Nucleon-Nucleon Scattering in Multiple Partial Waves from Lattice QCD

E. Berkowitz[1], T. Kurth[2], A. Nicholson[3], B. Joó[4], E. Rinaldi[1], M. Strother[2], P. Vranas[1], and A. Walker-Loud[2]

[1]Lawrence Livermore National Laboratory, [2]Lawrence Berkeley National Laboratory, [3]UC Berkeley, [4]Jefferson Laboratory [5]College of William and Mary

Introduction

- QCD is non-perturbative at low-energies
- Nuclear physics tools:
  - Models
    - not based directly on QCD
    - uncontrolled systematics
  - Effective field theories (EFTs)
    - incorporate symmetries of QCD
    - controlled systematics (in practice, poor convergence)
    - require potentially large number of parameters to be determined
  - Lattice QCD
    - first-principles QCD calculations
    - controlled systematics
- Nucleon-nucleon scattering from the lattice
  - can understand hadronic interactions directly from QCD
  - gives important input for EFTs
  - explore experimentally inaccessible setups, such as different quark masses

Constructing partial waves in a box

- Lattice calculations performed in a finite volume - generally, a periodic box
- Reduced symmetry of the box mixes different angular momenta
- Good quantum numbers are those of the cubic group

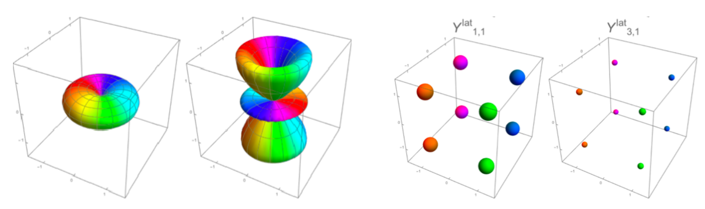


Figure: Infinite volume P- and F-waves (left) look the same when projected onto cubic irreps (right).

- One method for creating operators with good quantum numbers:
  - begin with two nucleons displaced by some distance x_0 (alternately, with some relative momentum, k), combined into desired total spin S
  - Use spherical harmonics to project onto desired orbital angular momentum L
  - Use Clebsch-Gordan coefficients to project onto desired total J

$$\mathcal{O}_{Jm_J lm_l, S\ell}(x_0, |\mathbf{k}|) = \sum_{m_S, m_\ell} \sum_{m_{s_1}, m_{s_2}} \sum_{m_{l_1}, m_{l_2}} CG(Jm_J|\ell m_\ell, Sm_S)CG(Sm_S|s_2 m_{s_2}, s_1 m_{s_1})$$

$$\times CG(lm_l|1/2\ m_{l_1}, 1/2\ m_{l_2}) \sum_{R\in\mathcal{O}_h} Y_{\ell m_\ell}(\widehat{R\mathbf{k}})\ N_{m_{s_1}, m_{l_1}}(x_0, R\mathbf{k}) N_{m_{s_2}, m_{l_2}}(x_0, -R\mathbf{k})\ ,$$

  where CG represents the Clebsch-Gordan coefficient and R is a rotation matrix belonging to the cubic group, O_h.
  - Use subduction matrices to project a given J onto desired cubic irrep

Luscher's method for extracting scattering phase shifts

- Energies of two-particle systems in a periodic box are quantized
- Finite volume spectrum can be related to scattering phase shift
- For low-energy s-wave scattering:

$$q \cot\delta = \frac{1}{\pi L} S\left(\left(\frac{qL}{2\pi}\right)^2\right)$$

where q is obtained from the energy, $E = 2\sqrt{q^2 + M_N^2}$, L is the length of the box, and S is related to the Riemann zeta function.
- For higher partial waves:
  - partial wave mixing (both physical mixing and mixing due to the cubic symmetry) leads to a matrix eigenvalue equation
  - for low energies, mixing from higher partial waves may be neglected, and the quantization condition reduces to (for $l \le 4$)

$$q\cot\delta_\Lambda(q) = 4\pi\left(c_{00}(q^2) + \alpha_{4,\Lambda}\frac{c_{40}(q^2)}{q^4} + \alpha_{6,\Lambda}\frac{c_{60}(q^2)}{q^6}\right),$$

  where δ_Λ and α_ℓ are given in the Table below, and the $c_{\ell m_\ell}$ are kinematic functions

$$c_{\ell m_\ell}(q^2) = \frac{\sqrt{4\pi}}{L^3}\left(\frac{2\pi}{L}\right)^{\ell-2} \sum_{\mathbf{r}\in Z^3} \frac{|\mathbf{r}|^\ell Y_{\ell m_\ell}(\mathbf{r})}{(r^2 - q^2)}.$$

Isospin	Spin	Parity	Λ	δ_Λ	$\alpha_{4,\Lambda}$	$\alpha_{6,\Lambda}$
Triplet	Singlet	Positive	A_1^+	$\delta_{^1S_0}$	0	0
			T_2^+	$\delta_{^1D_2}$	$-4/7$	0
Singlet	Singlet	Negative	T_1^-	$\delta_{^1P_1}$	0	0
			A_2^-	$\delta_{^1F_3}$	$-12/11$	$80/11\sqrt{13}$
Singlet	Triplet	Positive	T_1^+	$\delta_{^3S_1}$	0	0
			A_2^+	$\delta_{^3D_3}$	-4/7	0
Triplet	Triplet	Negative	A_1^-	$\delta_{^3P_0}$	0	0
			T_1^-	$\delta_{^3P_1}$	0	0
			T_2^-	$\delta_{^3P_2}$	0	0
			E^-	$\delta_{^3P_2}$	2/7	0

Spectrum from the lattice

- Calculations performed at $m_\pi \sim 800$ MeV and lattice spacing $a \sim 0.15$ fm
- Cubic operators used as sources and sinks for two-nucleon systems and form correlation functions
  - position space operators at the source
  - at the sink, project onto relative momentum shells corresponding to non-interacting states in a box
- Lattice calculation performed in Euclidean space
  - large Euclidean time limit gives lowest energy state that couples to operator, $C(t) \sim e^{-E_0 t}$
  - good overlap of operators with a given momentum shell gives us access to excited states at intermediate times

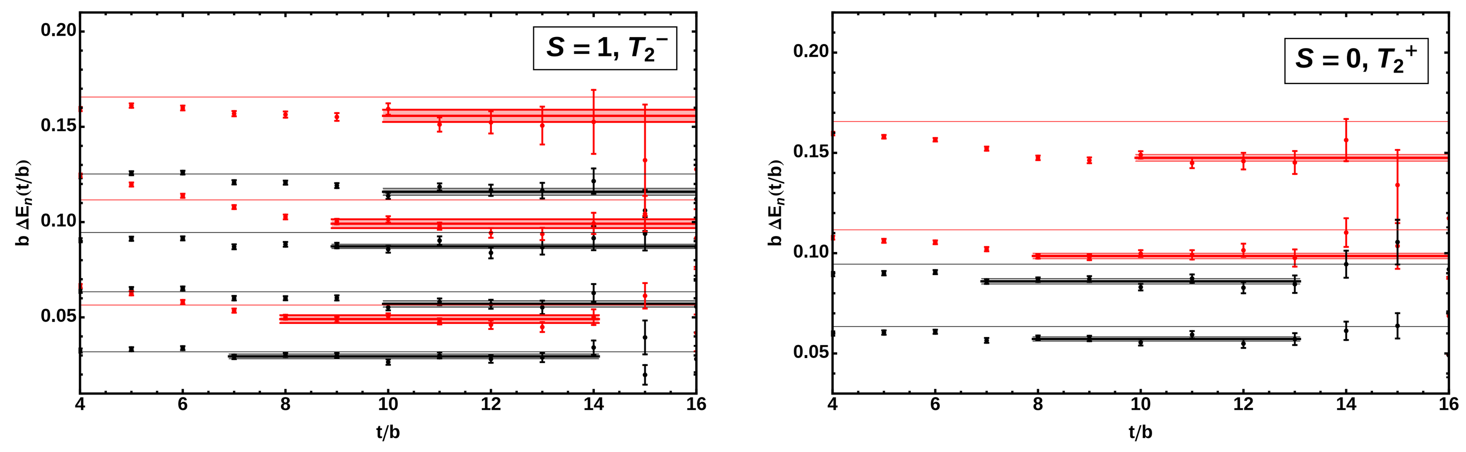
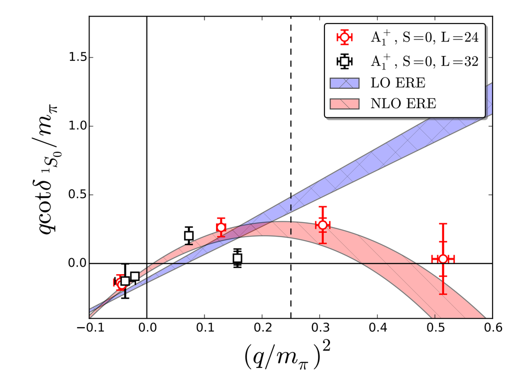


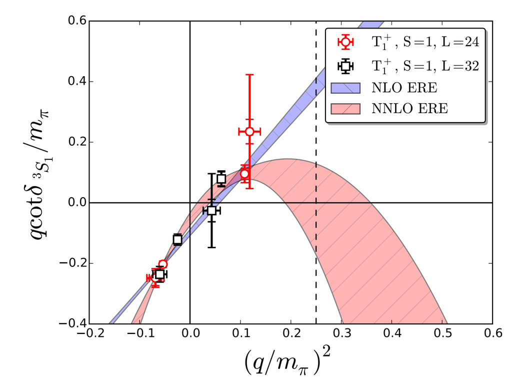
Figure: Effective mass plots (data points) and fits (bands) to the energy shift values, $\Delta E = E_{NN} - 2m_N$ for NN-systems in the T_2^- (left) and T_2^+ cubic irreps. Solid lines represent non-interacting energy levels. Red: $L = 24$, Black: $L = 32$.

Phase Shifts

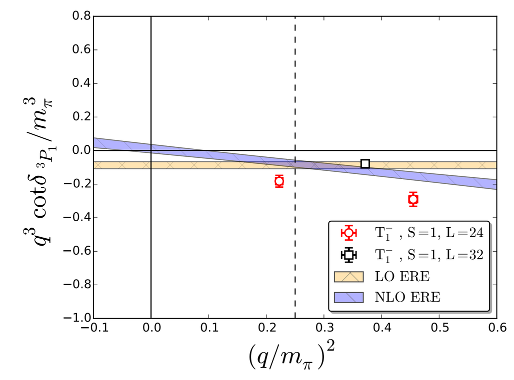
Here we show plots of the scattering phase shifts in a particular angular momentum channel as a function of the scattering momentum, q. Fits to an effective range expansion at a given order in q^2 are shown as colored bands.

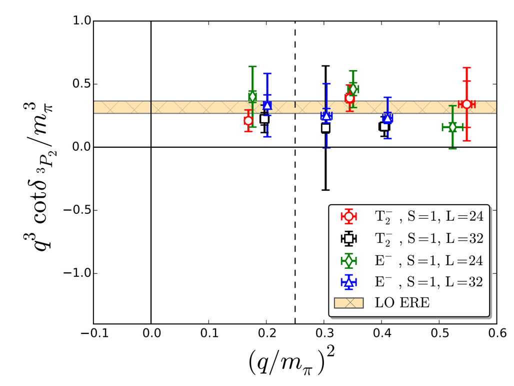
- S-wave

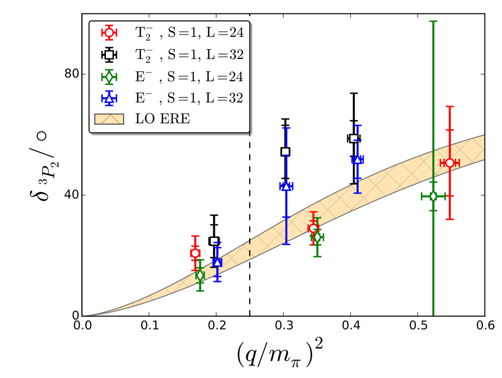




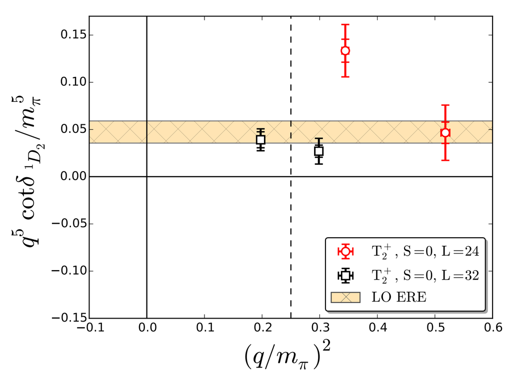
- P-wave

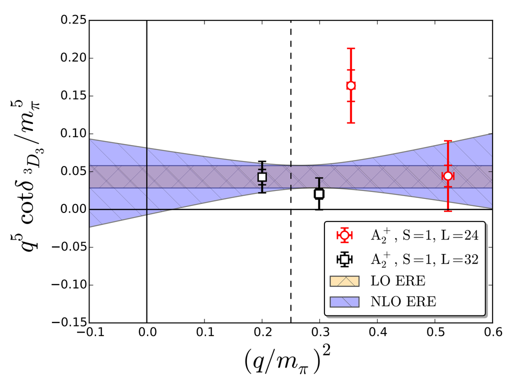




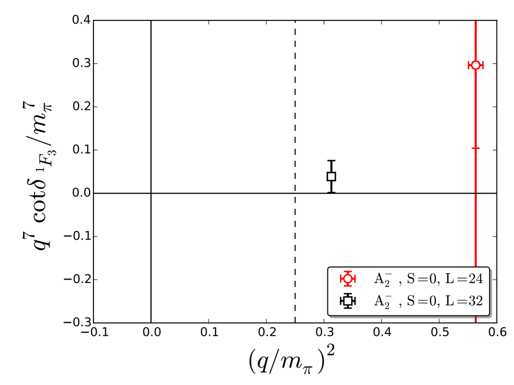


- D-wave





- F-wave



Conclusions

- Lattice QCD gives reliable tool for first-principles QCD calculations in non-perturbative regimes
- Extracted finite volume energy levels for two-nucleon systems at $m_\pi \sim 800$ MeV
- Calculated scattering phase shifts for two nucleon systems scattering in S, P, D, and F partial wave channels

Acknowledgements

- Support for this work was provided through Scientific Discovery through Advanced Computing (SciDAC) program funded by U.S. Department of Energy, Office of Science, Advanced Scientific Computing Research and Nuclear Physics.
- Prepared by LLNL under Contract DE-AC52-07NA27344. LLNL-POST-