Proton Cumulants, Correlation Functions, and Hydrodynamics

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RHIC Beam Energy Scan and Beyond

August 17, 2021

- Proton number cumulants at RHIC-BES from hydrodynamics (non-critical scenario)
- Factorial cumulants vs ordinary cumulants

V.V., C. Shen, V. Koch, arXiv:2107.00163

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QCD phase diagram with heavy-ion collisions

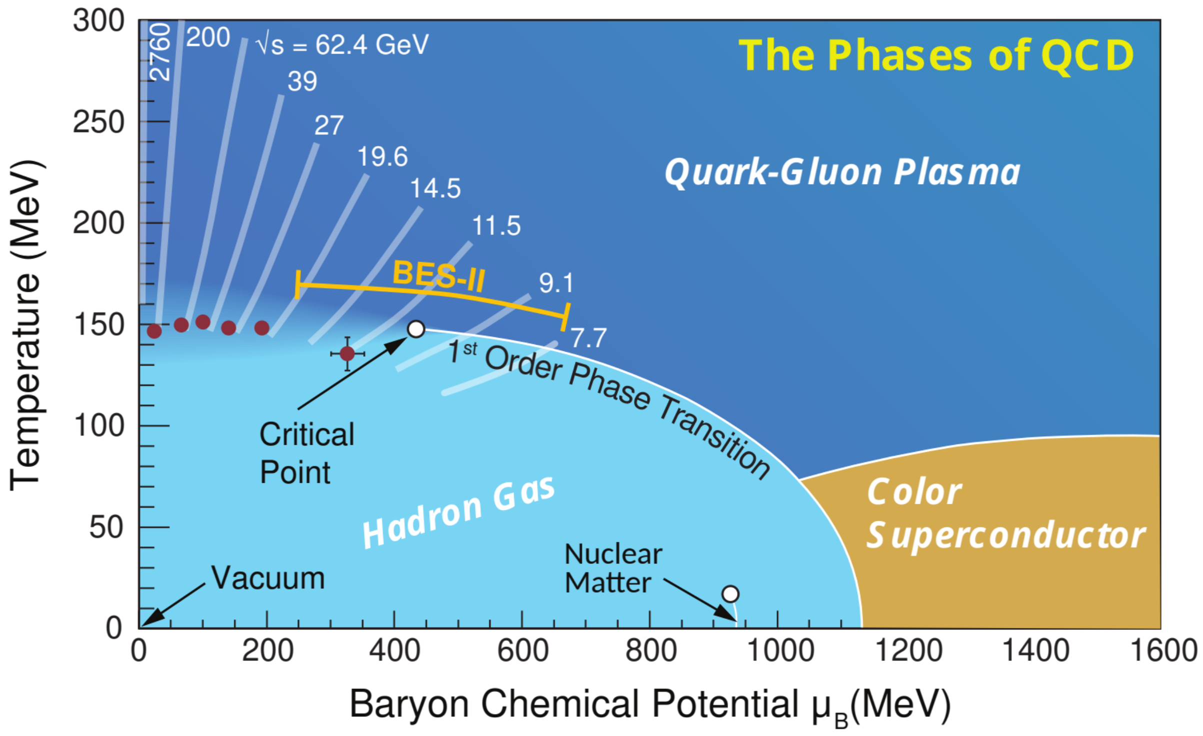
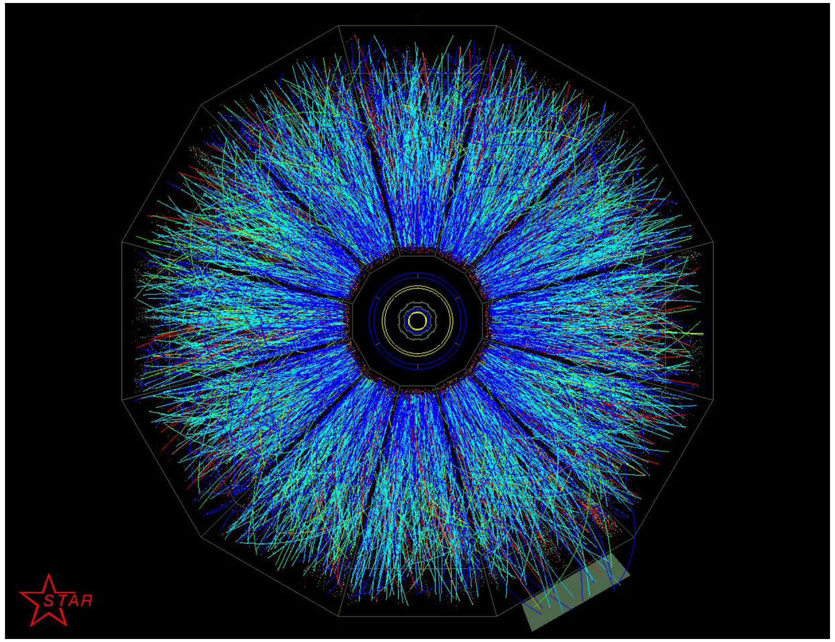


Figure from Bzdak et al., Phys. Rept. '20



STAR event display

Thousands of particles created in relativistic heavy-ion collisions

↓

Apply concepts of statistical mechanics

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

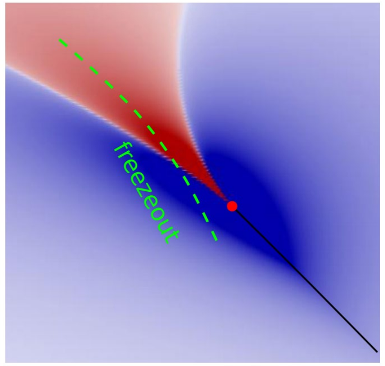
Grand partition function

$$\ln Z^{gce}(T, V, \mu) = \ln\left[\sum_N e^{\mu N/T} Z^{ce}(T, V, N)\right]$$

$$\kappa_n \propto \frac{\partial^n (\ln Z^{gce})}{\partial \mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- QCD critical point
- Test of (lattice) QCD at $\mu_B \approx 0$
- Freeze-out from fluctuations



M. Stephanov, PRL '09
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

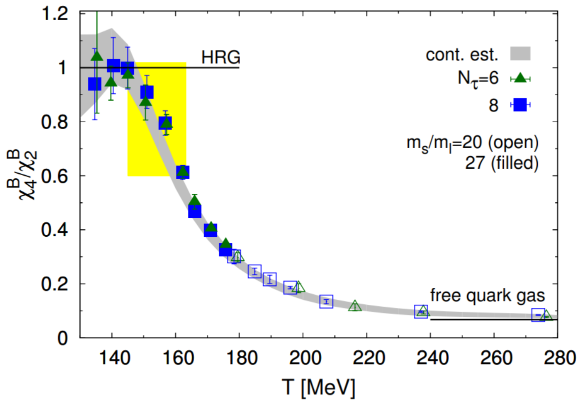
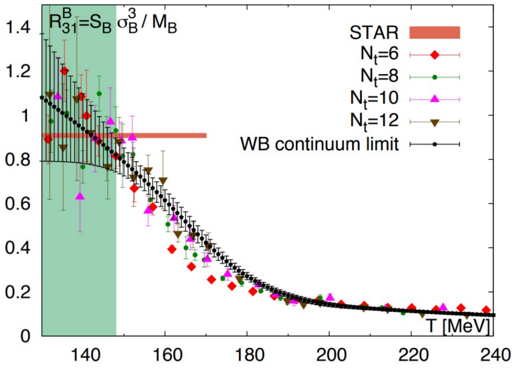


Figure from Bazavov et al. PRD 95, 054504 (2017)
Probed by LHC and top RHIC



Borsanyi et al. PRL 113, 052301 (2014)
Bazavov et al. PRL 109, 192302 (2012)
...

Theory vs experiment: Caveats

- **accuracy of the grand-canonical ensemble (global conservation laws)**
  - **subensemble acceptance method (SAM)**

  VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020); JHEP 089(2020); arXiv:2106.13775

- **coordinate vs momentum space**

  Ling, Stephanov, PRC 93, 034915 (2016); Ohnishi, Kitazawa, Asakawa, PRC 94, 044905 (2016)

- **proxy observables in experiment (net-proton, net-kaon) vs conserved charges in QCD (net-baryon, net-strangeness)**

  Kitazawa, Asakawa, PRC 85, 021901 (2012); VV, Jiang, Gorenstein, Stoecker, PRC 98, 024910 (2018)

- volume fluctuations

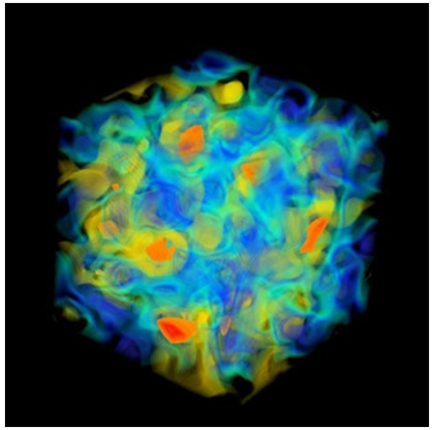
  Gorenstein, Gazdzicki, PRC 84, 014904 (2011); Skokov, Friman, Redlich, PRC 88, 034911 (2013)
  X. Luo, J. Xu, B. Mohanty, JPG 40, 105104 (2013); Braun-Munzinger, Rustamov, Stachel, NPA 960, 114 (2017)

- hadronic phase

  Steinheimer, VV, Aichelin, Bleicher, Stoecker, PLB 776, 32 (2018)

- non-equilibrium (memory) effects

  Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)



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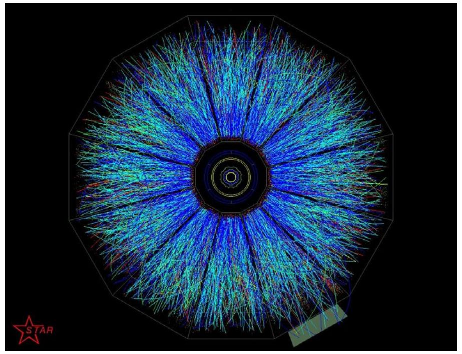
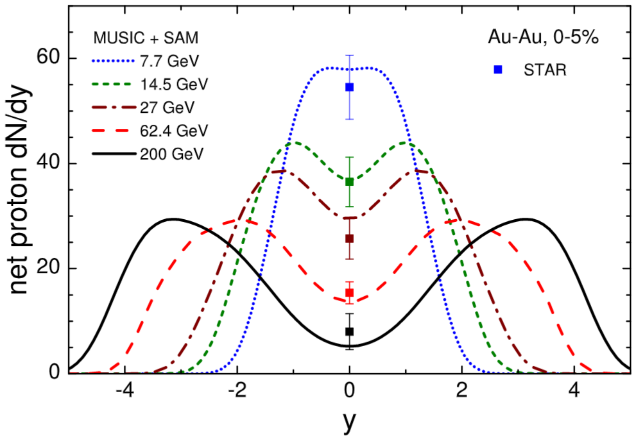


STAR event display

Need for ***dynamical description***

Hydrodynamic description within the non-critical scenario

VV, C. Shen, V. Koch, arXiv:2107.00163

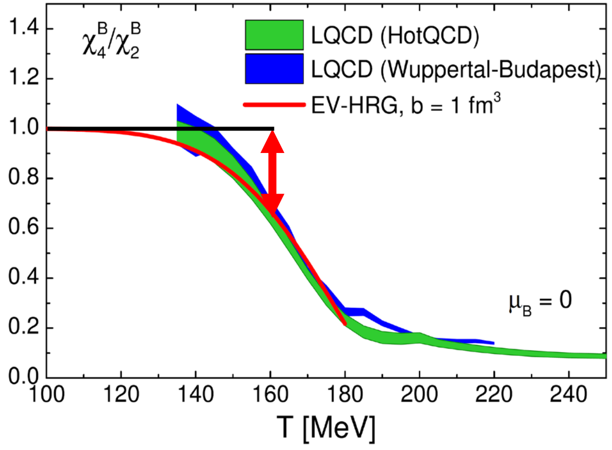


- Collision geometry based 3D initial state [Shen, Alzhrani, PRC '20]
  - Constrained to net proton distributions

- Viscous hydrodynamics evolution – MUSIC-3.0
  - Energy-momentum and baryon number conservation
  - NEOS-BQS crossover equation of state [Monnai, Schenke, Shen, PRC '19]
  - Shear viscosity via IS-type equation

- Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3

$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu \, \frac{d_j \, \lambda_j^{\rm ev}(x)}{(2\pi)^3} \, \exp\left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)}\right].$$



- Particlization includes QCD-based baryon number distribution
  - Here incorporated via baryon excluded volume

[VV, Pasztor, Fodor, Katz, Stoecker, PLB 775, 71 (2017)]

Calculating cumulants at particlization

- Strategy:
  1. Calculate proton cumulants in the experimental acceptance in the grand-canonical limit
  2. Apply correction for the exact global baryon number conservation

First step:

- Sum contributions from each hypersurface element x_i
  - Cumulants of joint (anti)proton/(anti)baryon distribution
  - Assumes small correlation length $\xi \to 0$

$$\kappa_{n,m}^{B^\pm,p^\pm,\mathrm{gce}}(\Delta p_{\mathrm{acc}}) = \sum_{i\in\sigma} \delta\kappa_{n,m}^{B^\pm,p^\pm,\mathrm{gce}}(x_i;\Delta p_{\mathrm{acc}})$$

- To compute each contribution
  - GCE susceptibilities $\chi^{B^\pm}(x_i)$ define the distribution of the emitted (anti)baryons
  - Each baryon ends up in acceptance Δp_{acc} with binomial probability
  - Each baryon is a proton with probability $q(x_i) = \langle N_p(x_i)\rangle / \langle N_B(x_i)\rangle$

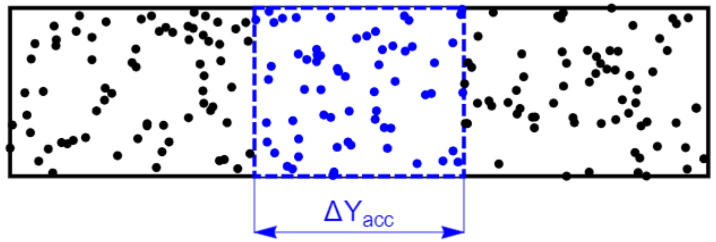
$$p_{\mathrm{acc}}(x_i;\Delta p_{\mathrm{acc}}) = \frac{\int_{p\in\Delta p_{\mathrm{acc}}} \frac{d^3p}{\omega_p}\delta\sigma_\mu(x_i)\, p^\mu\, f[u^\mu(x_i)p_\mu;\, T(x_i), \mu_j(x_i)]}{\int \frac{d^3p}{\omega_p}\delta\sigma_\mu(x_i)\, p^\mu\, f[u^\mu(x_i)p_\mu;\, T(x_i), \mu_j(x_i)]}.$$

[Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]

*Cross-check with the Monte Carlo results of a canonical Cooper-Frye sampler from https://github.com/vlvovch/CooperFryeSampler

Correcting for baryon number conservation

$$P_1^{\rm ce}(B_1) \propto \sum_{B_1,B_2} P_1^{\rm gce}(B_1) P_2^{\rm gce}(B_2) \times \delta_{B,B_1+B_2}$$



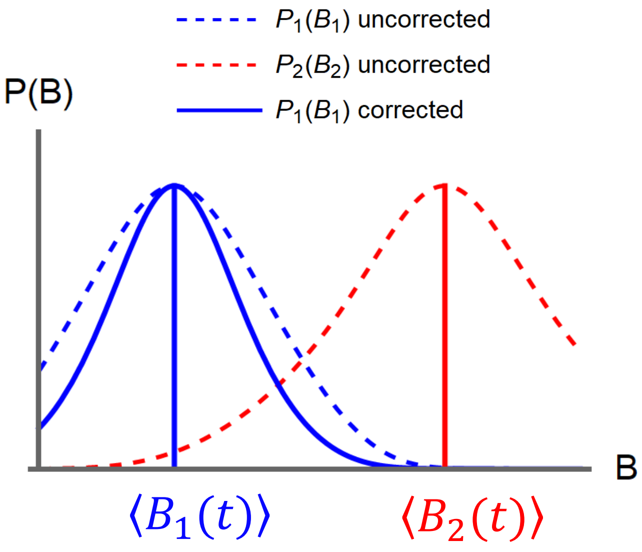
- Subensemble acceptance method (SAM)
  - Corrects *any* equation of state for global BQS-charge conservation
  - Canonical ensemble cumulants in terms of grand-canonical ones

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, Phys. Lett. B 811, 135868 (2020) [arXiv:2003.13905]

VV, Poberezhnyuk, Koch, JHEP 10, 089 (2020) [arXiv:2007.03850]

- SAM-2.0 VV, arXiv:2106.13775
  - Non-conserved quantities (e.g. proton number)
  - Spatially inhomogeneous systems
  - Momentum space
  - Maps "grand-canonical" cumulants inside and outside the acceptance to the "canonical" cumulants inside the acceptance:*

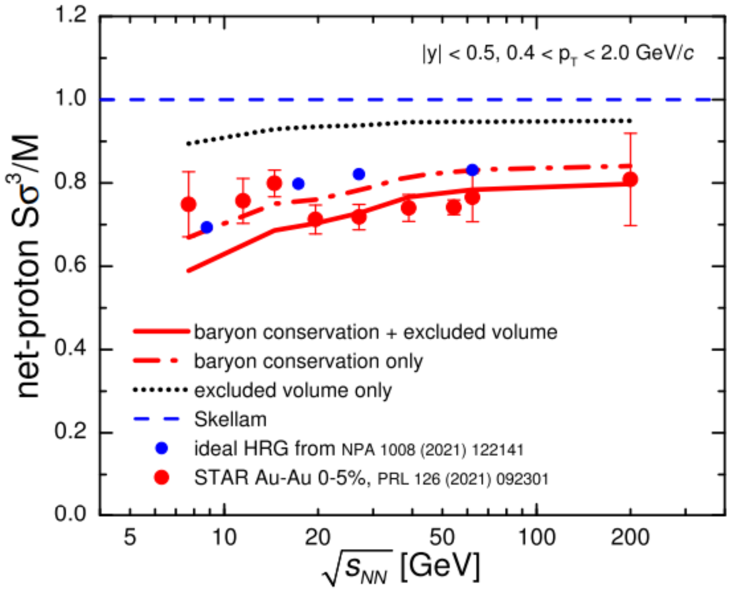
$$\kappa_{p,B}^{\rm in,ce} = {\rm SAM}\left[\kappa_{p,B}^{\rm in,gce}, \kappa_{p,B}^{\rm out,gce}\right]$$



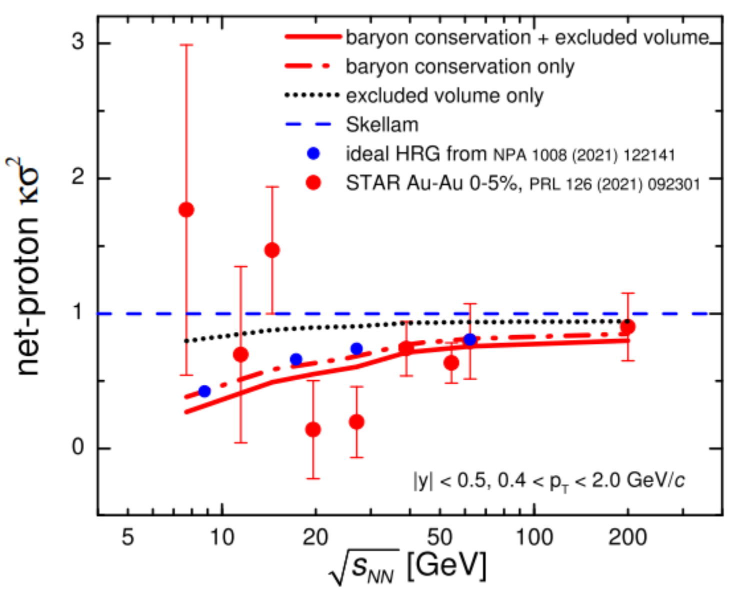
*Explicit expressions for any cumulant available via a Mathematica notebook at https://github.com/vlvovch/SAM

Net proton cumulant ratios

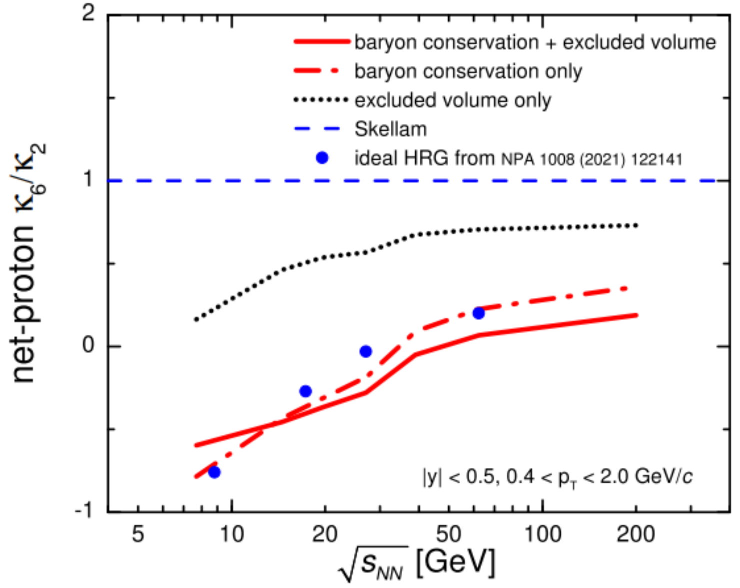
κ_3/κ_1



κ_4/κ_2



κ_6/κ_2



- Both the baryon conservation and repulsion needed to describe data at $\sqrt{s_{NN}} \geq 20$ GeV quantitatively
- Effect from baryon conservation is larger than from repulsion
- Canonical ideal HRG limit is consistent with the data-driven study of [Braun-Munzinger et al., NPA 1008 (2021) 122141]
- κ_6/κ_2 turns negative at $\sqrt{s_{NN}}$~50 GeV

Cumulants vs Correlation Functions

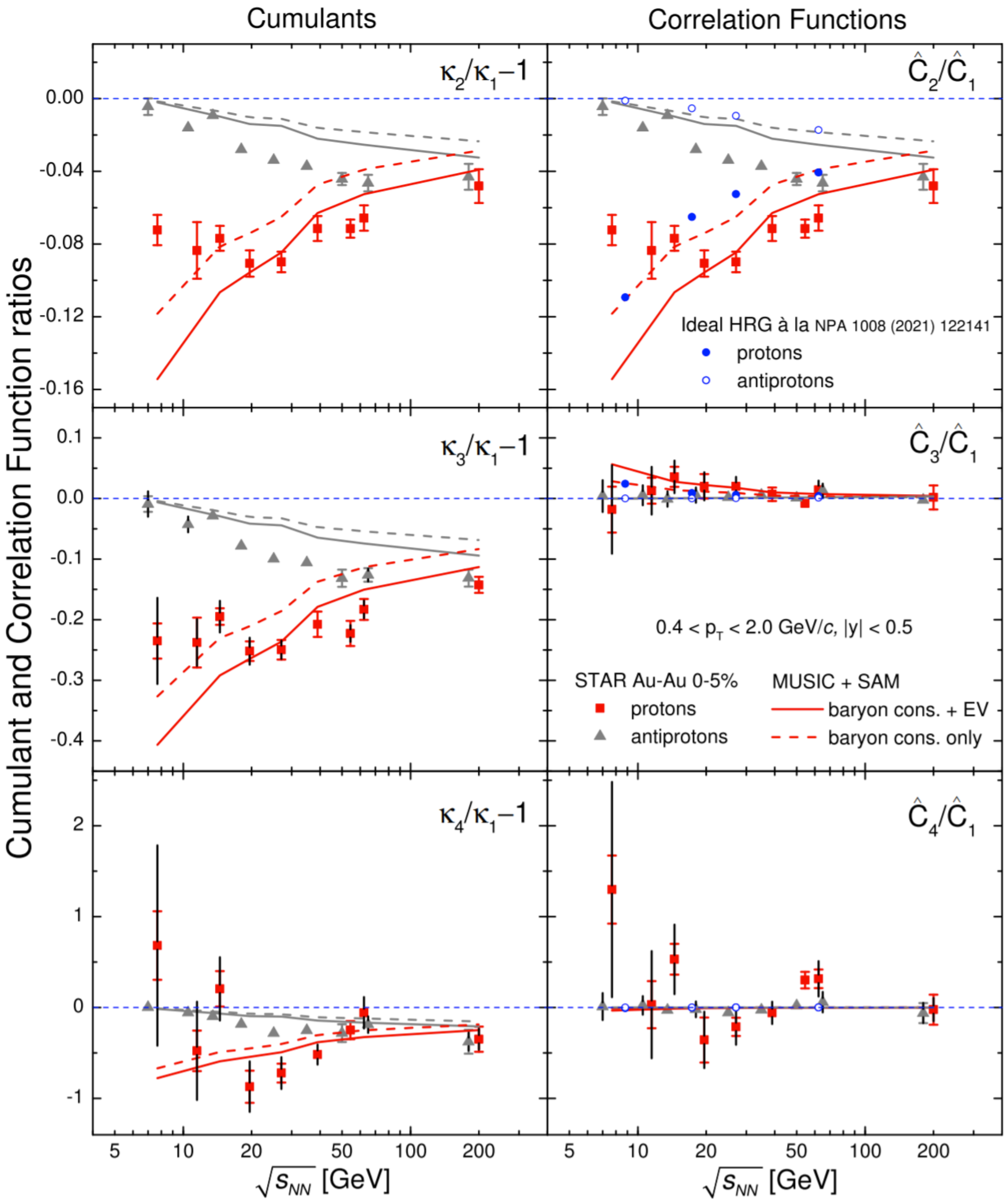
- Analyze genuine multi-particle correlations via **factorial cumulants** [Bzdak, Koch, Strodthoff, PRC '17]

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \qquad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

$$\hat{C}_n^{\text{cons}} \propto \alpha^n, \qquad \hat{C}_n^{\text{EV}} \propto b^n$$

[Bzdak, Koch, Skokov, EPJC '17] [VV et al, PLB '17]

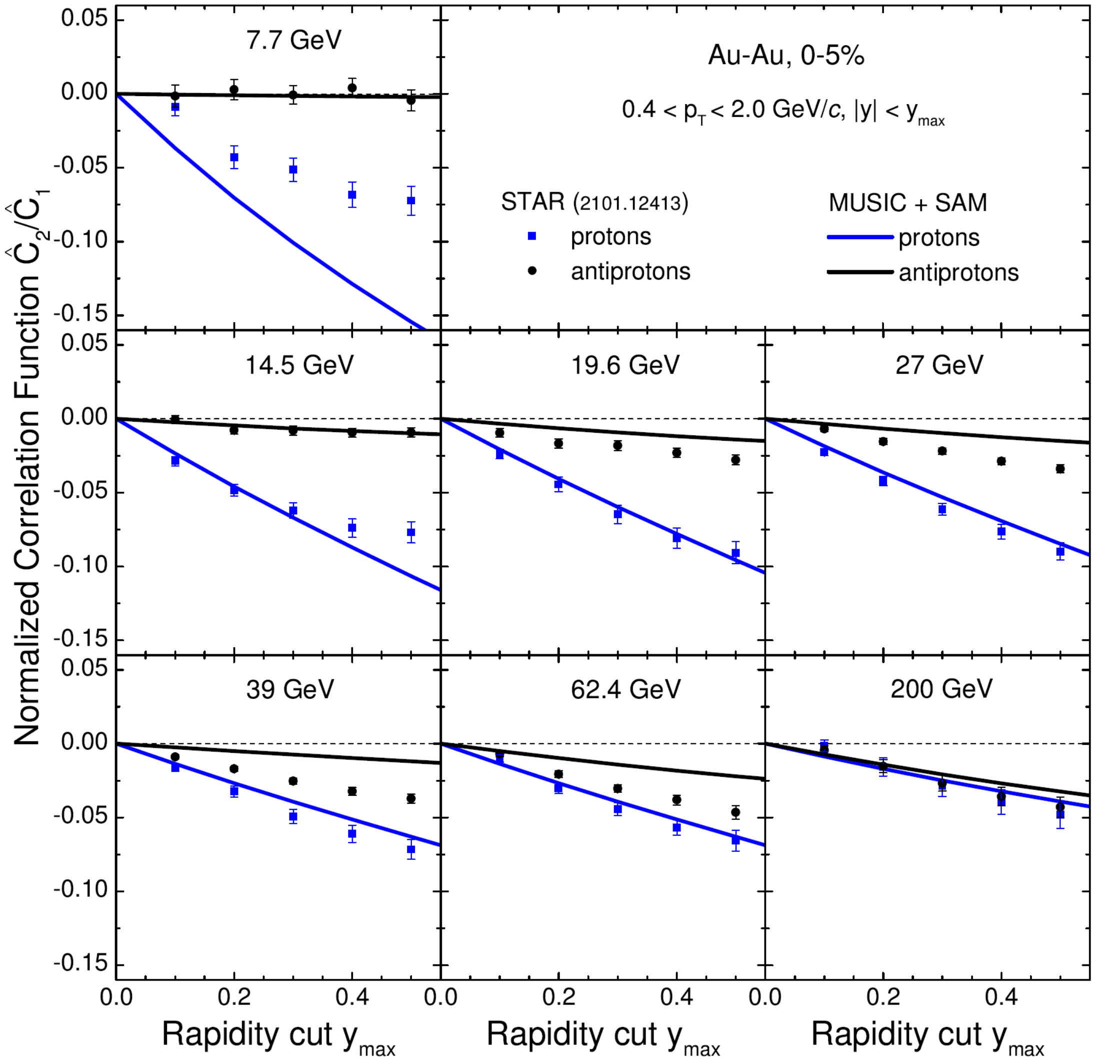
- Three- and four-particle correlations are small
  - Small positive $\hat{C}_3/\hat{C}_1$ in the data is explained by baryon conservation + excluded volume
  - Strong multi-particle correlations would be expected near the critical point [Ling, Stephanov, 1512.09125]
- Two-particle correlations are negative
  - Protons at $\sqrt{s_{NN}} \leq 14.5$ GeV overestimated
  - Antiprotons at $19.6 \leq \sqrt{s_{NN}} \leq 62.4$ GeV underestimated



*We use the notation for (factorial) cumulants from Bzdak et al., Phys. Rept. '20. This is different from STAR's 2101.12413 where it is reversed

Acceptance dependence of two-particle correlations

- Changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV?



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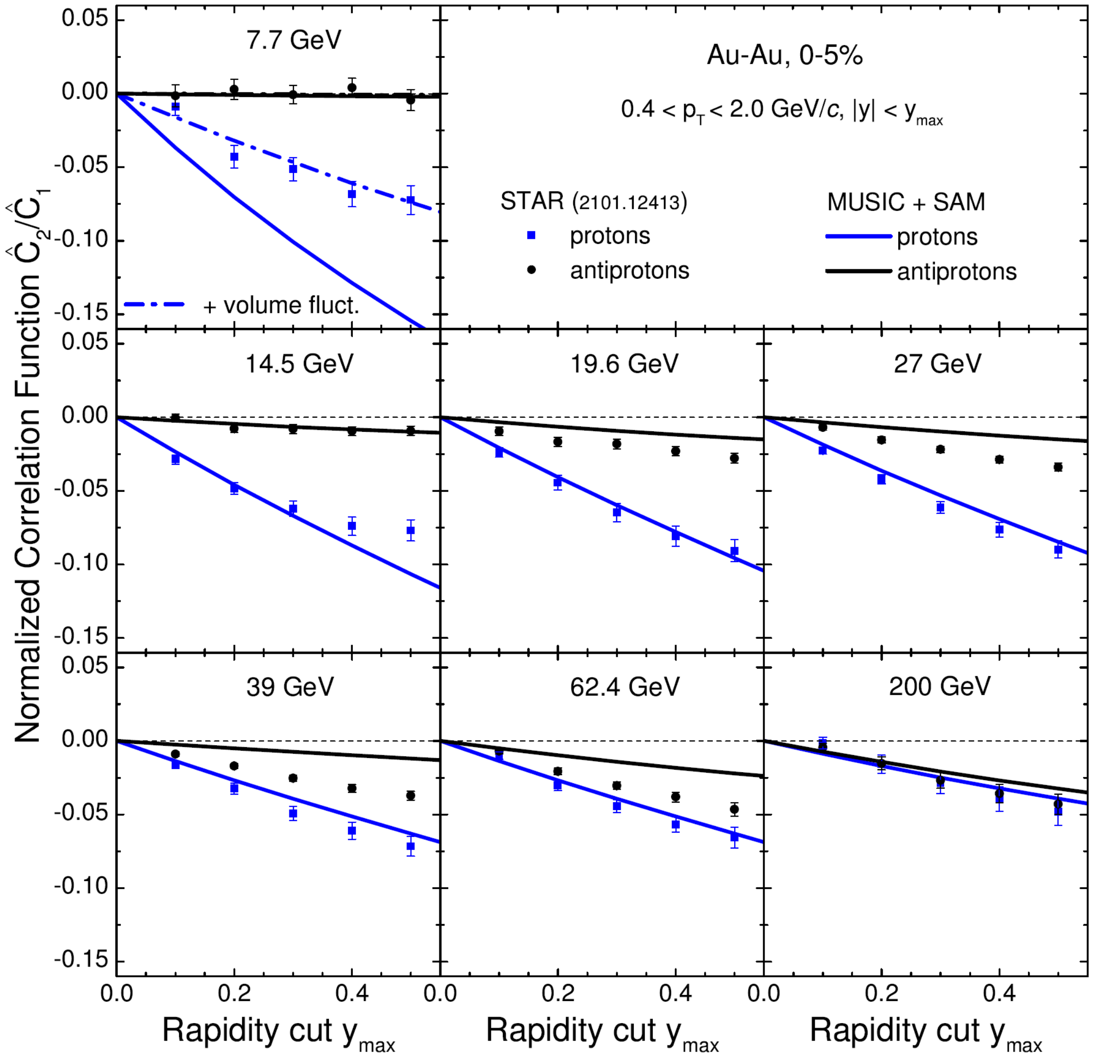
- **Volume fluctuations?** [Skokov, Friman, Redlich, PRC ‘13]
  - $C_2/C_1 \mathrel{+}= C_1 * v_2$



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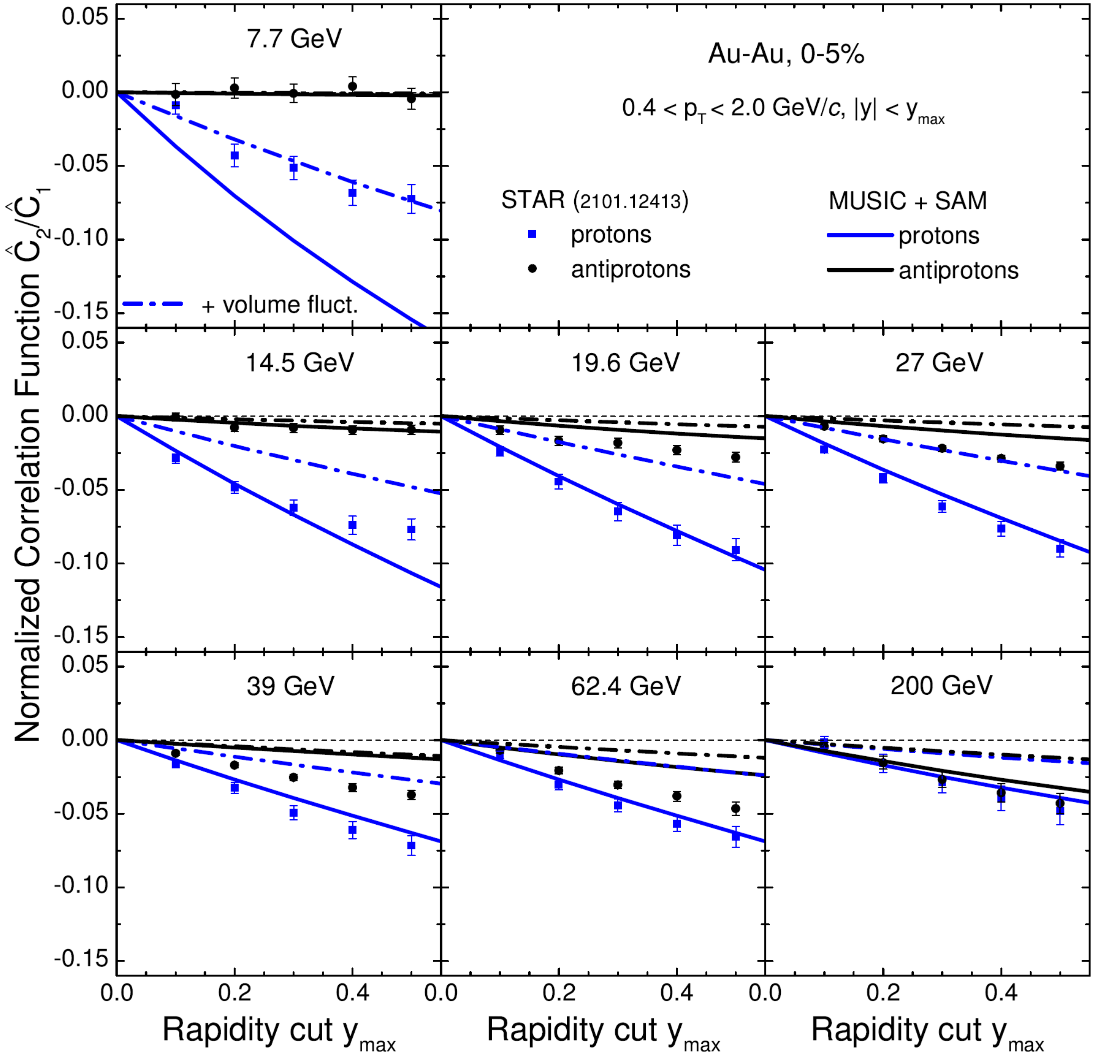

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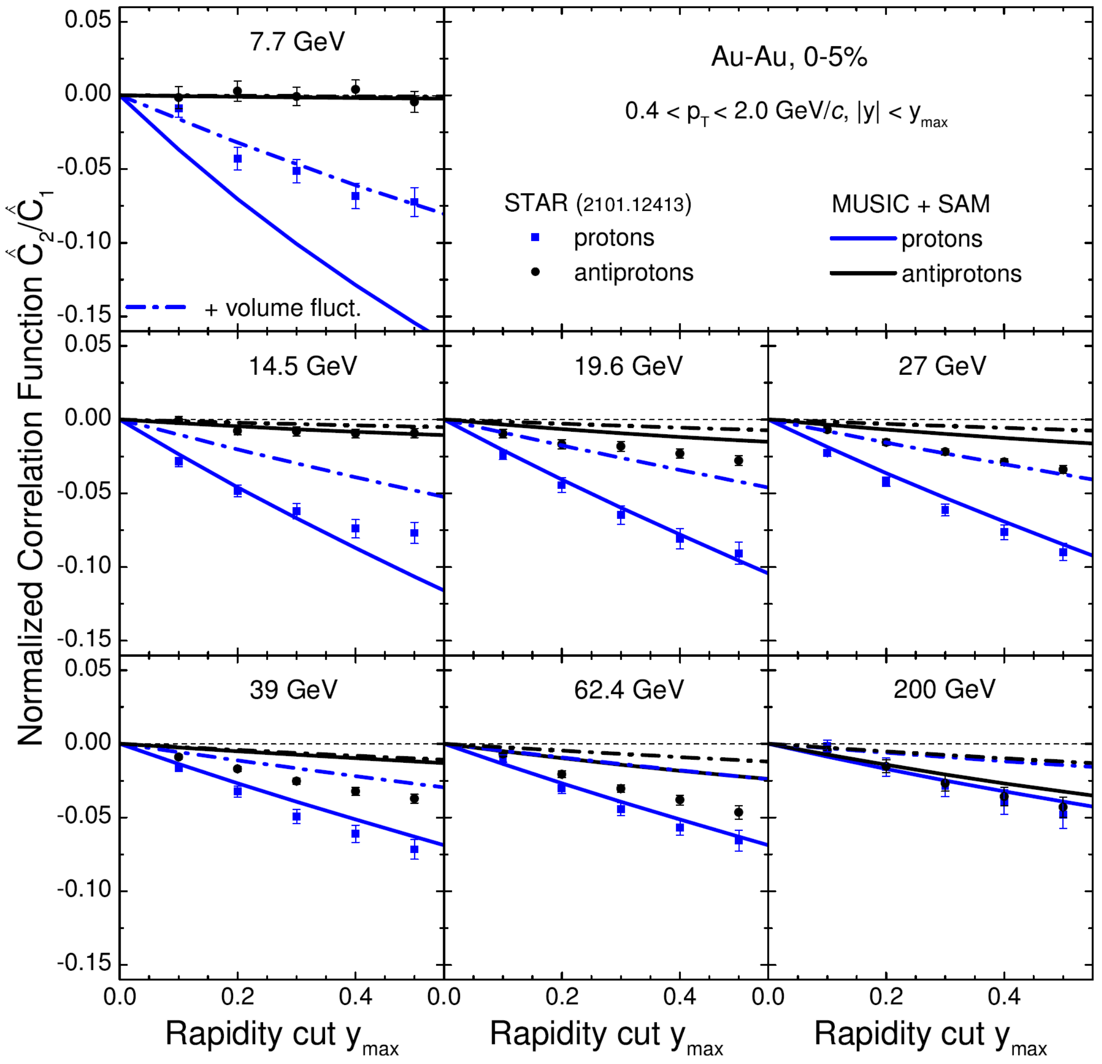
- **Exact electric charge conservation?**
  - Worsens the agreement at $\sqrt{s_{NN}} \leq 14.5$, higher energies virtually unaffected



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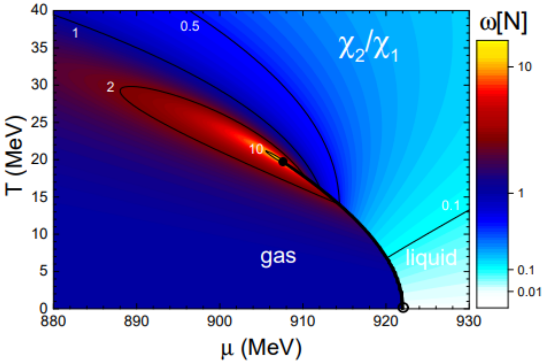
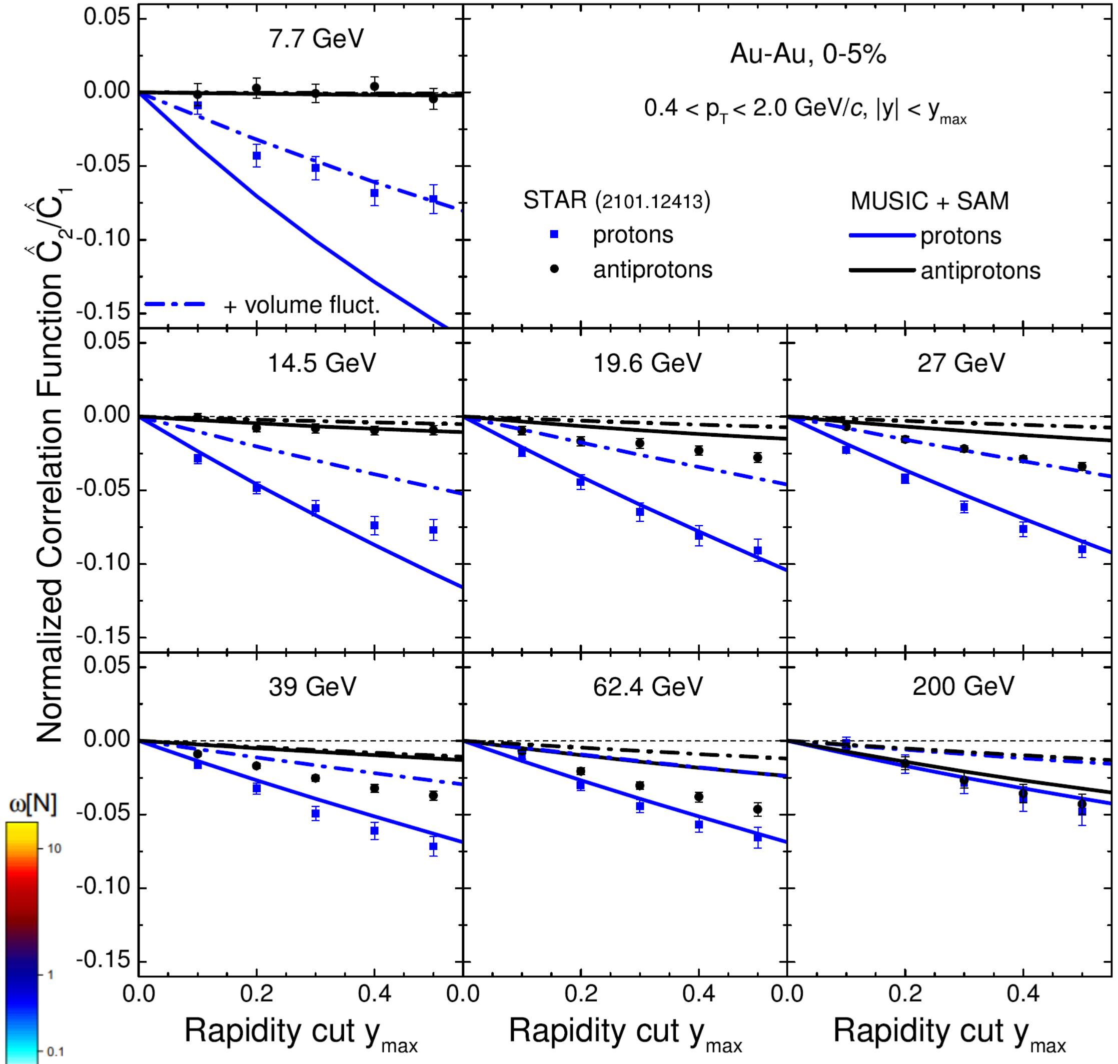
- **Attractive interactions?**
  - Could work if baryon repulsion turns into attraction in the high-μ_B regime
  - **Critical point?**

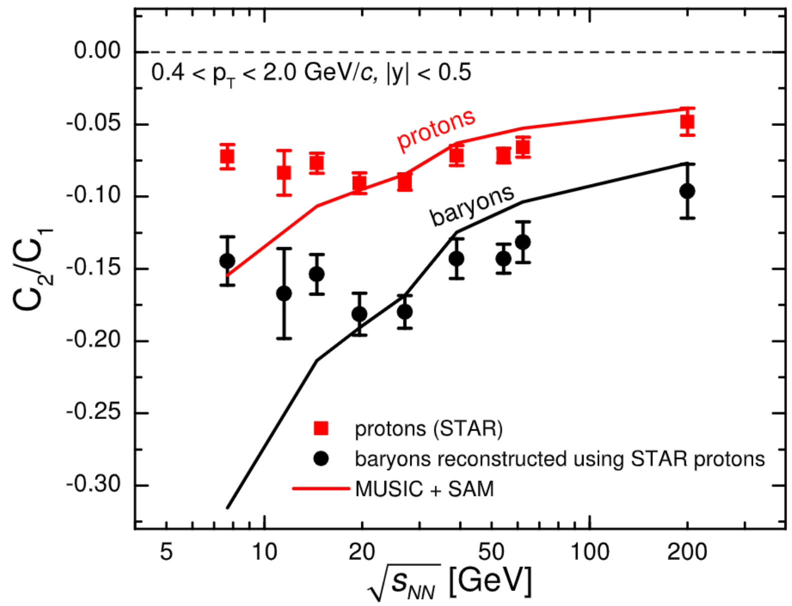




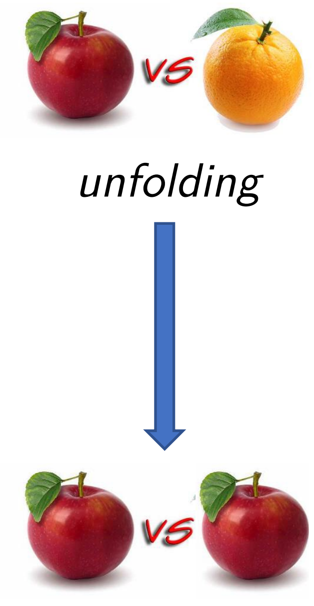
Net baryon vs net proton

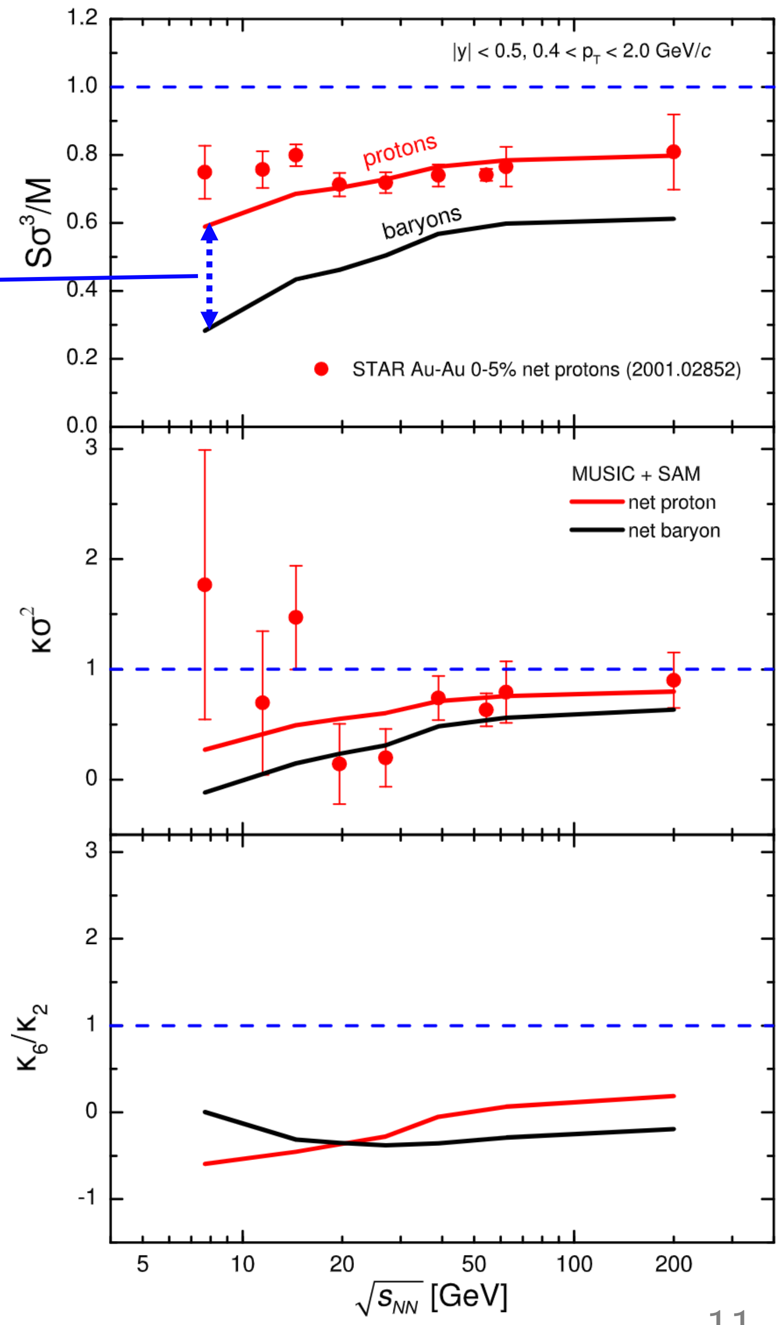
- **net baryon $\neq$ net proton**

- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
  - Amounts to an additional "efficiency correction" and requires the use of joint factorial moments, only experiment can do it model-independently



$$\frac{\hat{C}_2^B}{\hat{C}_1^B} \approx 2\,\frac{\hat{C}_2^p}{\hat{C}_1^p}$$

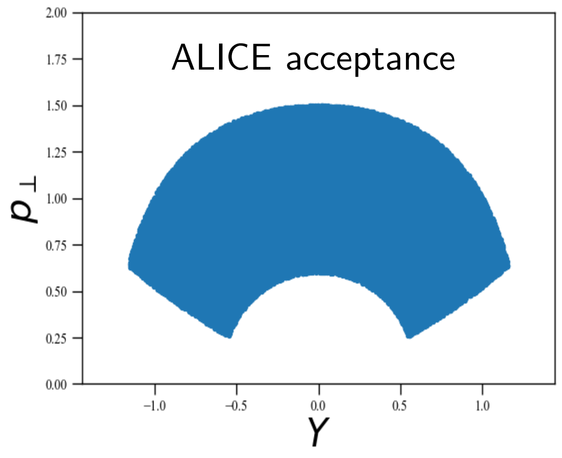




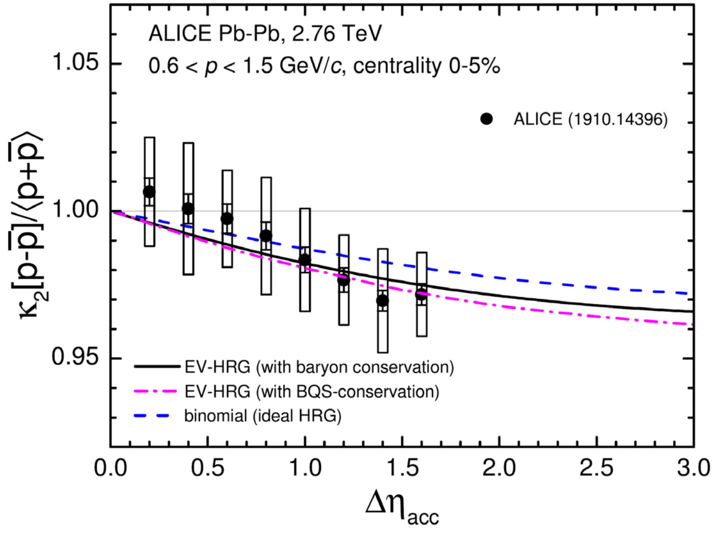
Net-particle fluctuations at the LHC

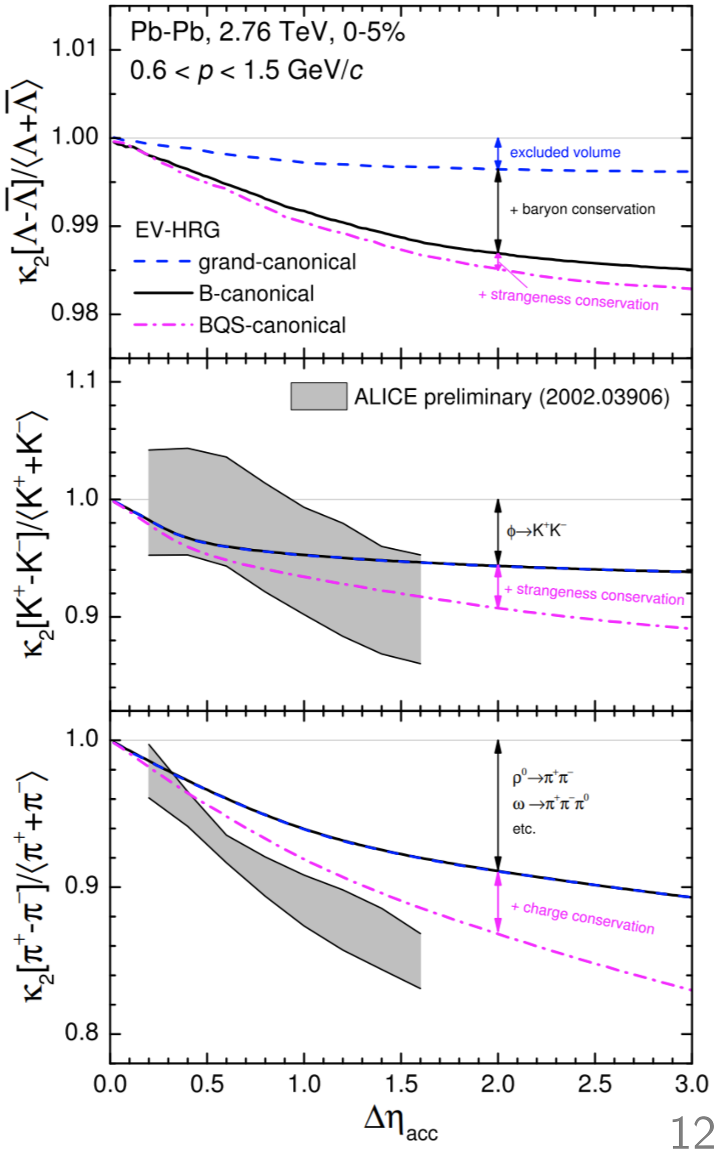
VV, Koch, Phys. Rev. C 103, 044903 (2021)

- Net protons described within errors but not sensitive to the equation of state for the present experimental acceptance
- Large effect from resonance decays for lighter particles + conservation of electric charge/strangeness
- Future measurements will require larger acceptance



$0.6 < p < 1.5$ GeV$/c$, $\Delta\eta_{acc} = 1.6$



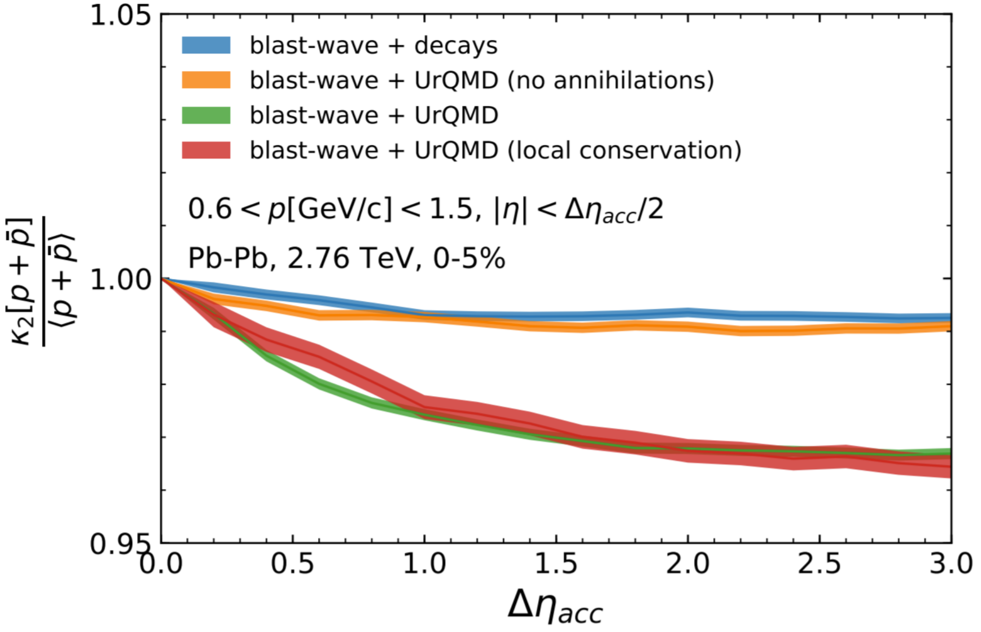


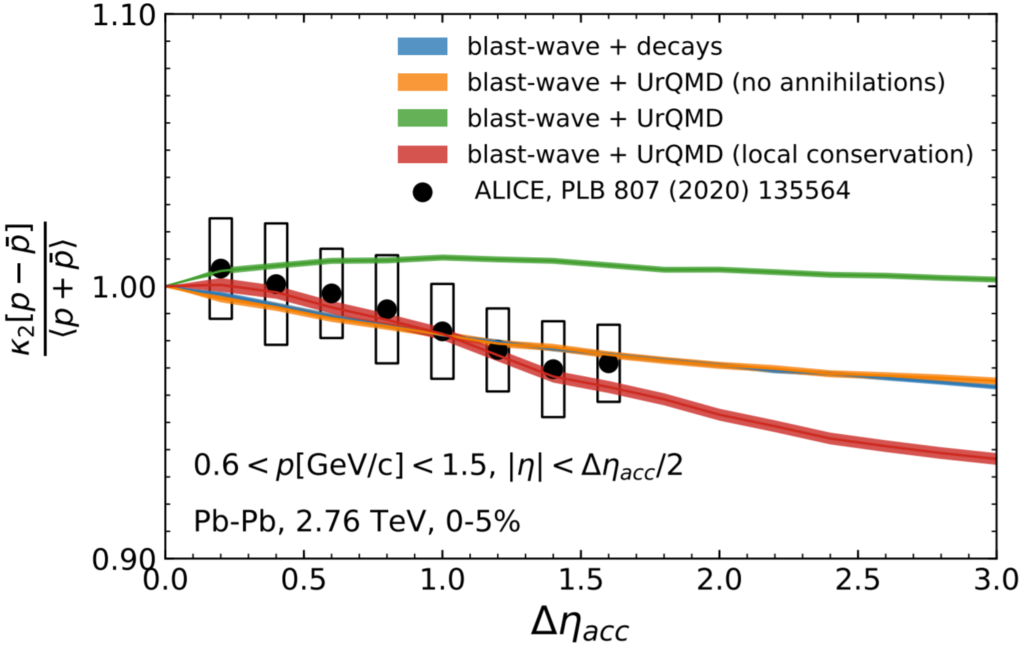
Effects of baryon annihilation and local conservation

O. Savchuk, V.V., V. Koch, J. Steinheimer, H. Stoecker, arXiv:2106.08239

Baryon annihilation $B\bar{B} \to n\pi$ in afterburners (UrQMD, SMASH) suppresses baryon yields

$\langle p + \bar{p} \rangle \searrow$ → $\dfrac{\kappa_2[p - \bar{p}]}{\langle p + \bar{p} \rangle} \nearrow$

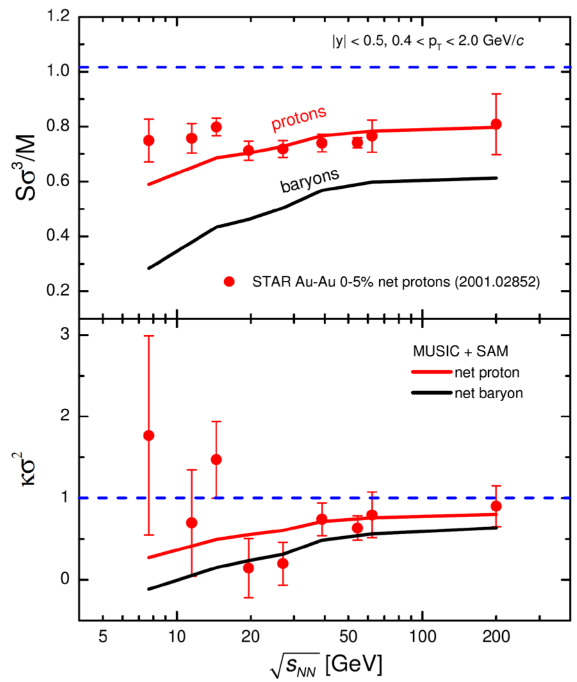


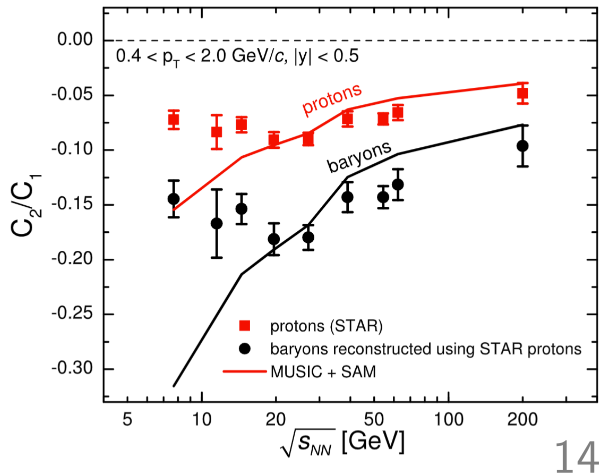


- ALICE data requires local baryon conservation across $\Delta y \sim \pm 1.5$ with UrQMD annihilations
- Local conservation and $B\bar{B}$ annihilation can be constrained from data through combined analysis of $\kappa_2[p - \bar{p}]$ and $\kappa_2[p + \bar{p}]$

Summary

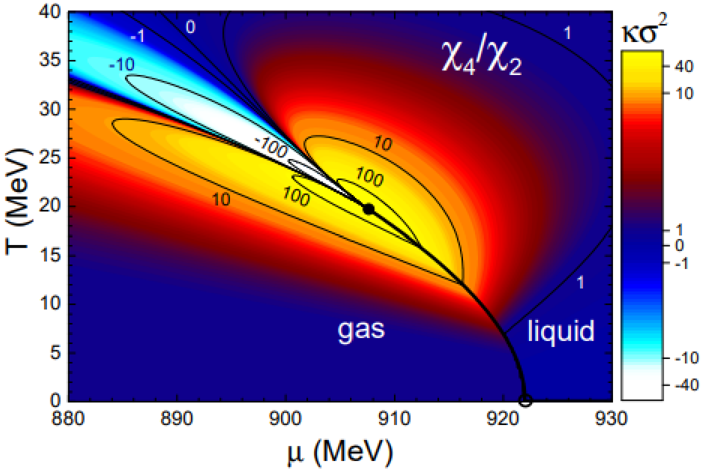
- (Net-)(anti-) proton cumulants calculated in a hydro description
  - momentum space acceptance instead of coordinate space
  - simultaneous effects of baryon conservation and repulsive interactions
- Quantitative analysis of Au-Au collisions at $\sqrt{s_{NN}}$=7.7-200 GeV
  - STAR protons are described quantitatively at $\sqrt{s_{NN}} \geq 20$ GeV
  - Significant difference between protons and baryons
- Factorial cumulants carry rich information
  - Small three- and four-particle correlations in absence of critical point effects
  - Possible evidence for attractive proton interactions at $\sqrt{s_{NN}} \leq 14.5$ GeV
  - No quantitative description of antiprotons at $19.6 \leq \sqrt{s_{NN}} \leq 62.4$ GeV



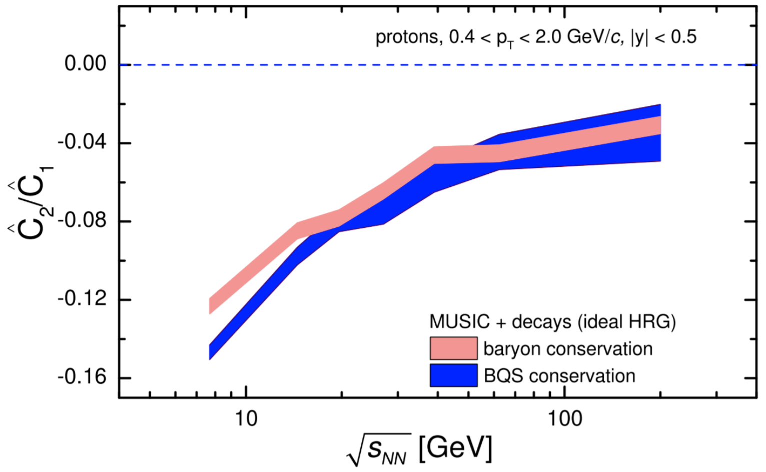


Outlook

- Incorporate the effect of critical point
  - can be achieved by using baryon number susceptibilities from equation of state with critical point



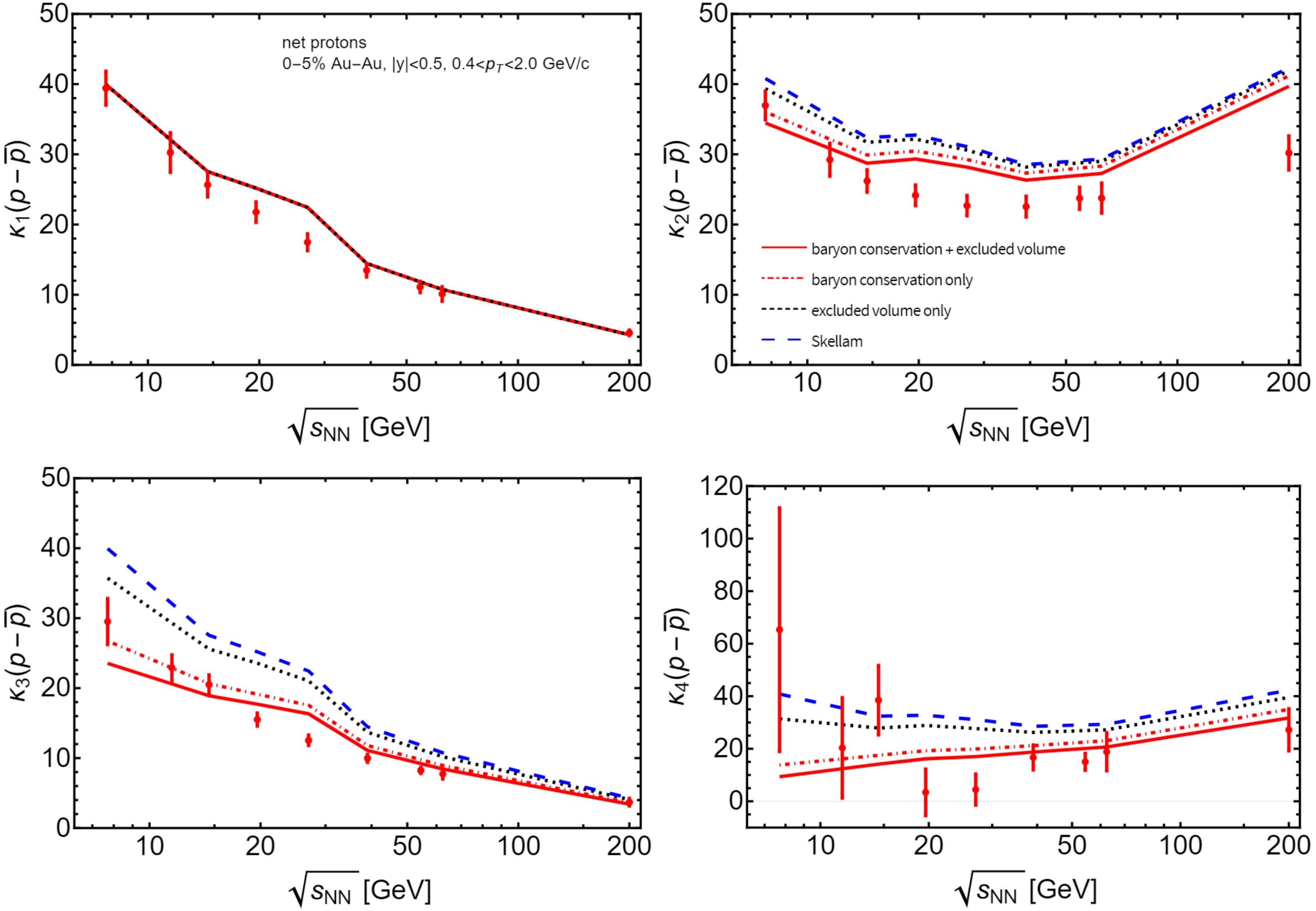
- Proton fluctuations at $\sqrt{s_{NN}} \leq 7.7$ GeV (STAR FXT and HADES)
  - Needs hydrodynamic input
  - Electric charge conservation becomes as important as baryon conservation
  - Impact of light nuclei
  - Nuclear liquid-gas transition



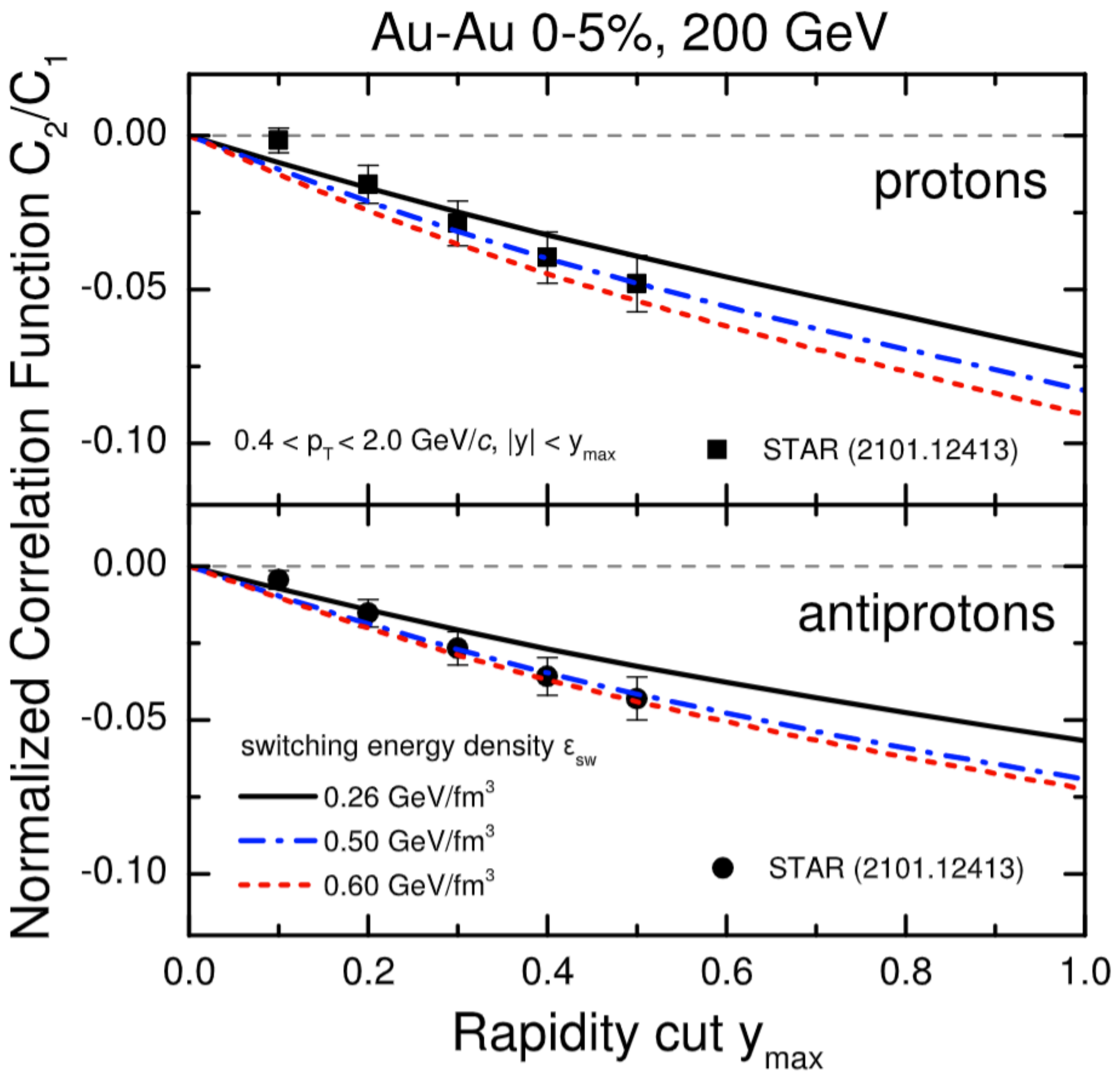
Thanks for your attention!

Backup slides

Net proton cumulants at RHIC

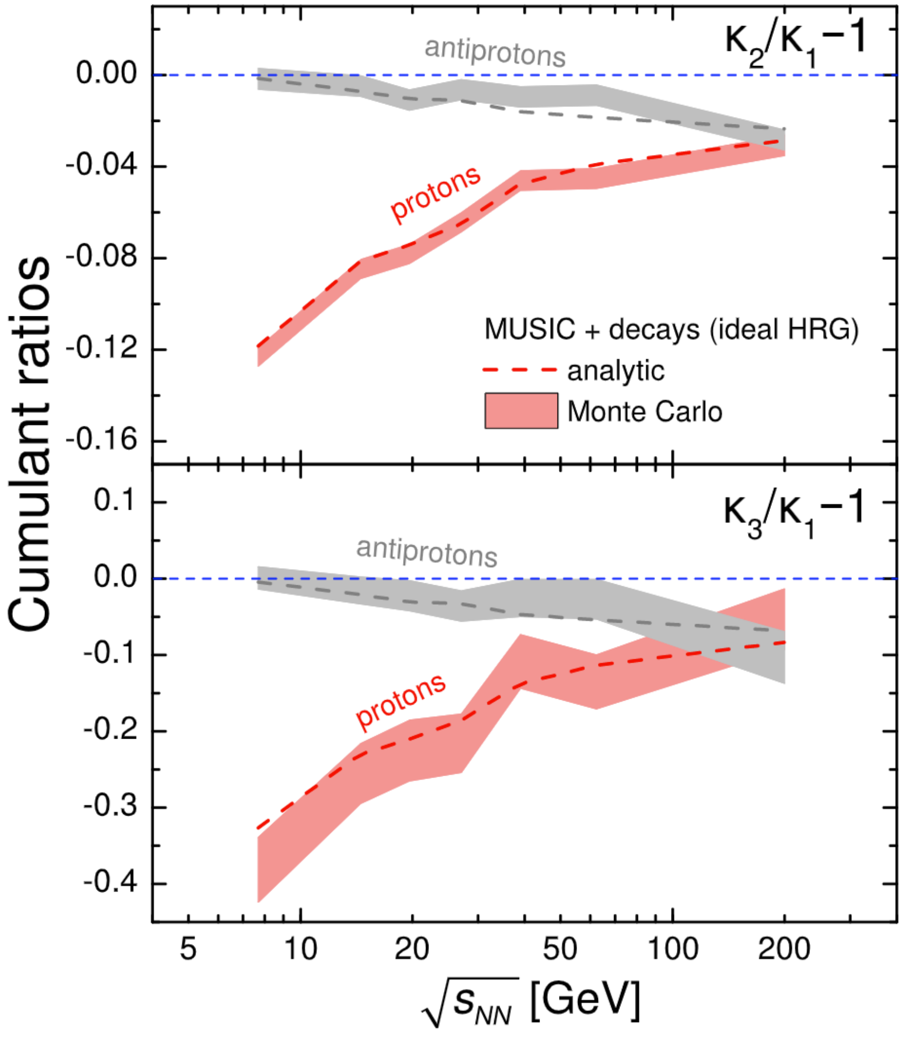


Dependence on the switching energy density



Cross-checking the cumulants with Monte Carlo

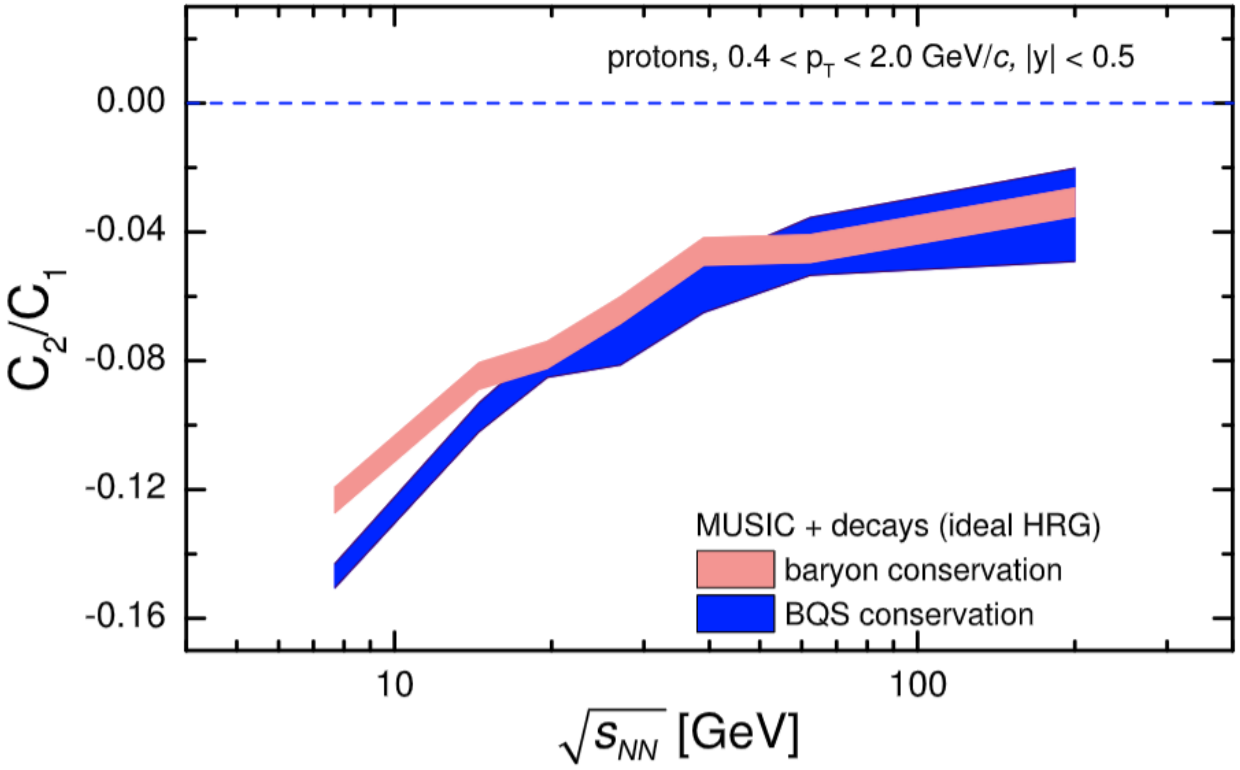
- Sample canonical ideal HRG model at particlization with Thermal-FIST
- Analytic results agree with Monte Carlo within errors



Exact conservation of electric charge

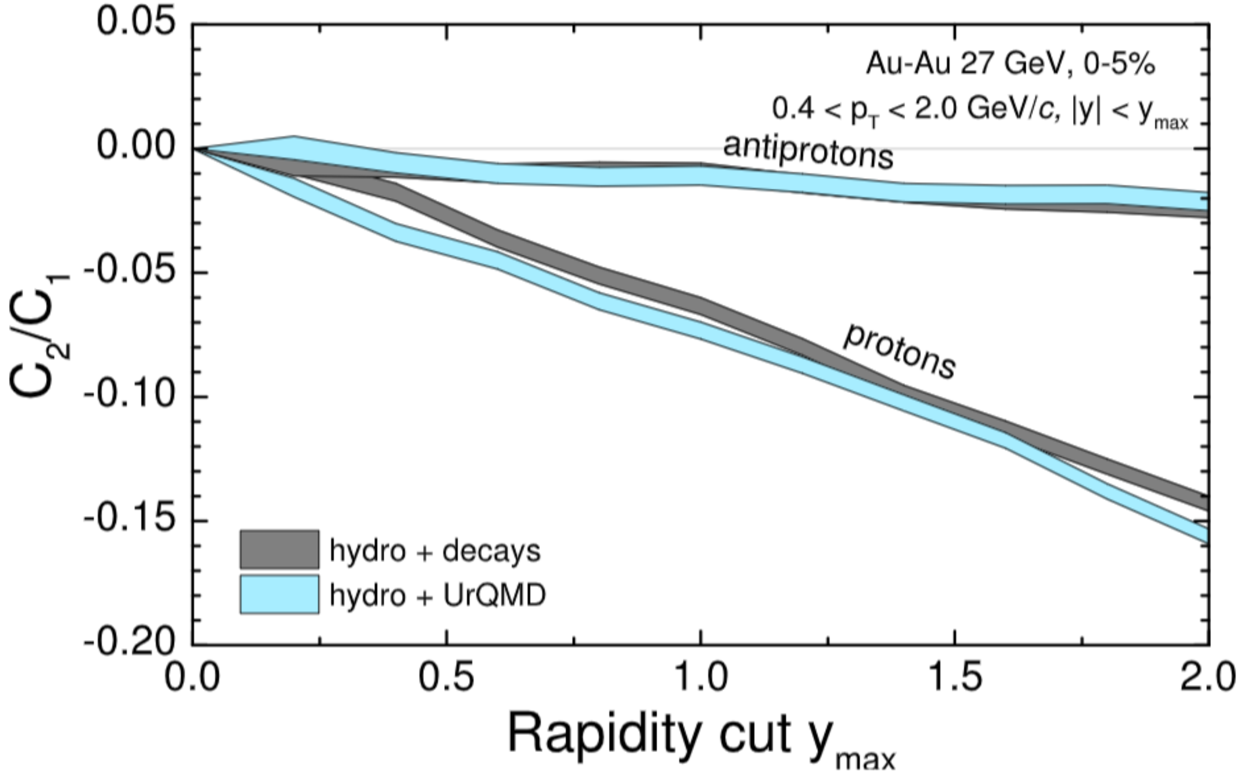
- Sample ideal HRG model at particlization with exact conservation of baryon number, electric charge, and strangeness using Thermal-FIST

- Protons are affected by electric charge conservation at $\sqrt{s_{NN}} \leq 14.5$ GeV



Effect of the hadronic phase

Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD



Net baryon fluctuations at LHC

- Global baryon conservation distorts the cumulant ratios already for one unit of rapidity acceptance

e.g. $\left.\frac{\chi_4^B}{\chi_2^B}\right|_{T=160MeV}^{\text{GCE}} \simeq 0.67 \neq \left.\frac{\chi_4^B}{\chi_2^B}\right|_{\Delta Y_{\text{acc}}=1}^{\text{HIC}} \simeq 0.56$

"lattice QCD" *experiment*

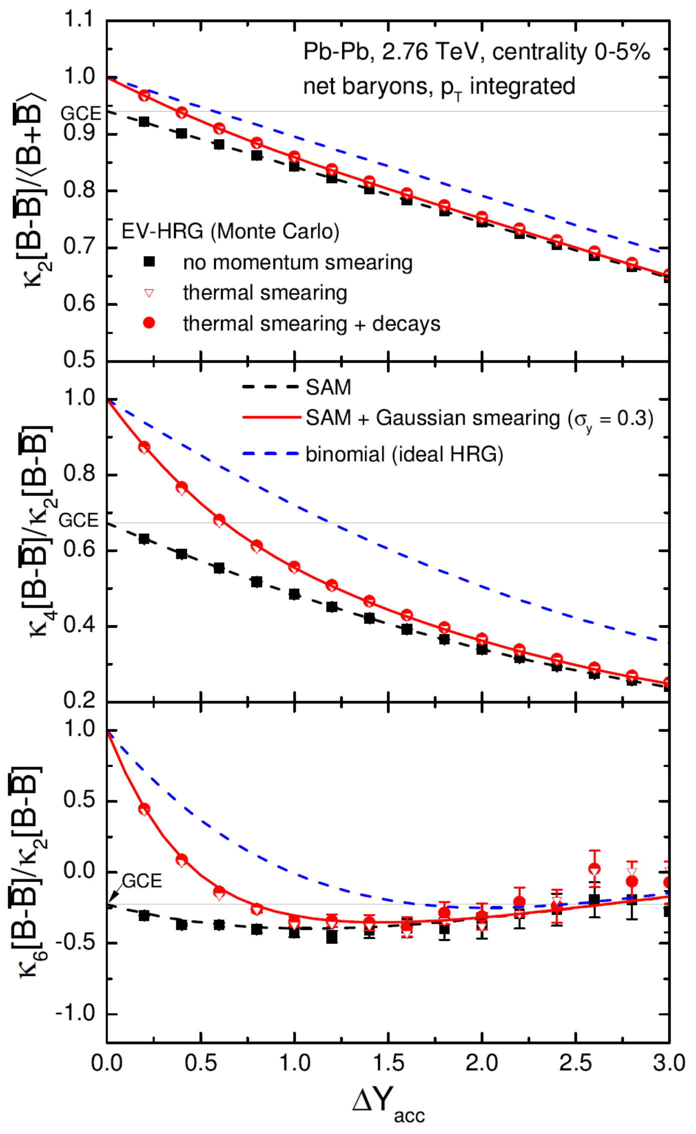
- Neglecting thermal smearing, effects of global conservation can be described analytically via SAM

$$\frac{\kappa_2}{\langle B+\bar{B}\rangle} = (1-\alpha)\frac{\kappa_2^{\text{gce}}}{\langle B+\bar{B}\rangle}, \qquad \alpha = \frac{\Delta Y_{\text{acc}}}{9.6}, \quad \beta \equiv 1-\alpha$$

$$\frac{\kappa_4}{\kappa_2} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B},$$

$$\frac{\kappa_6}{\kappa_2} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$

- Effect of resonance decays is negligible



VV, Koch, arXiv:2012.09954

Net baryon fluctuations at LHC

- Global baryon conservation distorts the cumulant ratios already for one unit of rapidity acceptance

e.g. $\left.\frac{\chi_4^B}{\chi_2^B}\right|_{T=160MeV}^{GCE} \simeq 0.67$ *"lattice QCD"* $\neq$ $\left.\frac{\chi_4^B}{\chi_2^B}\right|_{\Delta Y_{acc}=1}^{HIC} \simeq 0.56$ *experiment*

- Neglecting thermal smearing, effects of global conservation can be described analytically via SAM

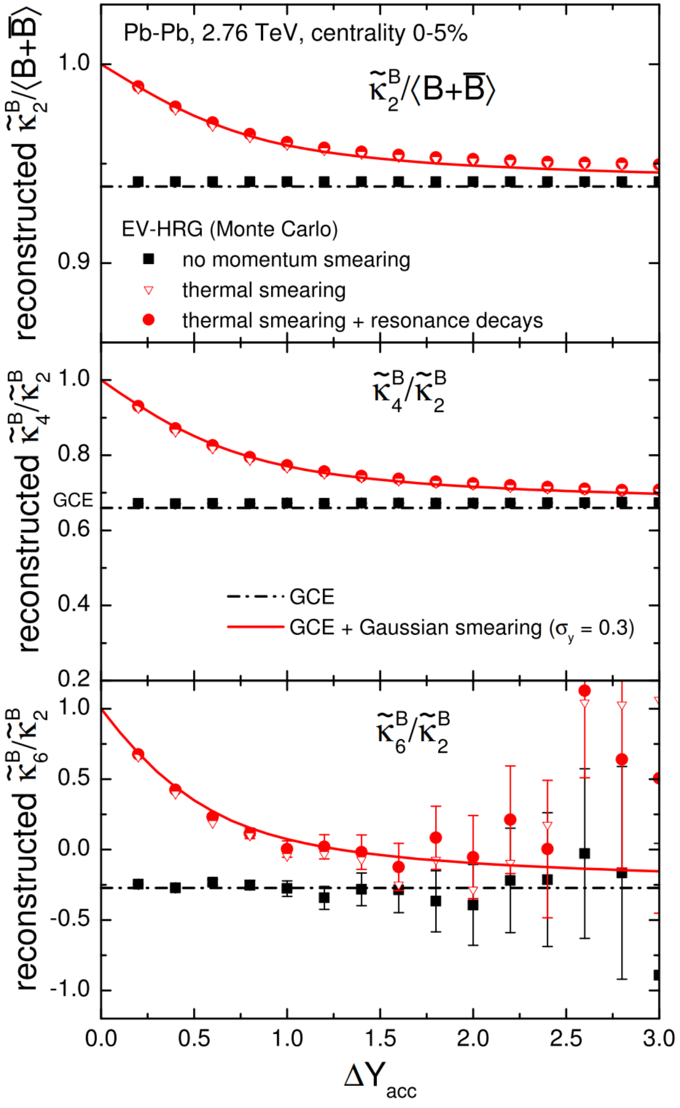
$$\frac{\kappa_2}{\langle B+\bar{B}\rangle} = (1-\alpha)\frac{\kappa_2^{gce}}{\langle B+\bar{B}\rangle}, \qquad \alpha = \frac{\Delta Y_{acc}}{9.6}, \quad \beta \equiv 1-\alpha$$

$$\frac{\kappa_4}{\kappa_2} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B},$$

$$\frac{\kappa_6}{\kappa_2} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$

- Effect of resonance decays is negligible

Cumulants corrected for baryon conservation



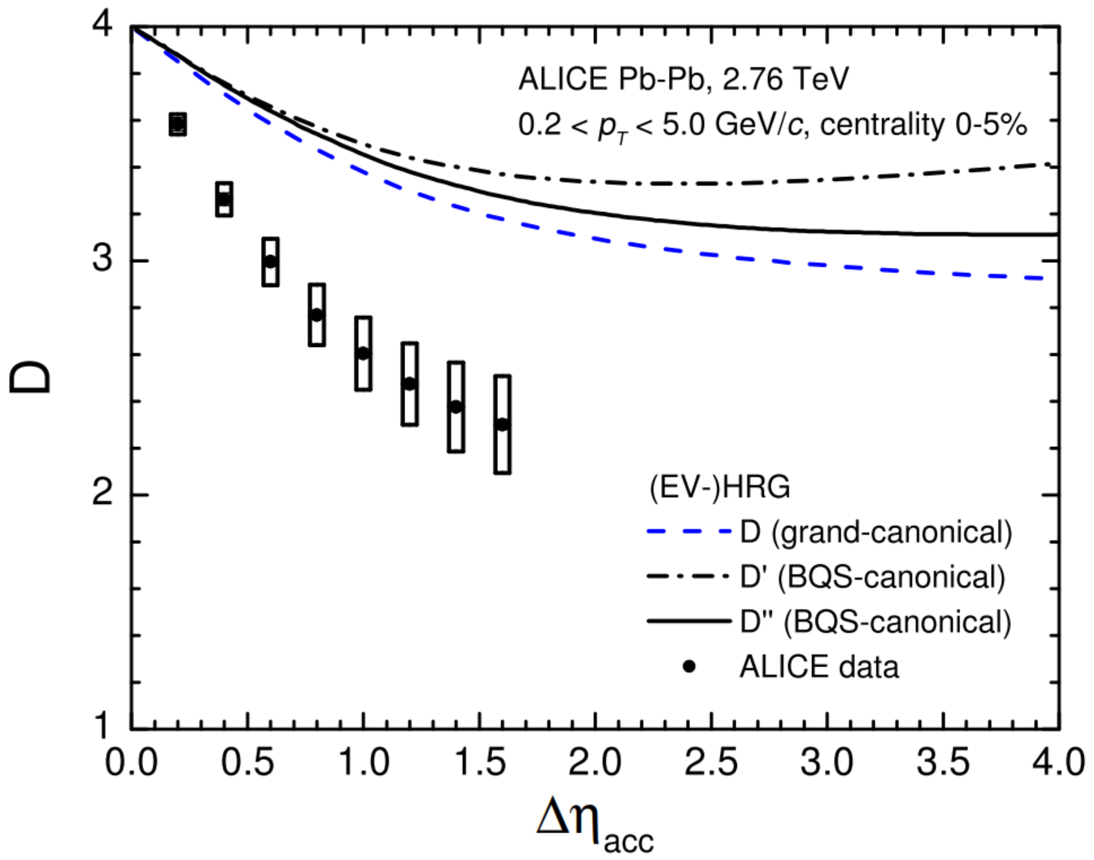
VV, Koch, arXiv:2012.09954

The D-measure

$$D = \frac{\langle \delta Q^2 \rangle}{\langle N_{\text{ch}} \rangle}$$

Jeon, Koch, PRL85, 2076 (2000)

QGP: $D \sim 1 - 1.5$ **HRG:** $D \sim 3 - 4$



VV, Koch, arXiv:2012.09954

Volume fluctuations

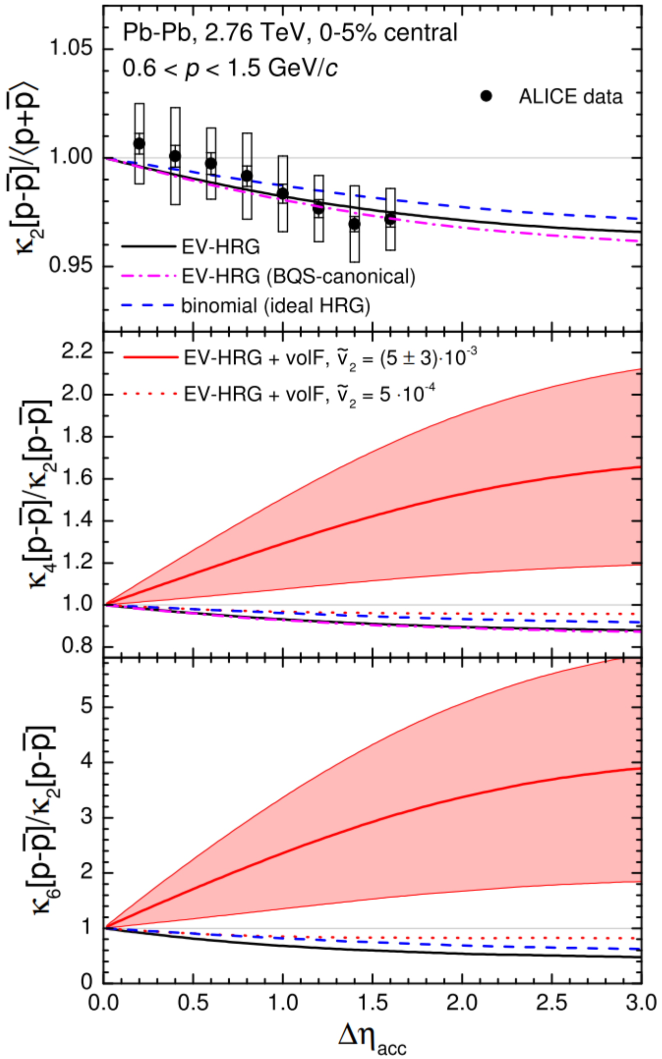
$$\tilde{\kappa}_n = \sum_{l=1}^{n} V_l \, B_{n,l}(\kappa_1/V, \kappa_2/V, \ldots, \kappa_{n-l+1}/V)$$

Net-protons at LHC:

$$\tilde{\kappa}_2 = \kappa_2,$$
$$\tilde{\kappa}_4 = \kappa_4 + 3\kappa_2^2 \, \tilde{v}_2,$$
$$\tilde{\kappa}_6 = \kappa_6 + 15\kappa_2 \, \kappa_4 \, \tilde{v}_2 + 15\kappa_2^3 \, \tilde{v}_3 \ .$$

Protons at LHC:

$$\frac{\tilde{\kappa}_2^p}{\langle p \rangle} = \frac{\kappa_2^p}{\langle p \rangle} + \langle p \rangle \, \tilde{v}_2$$



VV, Koch, arXiv:2012.09954