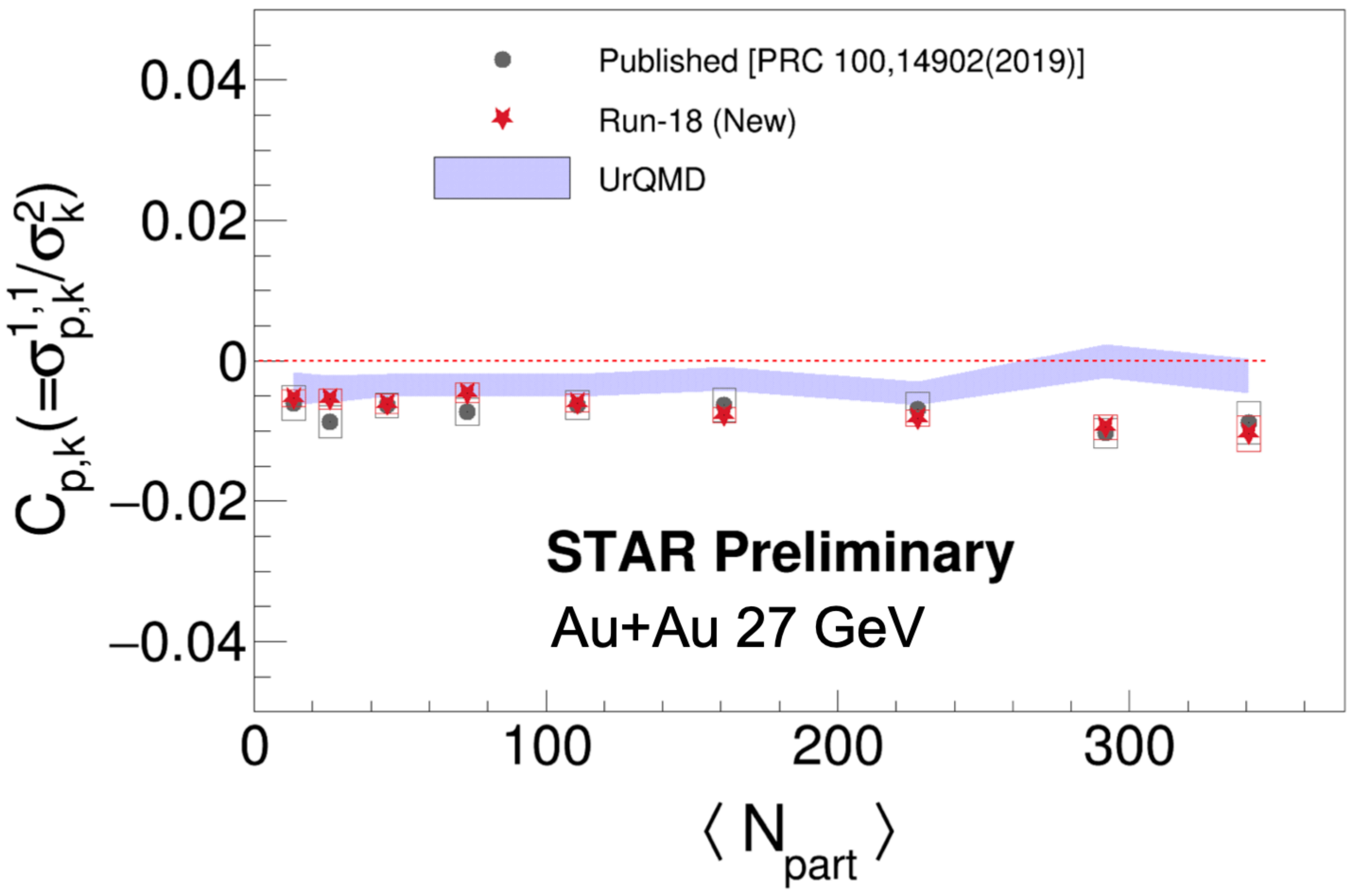
LatticeQCD



$$T_{pc}(eB=0) \approx 170 \text{ MeV}$$

$$M_\pi(eB=0) \approx 220 \text{ MeV}$$

Proxies of χ_{11}^{BS}/χ_2^S



Changfeng Li, QPT 2021

STAR, *Phys.Rev.C* 100 (2019) 1, 014902

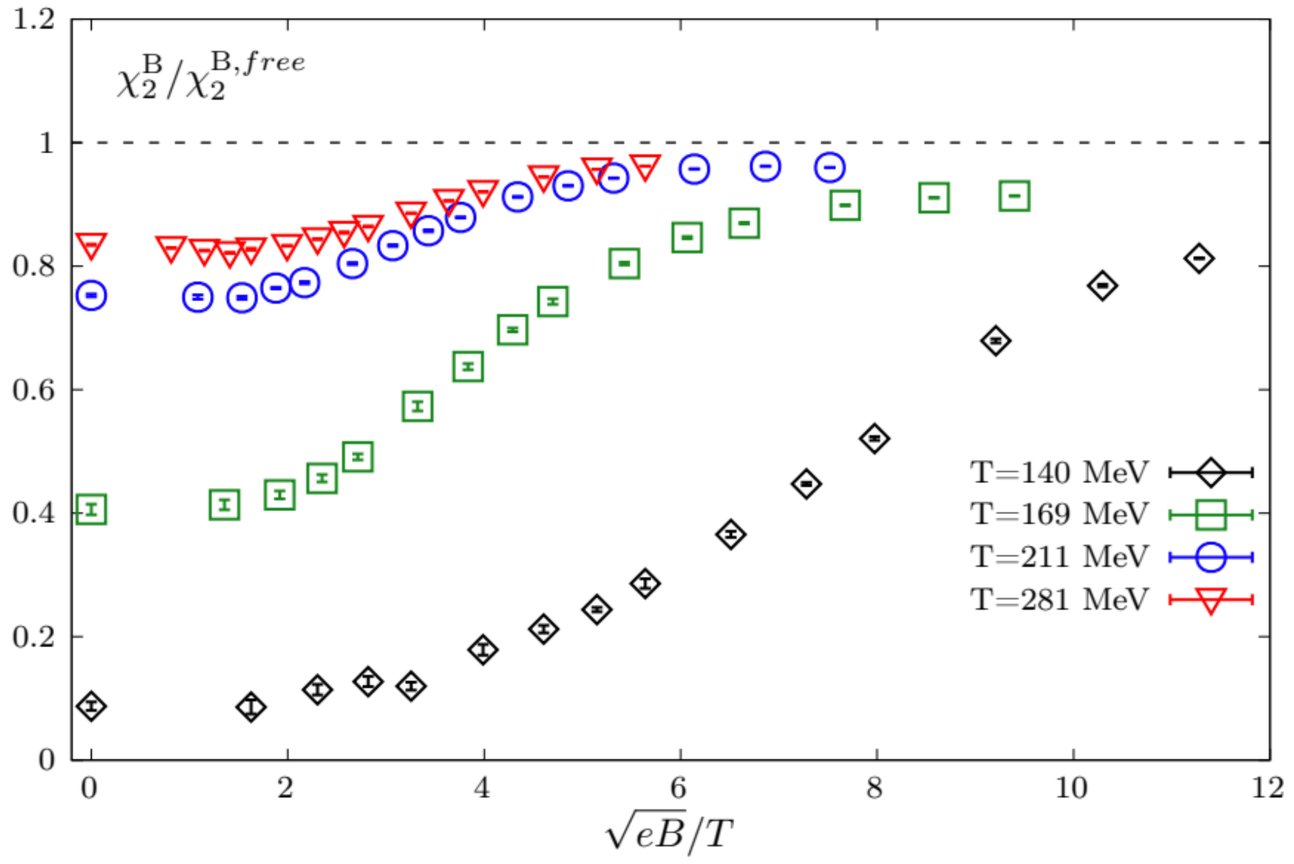
Isospin symmetry breaking at $eB\neq0$

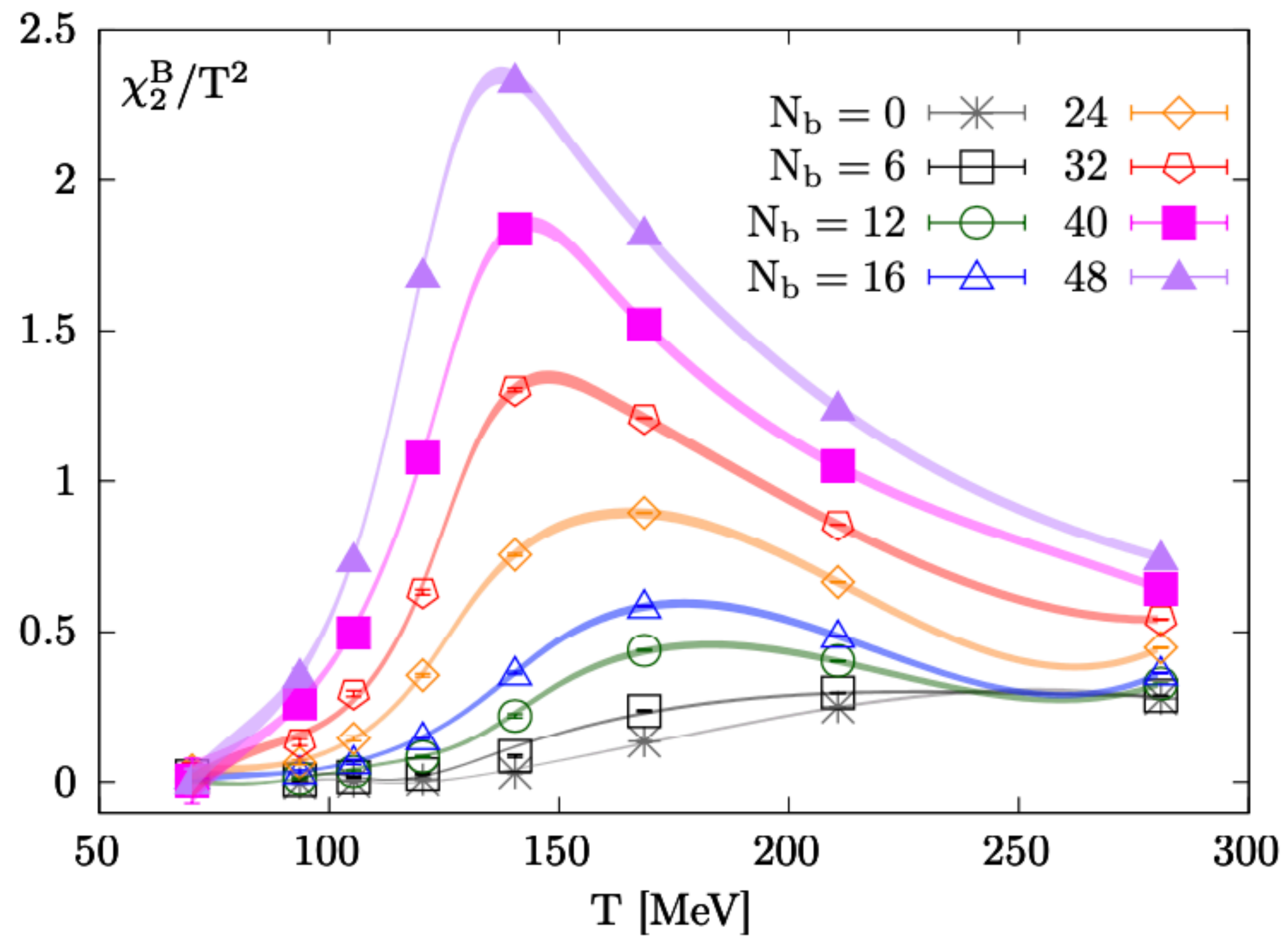
At eB=0

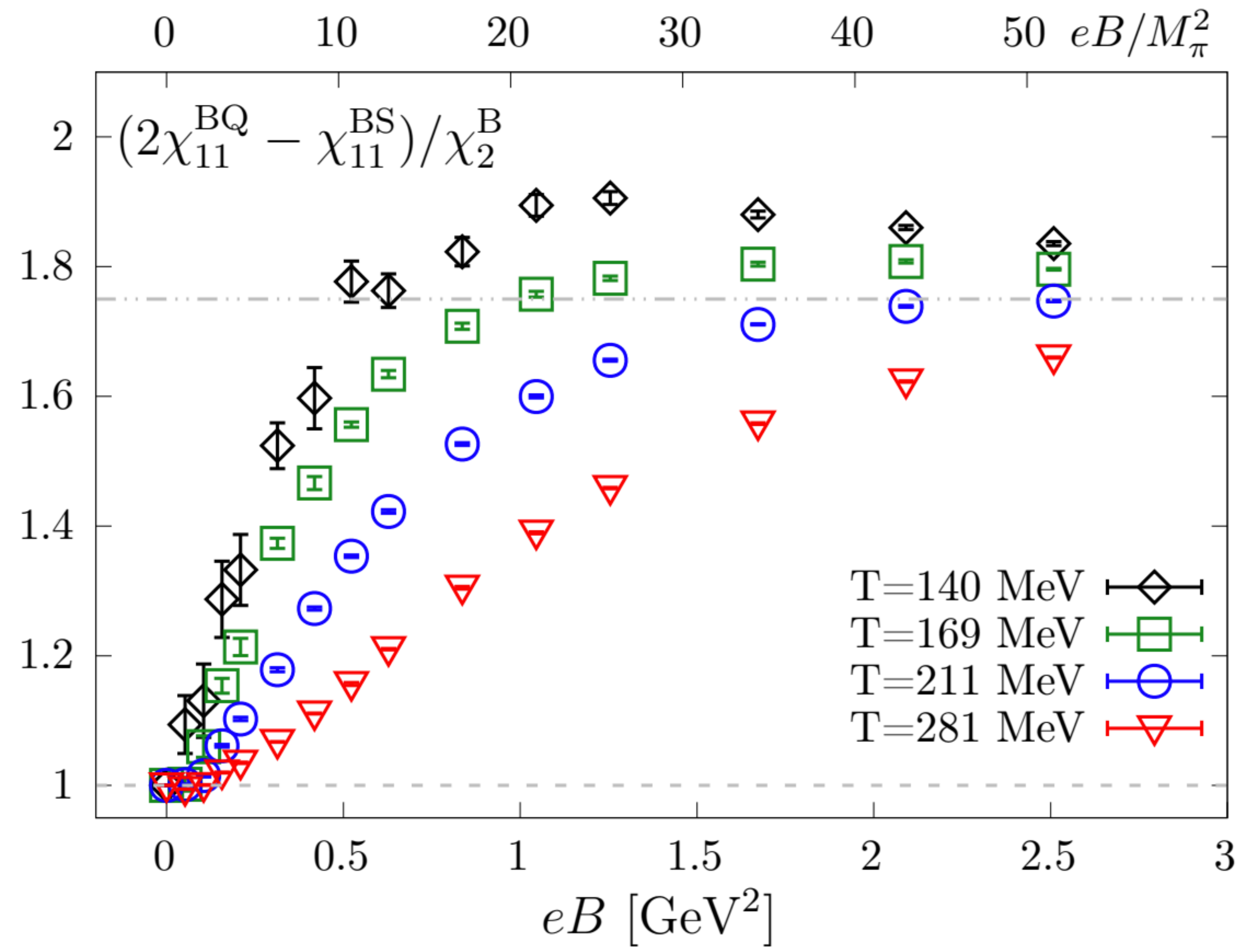
$$2\chi_{11}^{QS}-\chi_{11}^{BS}=\chi_2^S,$$
$$2\chi_{11}^{BQ}-\chi_{11}^{BS}=\chi_2^B.$$

Summary & Outlook

- The 2nd order fluctuations and correlations of B,Q & S are strongly affected by *eB*
- Could be useful to 1) detect the existence of a magnetic field in HIC; 2) analogy to study the QCD critical end point in the T-muB plane







- Lattice QCD computation at the physical pion mass of M_π=135 MeV is ongoing HTD, S.-T. Li, J.-H. Liu, X.-D. Wang in preparation