Workshop for Applied Nuclear Data Activities (WANDA)

United States Department of Energy, Virtual conference

DDFRG: Double Differential FRaGmentation models for proton and light ion production in high energy nuclear collisions: Closed form, analytic formulas for transport codes and other applications

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February 28 - March 4, 2022

Outline

Introduction

- HZETRN (High charge (Z) and Energy TRaNsport) is NASA's primary space radiation transport code
- Primary engine behind OLTARIS (On-Line Tool for the Assessment of Radiation In Space)
- HZETRN is very fast, deterministic code capable of efficient analysis of a wide variety of human space mission scenarios
- Extensively benchmarked against the majority of the world's other radiation transport codes - agrees with world codes as well as they agree with each other
- Many nuclear physics codes (DDFRG, NUCFRG, EMDFRG, RAADFRG, NUCDAT) have been developed "in-house" in order to enable fast & efficient calculations with HZETRN
- 3DHZETRN is the most recent development, which is fully 3-dimensional (3D)
- Light ions & neutrons account for large fraction of radiation dose received by astronauts - Because they are light, they scatter at large angles and require a 3D treatment
- DDFRG (Double-Differential FRaGmentation) is a new nuclear physics code which calculates double-differential cross sections for light ion (^{1,2,3}H, ^{3,4}He) production - Provides closed-form, analytic formulas - highly efficient

Introduction - References

1. *Double-Differential FRaGmentation (DDFRG) models for proton and light ion production in high energy nuclear collisions*
   J. Norbury
   Nuclear Instruments & Methods in Physics Research A, vol. 986, p. 164681, 2021

2. *Light ion double-differential cross section parameterizations and results from the SHIELD transport code*
   J. Norbury, L. Latysheva, N. Sobolevsky
   Nuclear Instruments & Methods in Physics Research A, vol. 947, p. 162576, 2019

3. *Double-Differential FRaGmentation (DDFRG) models for proton and light ion production in high energy nuclear collisions valid for both small and large angles*
   J. Norbury
   NASA Technical Publication 2020-5001740 http://ntrs.nasa.gov/

4. *Light ion double-differential cross sections for space radiation*
   J. Norbury
   NASA Technical Publication 2018-220077 http://ntrs.nasa.gov/

Proton model

- Thermal proton production model
  - Protons being produced from 4 separate sources:
  - Projectile, Target, Central fireball, Direct projectile knockout

Proton thermal model for *each* source

$$E\frac{d^3\sigma}{dp^3} = N\,e^{-T/\Theta}$$

- N determined from requirement that integral of double differential cross section gives total cross section σ

Proton model

- In terms of total 3-momentum and angle variables (p_{jL}, θ_{jL}) the final thermal plus direct knockout model is

DDFRG proton thermal model

$$E\frac{d^3\sigma}{dp^3}(p_{jL},\theta_{jL}) = \frac{\sigma}{4\pi m}\{\exp[-(\gamma_{\mathcal{P}L}\sqrt{p_{jL}^2+m^2} - \gamma_{\mathcal{P}L}\,\beta_{\mathcal{P}L}\,p_{jL}\cos\theta_{jL} - m)/\Theta_{\mathcal{P}}]$$

$$+\exp[-(\gamma_{\mathcal{C}L}\sqrt{p_{jL}^2+m^2} - \gamma_{\mathcal{C}L}\,\beta_{\mathcal{C}L}\,p_{jL}\cos\theta_{jL} - m)/\Theta_{\mathcal{C}}]$$

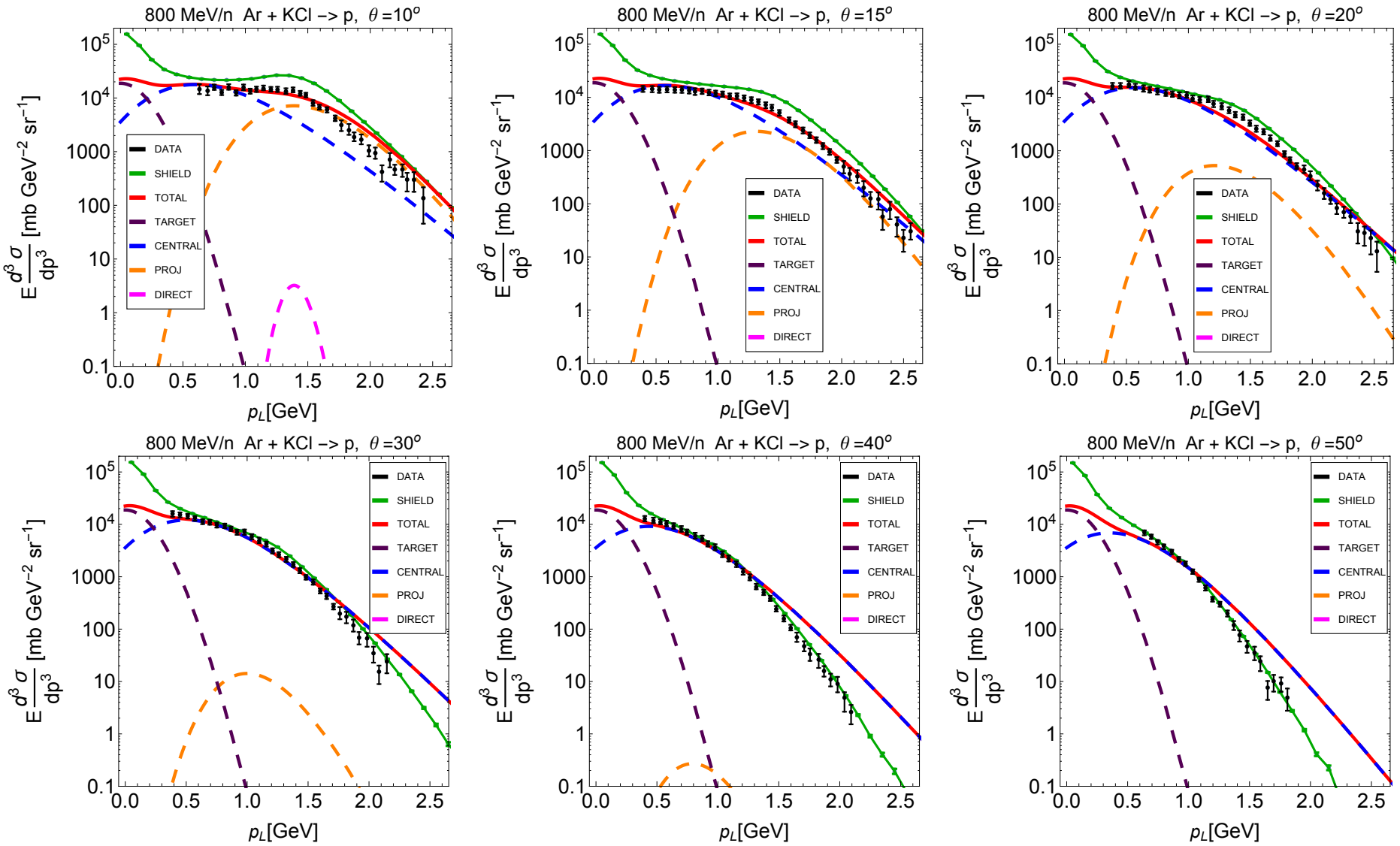
$$+\exp[-(\gamma_{\mathcal{T}L}\sqrt{p_{jL}^2+m^2} - \gamma_{\mathcal{T}L}\,\beta_{\mathcal{T}L}\,p_{jL}\cos\theta_{jL} - m)/\Theta_{\mathcal{T}}]$$

$$+w_{\mathcal{D}}^{(\mathrm{p})}\exp[-(\gamma_{\mathcal{P}L}\sqrt{p_{jL}^2+m^2} - \gamma_{\mathcal{P}L}\,\beta_{\mathcal{P}L}\,p_{jL}\cos\theta_{jL} - m)/\Theta_{\mathcal{D}}]\}$$

$$\times\left\{\Theta_{\mathcal{P}}e^{m/\Theta_{\mathcal{P}}}K_1(m/\Theta_{\mathcal{P}}) + \Theta_{\mathcal{C}}e^{m/\Theta_{\mathcal{C}}}K_1(m/\Theta_{\mathcal{C}}) + \Theta_{\mathcal{T}}e^{m/\Theta_{\mathcal{T}}}K_1(m/\Theta_{\mathcal{T}}) + w_{\mathcal{D}}^{(\mathrm{p})}\Theta_{\mathcal{D}}e^{m/\Theta_{\mathcal{D}}}K_1(m/\Theta_{\mathcal{D}})\right\}^{-1}$$

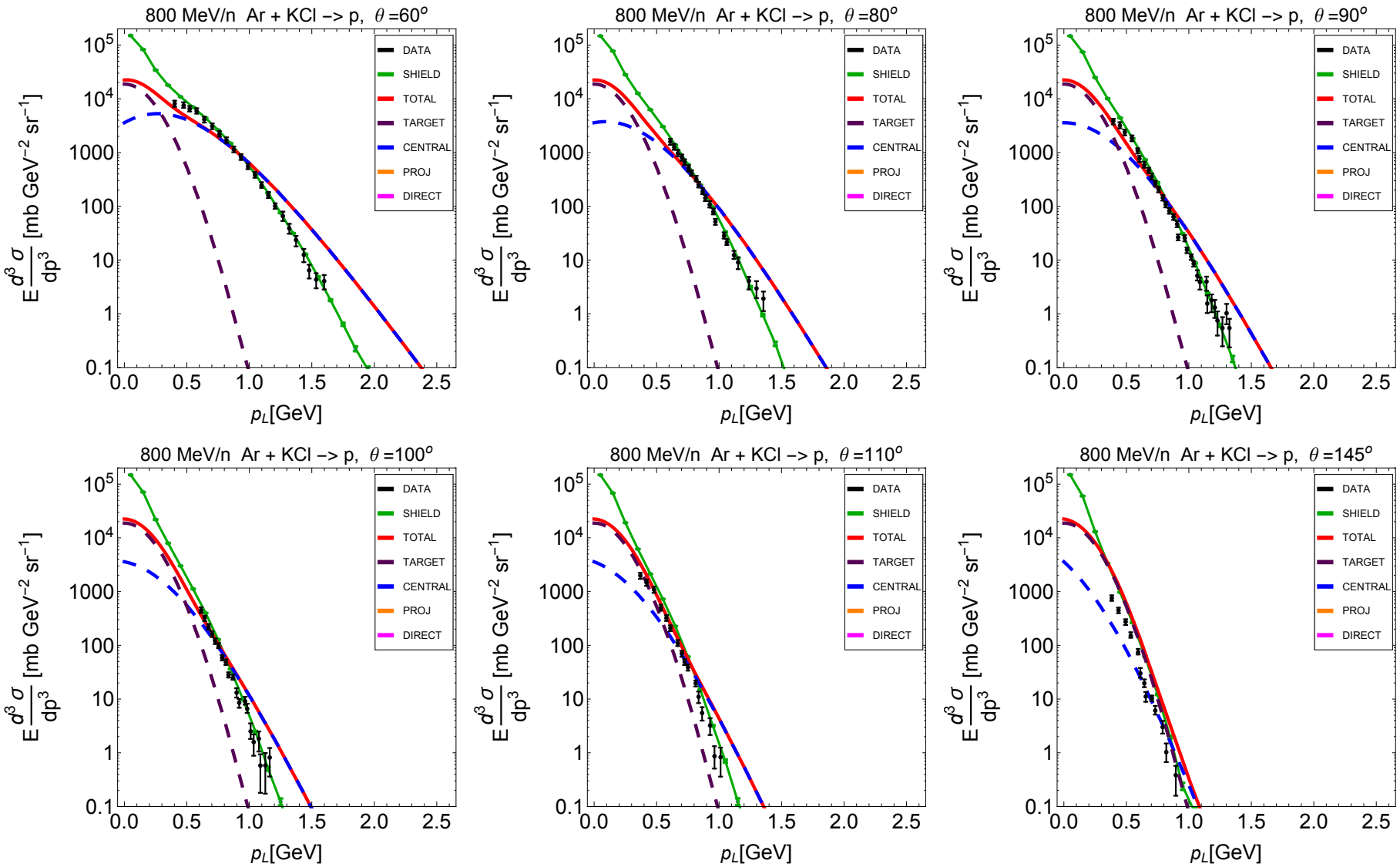
- Can be analytically integrated to give closed form analytic formula for $\frac{d\sigma}{dE}$ (see References)

Large Angle DDFRG 800 MeV/n Ar + KCl → p



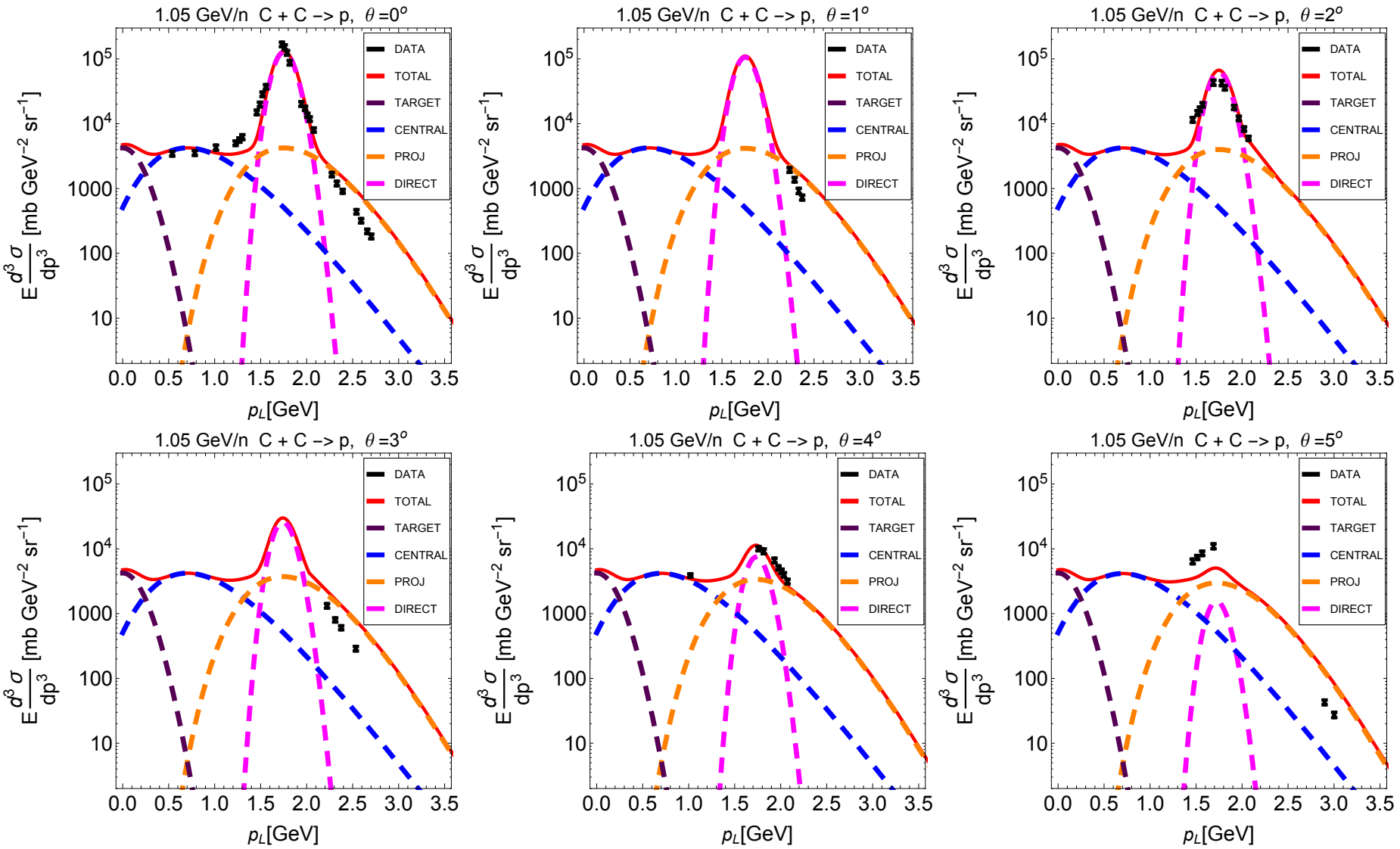
DDFRG model agrees well with data, but some differences

Large Angle DDFRG 800 MeV/n Ar + KCl → p



DDFRG model agrees well with data, but some differences

Small Angle DDFRG 1.05 GeV/n C + C → p



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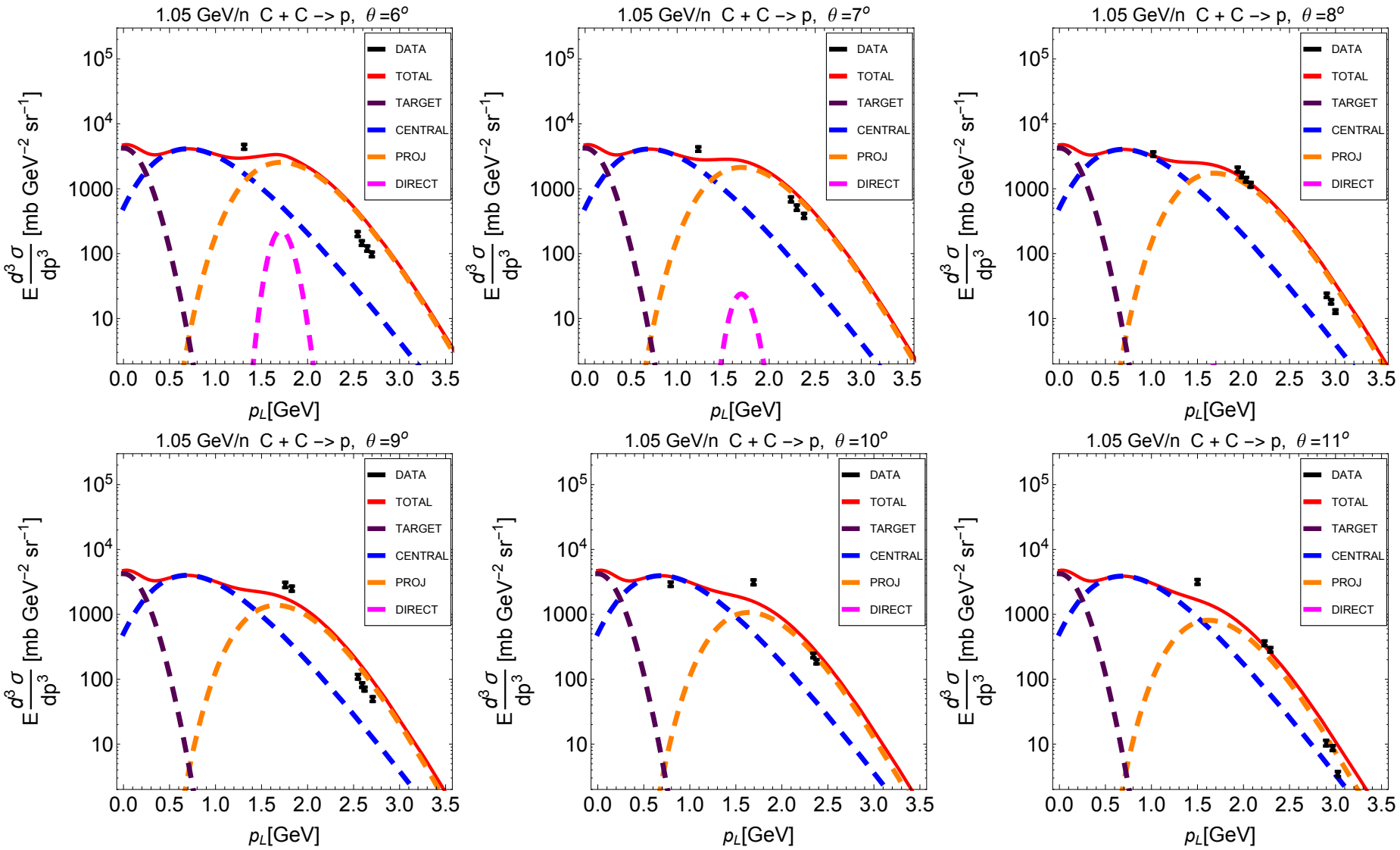


DDFRG model agrees well with data, but some differences

Coalescence Scaling

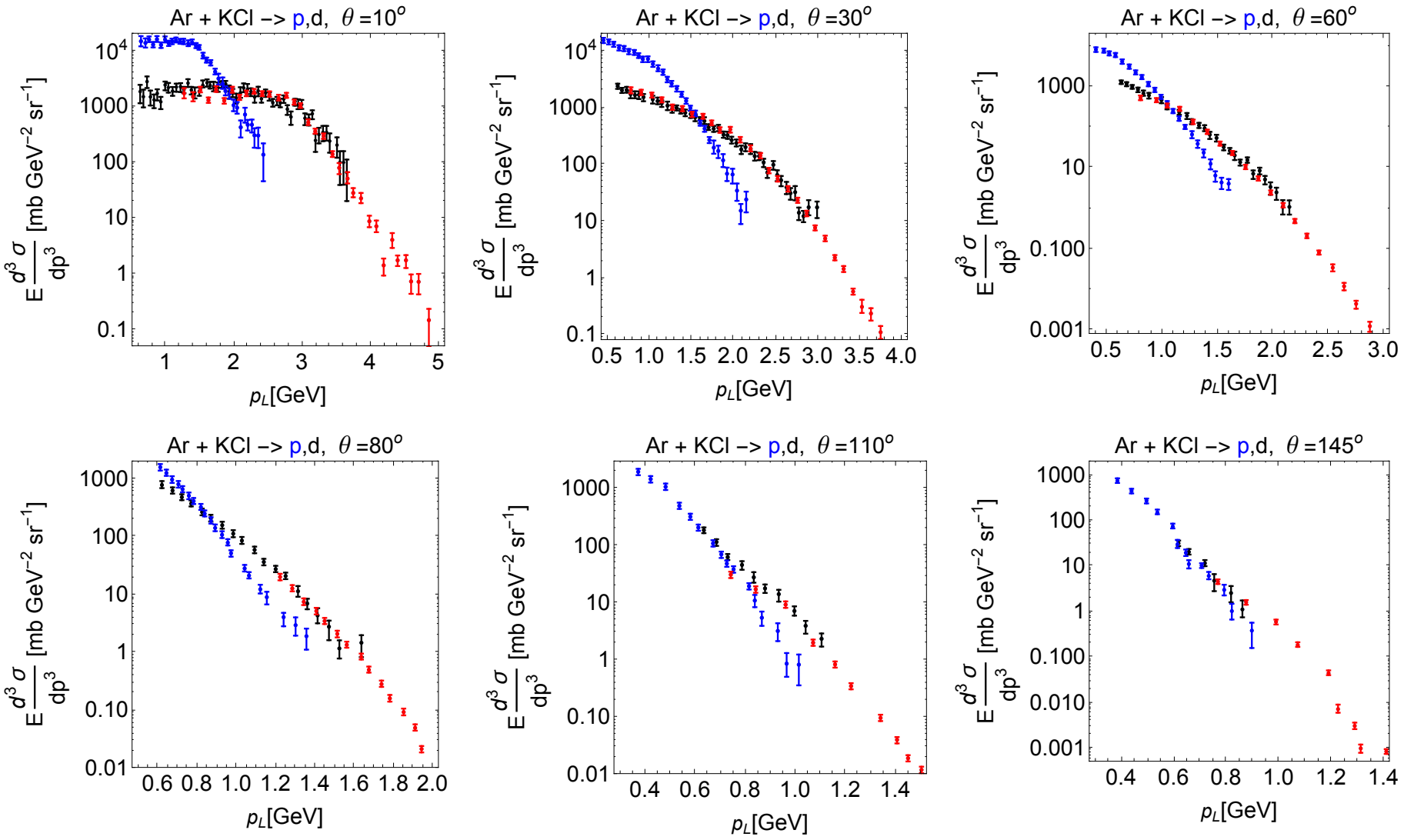
- Comparisons of double-differential cross section data between proton production and light ion production show that light ion data is very well represented by scaling proton data, assuming that light ions are produced via coalescence
- Comparisons done only using experimental proton data versus light ion data - no theoretical model used in comparing proton data to light ion data, except for simple scaling of proton data as

Coalescence model

$$E_A \frac{d^3\sigma_A}{dp_A^3} = C_A \left(E_\mathrm{p} \frac{d^3\sigma_\mathrm{p}}{dp_\mathrm{p}^3} \right)^A, \qquad p_A \equiv A p_\mathrm{p} \text{ and } E_A \equiv A E_\mathrm{p}$$

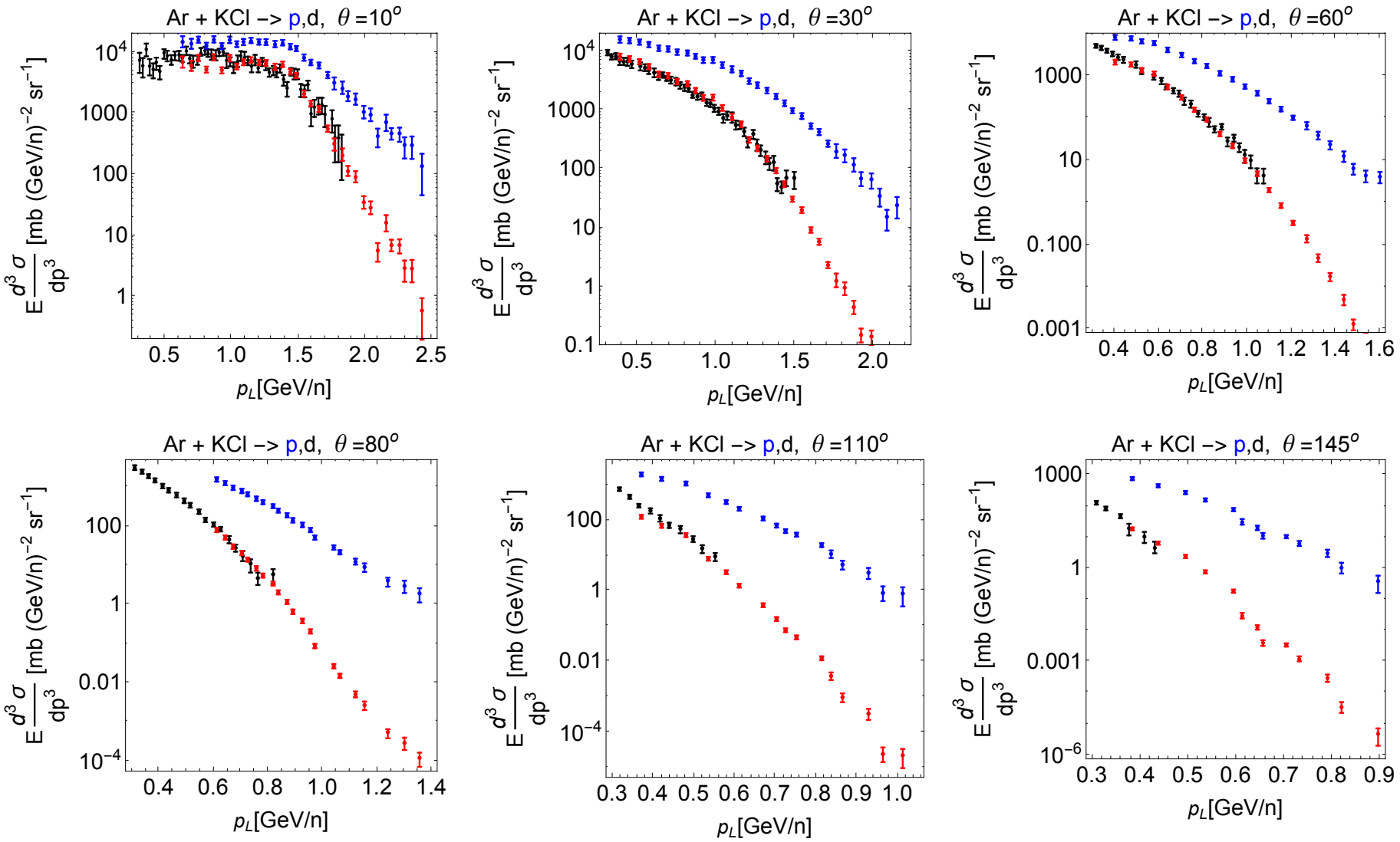
$C_A \equiv$ coalescence coefficient

Large Angle Scaling



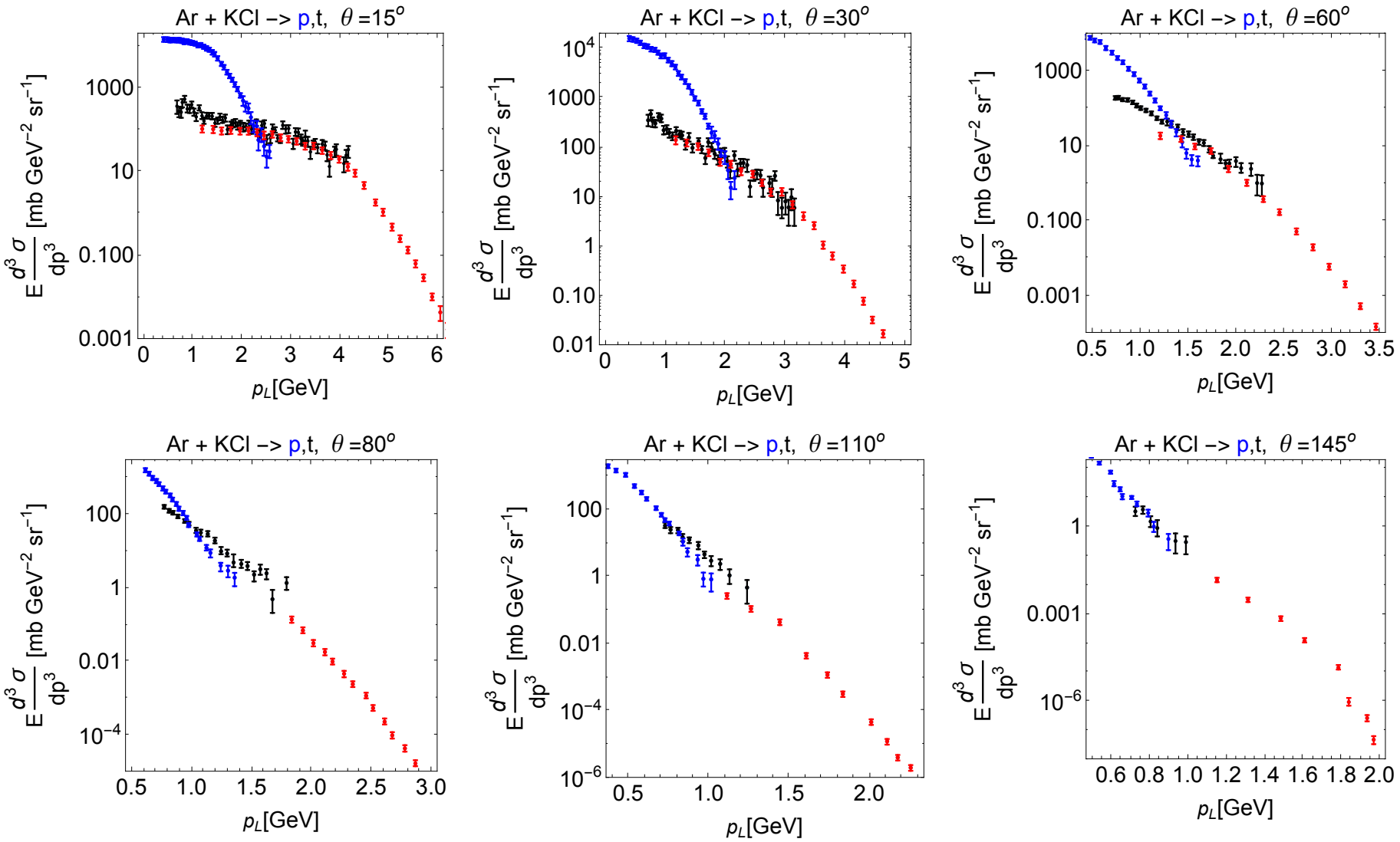
Proton data is scaled and then agrees very well with light ion data

Large Angle Scaling - GeV/n



Proton data is scaled and then agrees very well with light ion data

Large Angle Scaling



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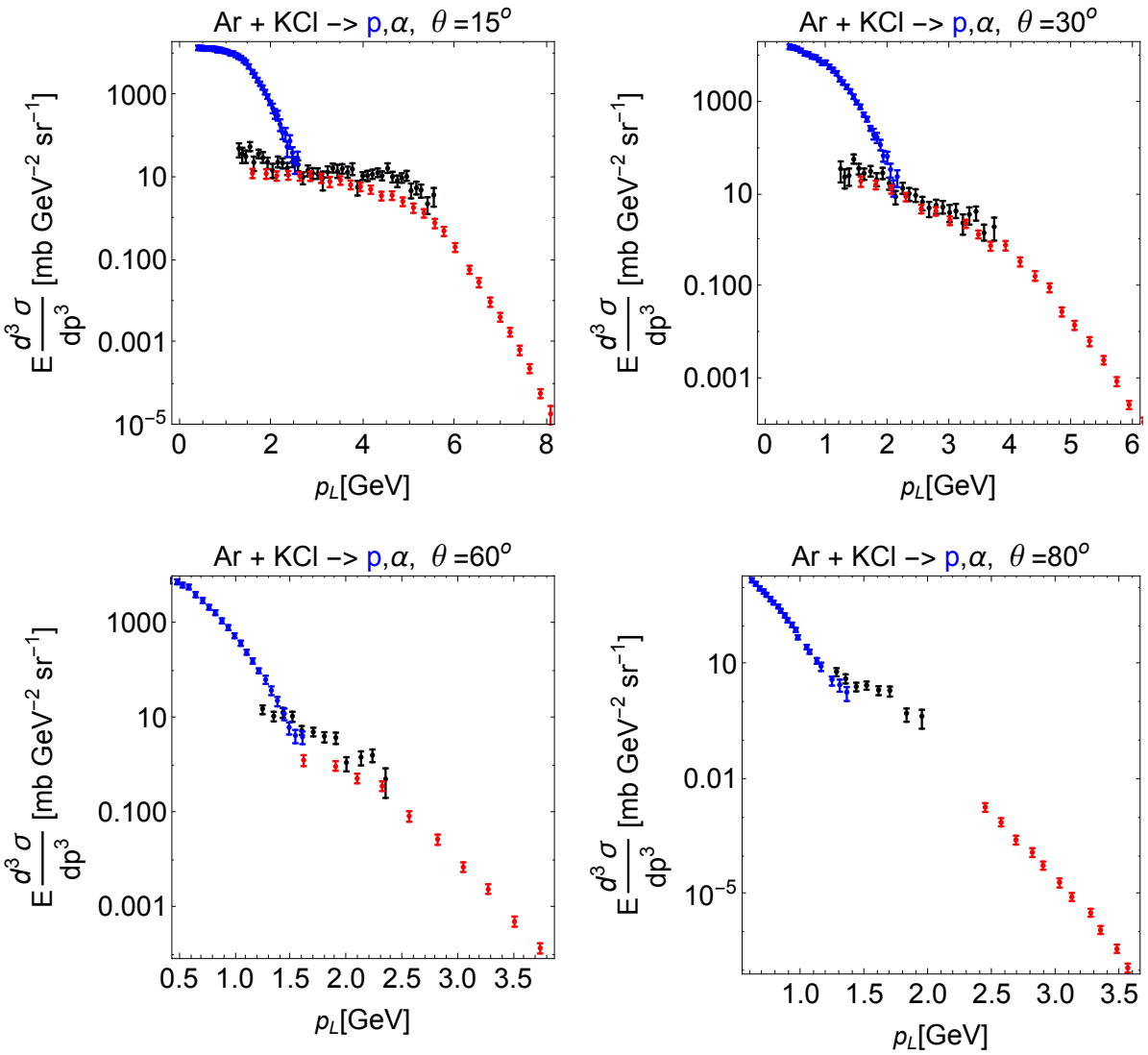


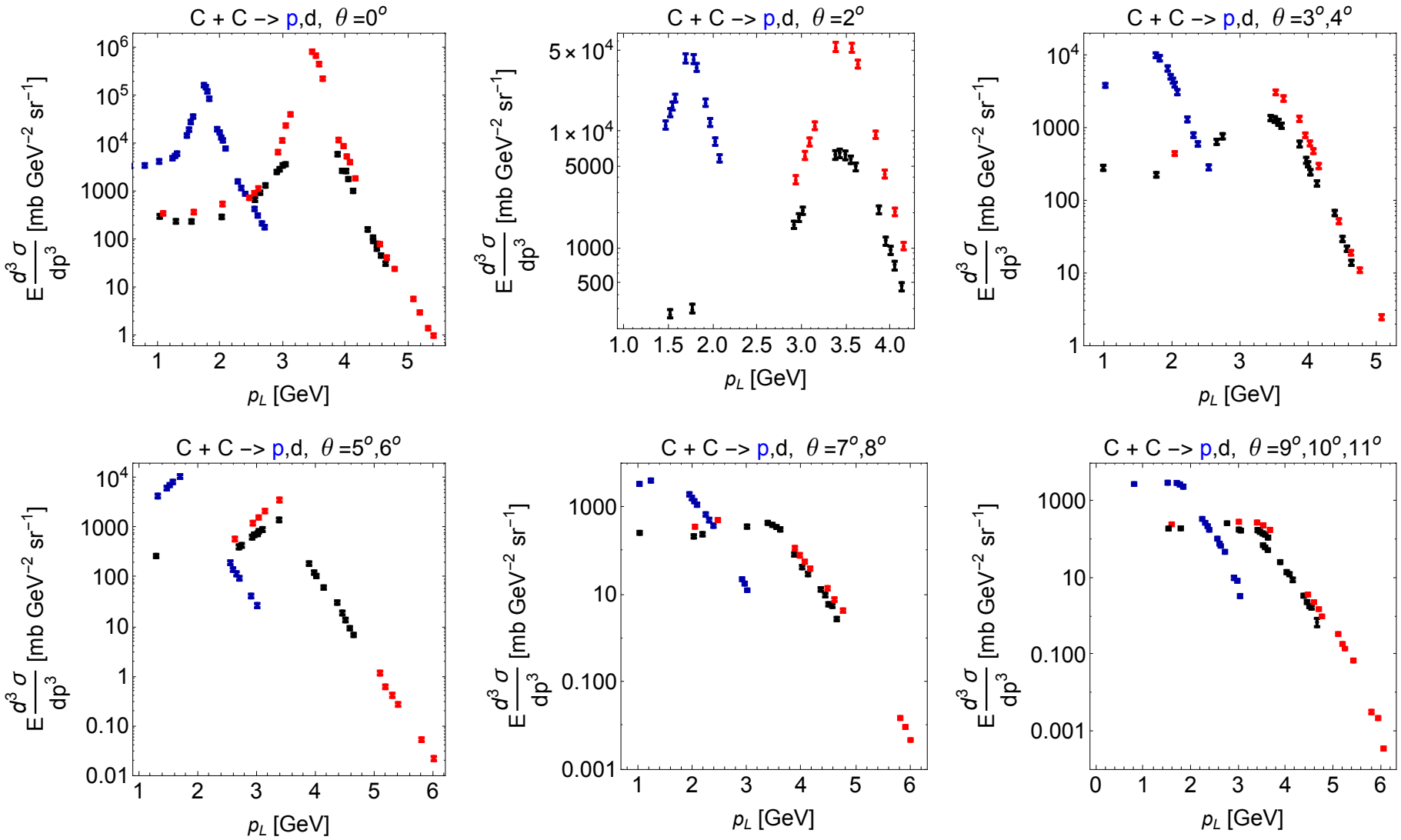
Proton data is scaled and then agrees very well with light ion data

Anderson small angle data

- Data looks nothing like previous large angle data of Nagamiya
  - now a giant peak!

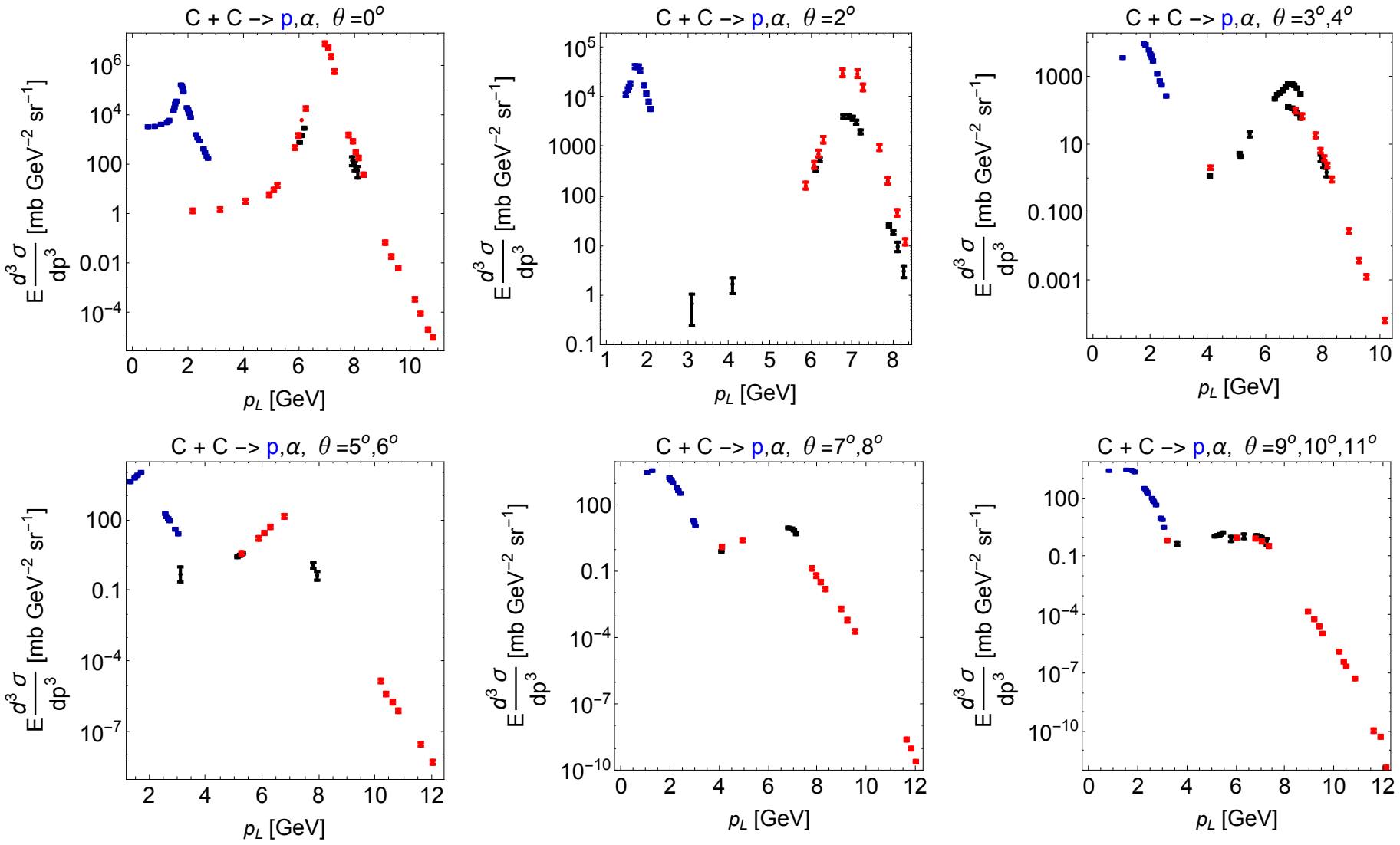
- Coalescence scaling fails!
  - at the smallest angles, but OK when angles increase

Small Angle Scaling 1.05 GeV/n C + C → d



Proton data is scaled and then agrees with large angle light ion data, but not small angle

Small Angle Scaling 1.05 GeV/n $C + C \rightarrow \alpha$



Proton data is scaled and then agrees with large angle light ion data, but not small angle

Light ion model

- Empirical relation between proton & light ion data implies not necessary for separate theoretical model for light ion production
- One only requires a model for proton production
- If proton model compares well to data, then scaled proton model will automatically compare well to light ion data
- Light ion model is obtained simply by scaling the proton model
- And this works for *all* composite light ions
- Separate light ion model not required
- However, the light ion model is not obtained without some effort
  - To demonstrate scaling of the experimental data, one typically uses a fitted coalescence coefficient, C_A for each light ion
  - To develop a fully predictive model, this coefficient must be calculated with theoretical model developed separately

Light ion model

$$E_A \frac{d^3\sigma_A}{dp_A^3} = C_A \left\{ w_{\mathcal{P}}^{(A)} \left[E \frac{d^3\sigma}{dp^3}(p_{jL}, \theta_{jL}) \right]_{\mathcal{P}} + w_{\mathcal{C}}^{(A)} \left[E \frac{d^3\sigma}{dp^3}(p_{jL}, \theta_{jL}) \right]_{\mathcal{C}} + \left[E \frac{d^3\sigma}{dp^3}(p_{jL}, \theta_{jL}) \right]_{\mathcal{T}} + w_{\mathcal{D}}^{(A)} \left[E \frac{d^3\sigma}{dp^3}(p_{jL}, \theta_{jL}) \right]_{\mathcal{D}} \right\}^A$$

- Complicated *algebraic* expression when expanded out and written in terms of σ and Lorentz transformed to lab frame variables
- Left hand side is DD cross section for light ion production
- Right hand side contains DD cross sections for proton production
- Modified "coalescence" model

Light ion model

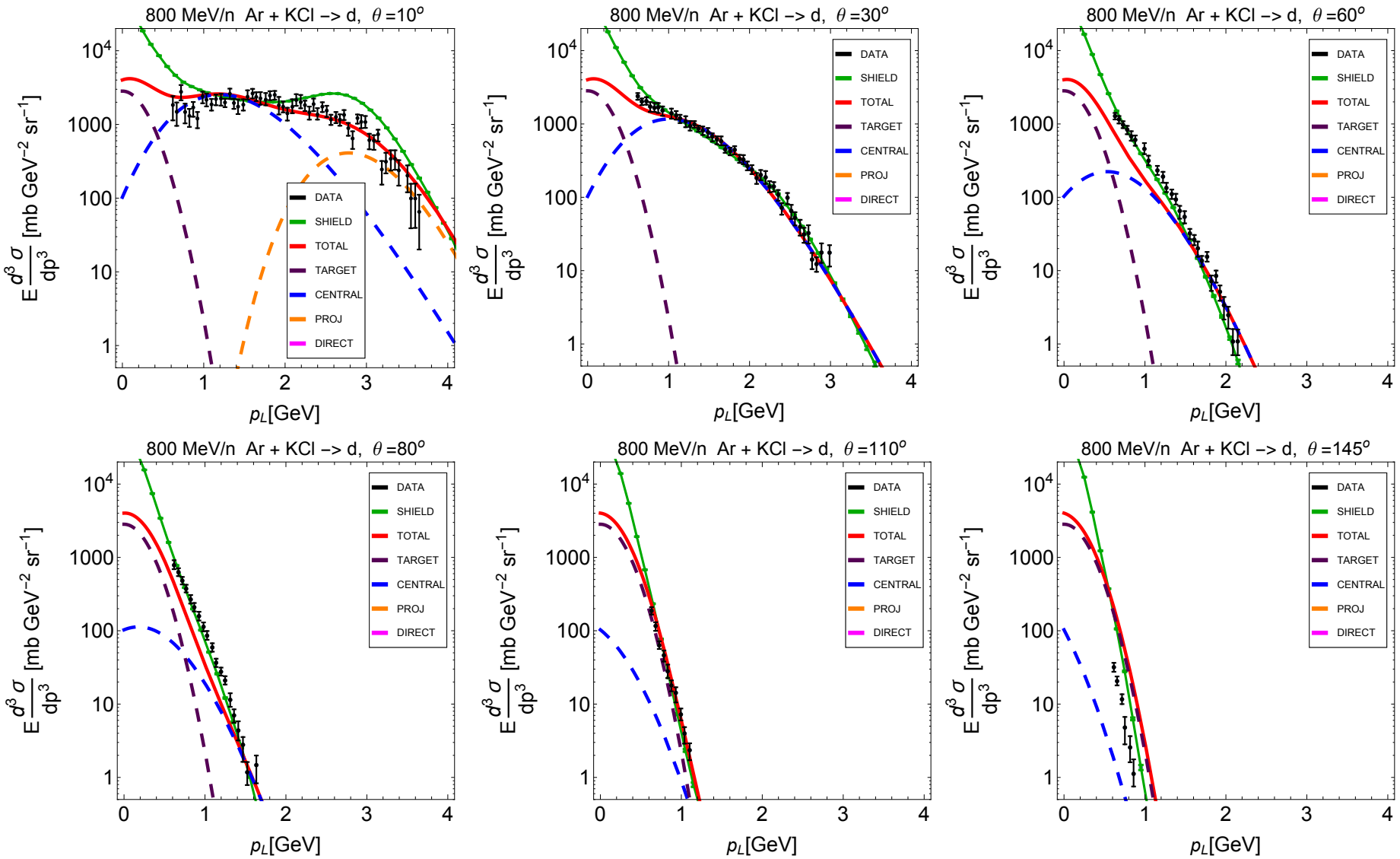
Thermal / Coalescence model for Light ion production

$$
\begin{aligned}
E_A \frac{d^3\sigma_A}{dp_A^3} &= C_A N_4^A \Big\{ w_{\mathcal{P}} \exp[(m_{\rm p} - \gamma_{\mathcal{P}L} \sqrt{p_{{\rm p}L}^2 + m_{\rm p}^2} + \gamma_{\mathcal{P}L}\, \beta_{\mathcal{P}L}\, p_{{\rm p}L} \cos\theta_{{\rm p}L})/\Theta_{\mathcal{P}}] \\
&\quad + w_{\mathcal{C}} \exp[(m_{\rm p} - \gamma_{\mathcal{C}L} \sqrt{p_{{\rm p}L}^2 + m_{\rm p}^2} + \gamma_{\mathcal{C}L}\, \beta_{\mathcal{C}L}\, p_{{\rm p}L} \cos\theta_{{\rm p}L})/\Theta_{\mathcal{C}}] \\
&\quad + w_{\mathcal{T}} \exp[(m_{\rm p} - \gamma_{\mathcal{T}L} \sqrt{p_{{\rm p}L}^2 + m_{\rm p}^2} + \gamma_{\mathcal{T}L}\, \beta_{\mathcal{T}L}\, p_{{\rm p}L} \cos\theta_{{\rm p}L})/\Theta_{\mathcal{T}}] \\
&\quad + w_{\mathcal{D}} w_{\mathcal{D}}^{({\rm p})} \exp[(m_{\rm p} - \gamma_{\mathcal{P}L} \sqrt{p_{{\rm p}L}^2 + m_{\rm p}^2} + \gamma_{\mathcal{P}L}\, \beta_{\mathcal{P}L}\, p_{{\rm p}L} \cos\theta_{{\rm p}L})/\Theta_{\mathcal{D}}] \Big\}^A
\end{aligned}
$$

$$
\begin{aligned}
N_4 &= \frac{\sigma_{\rm p}}{4\pi m_{\rm p}} \Big[\Theta_{\mathcal{P}}\, e^{\frac{m_{\rm p}}{\Theta_{\mathcal{P}}}}\, K_1\left(\frac{m_{\rm p}}{\Theta_{\mathcal{P}}}\right) + \Theta_{\mathcal{C}}\, e^{\frac{m_{\rm p}}{\Theta_{\mathcal{C}}}}\, K_1\left(\frac{m_{\rm p}}{\Theta_{\mathcal{C}}}\right) \\
&\qquad + \Theta_{\mathcal{T}}\, e^{\frac{m_{\rm p}}{\Theta_{\mathcal{T}}}}\, K_1\left(\frac{m_{\rm p}}{\Theta_{\mathcal{T}}}\right) + w_{\mathcal{D}}^{({\rm p})}\, \Theta_{\mathcal{D}}\, e^{\frac{m_{\rm p}}{\Theta_{\mathcal{D}}}}\, K_1\left(\frac{m_{\rm p}}{\Theta_{\mathcal{D}}}\right) \Big]^{-1}
\end{aligned}
$$

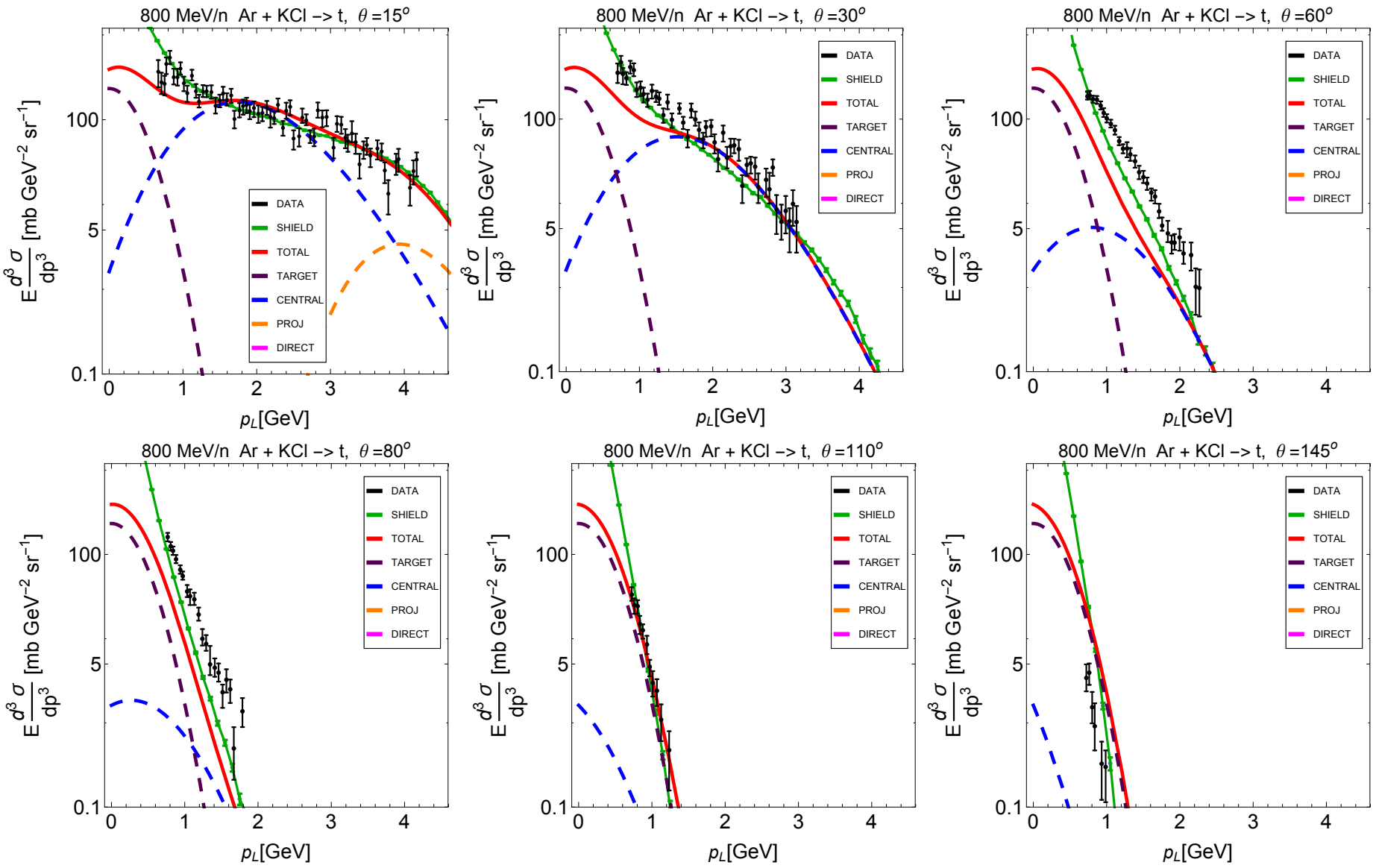
- Can be analytically integrated (for A = 2,3,4) to give closed form analytic formula for $\frac{d\sigma}{dE}$ (see References)

Large Angle DDFRG 800 MeV/n Ar + KCl → d



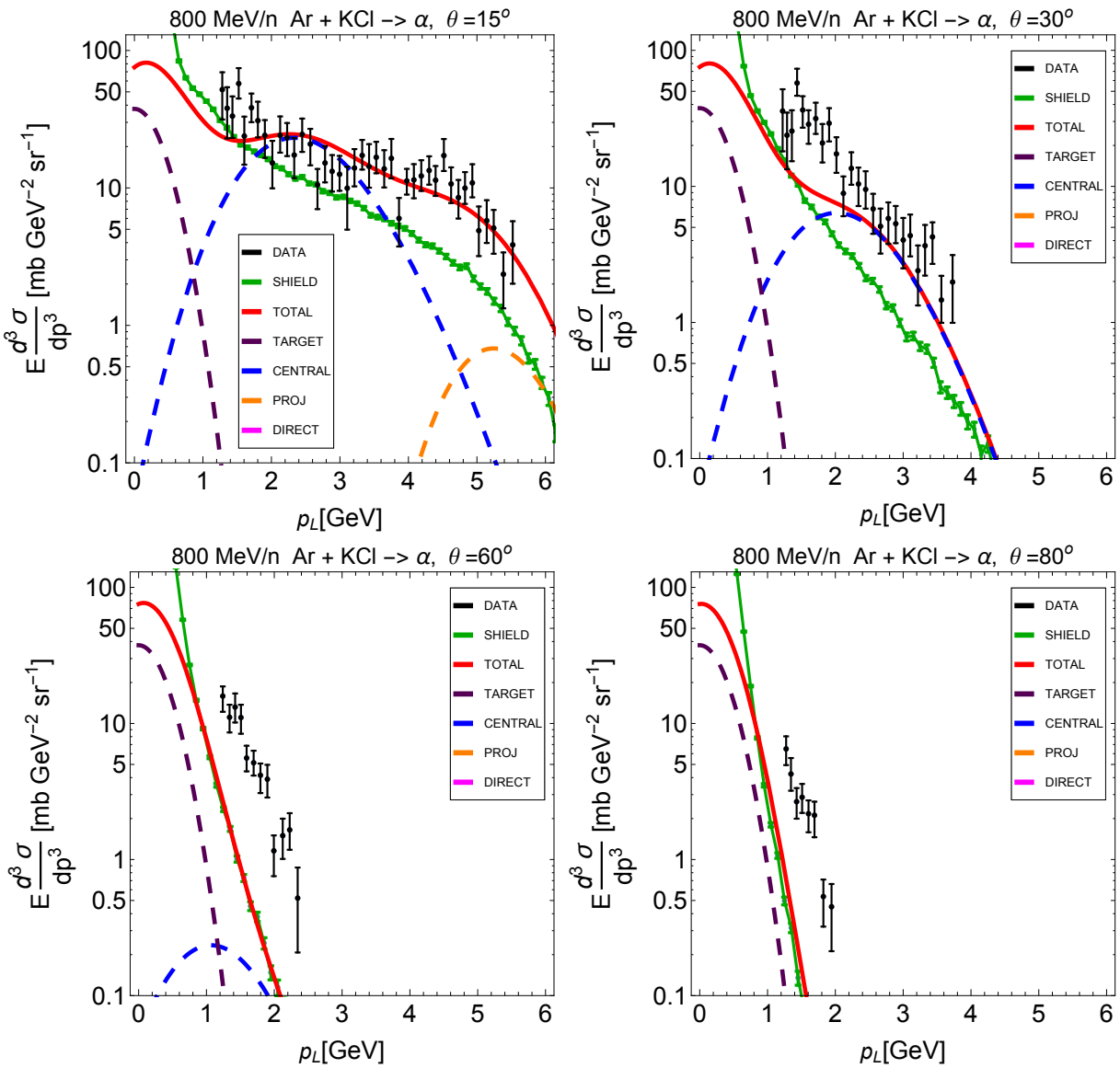
DDFRG model agrees well with data, but some differences

Large Angle DDFRG 800 MeV/n Ar + KCl → t



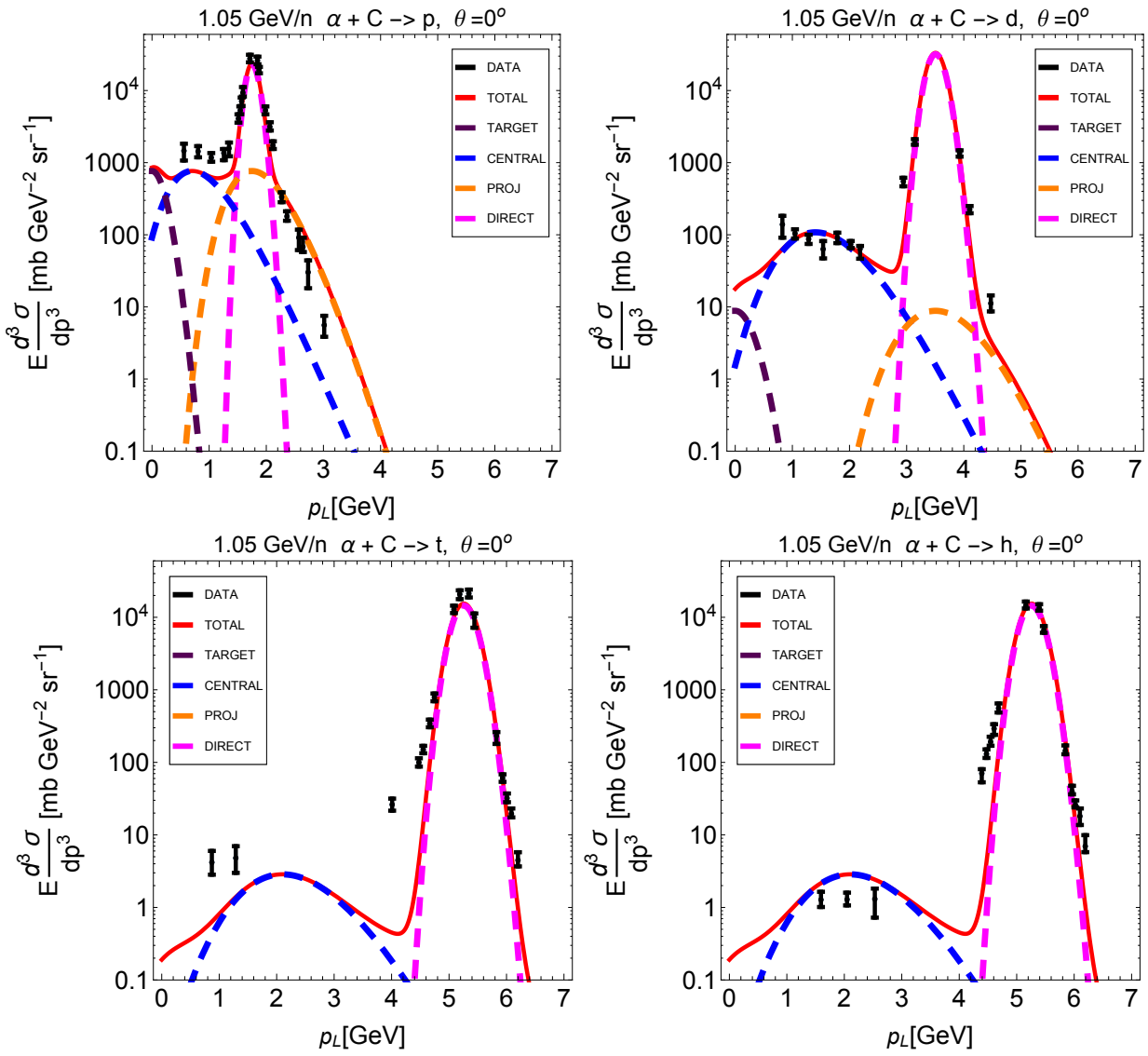
DDFRG model agrees well with data, but some differences

Large Angle DDFRG 800 MeV/n Ar + KCl → α



DDFRG model agrees with 15°data, but differences at larger angles

Small Angle DDFRG 1.05 GeV/n $\alpha + C \rightarrow p, d, t, h$



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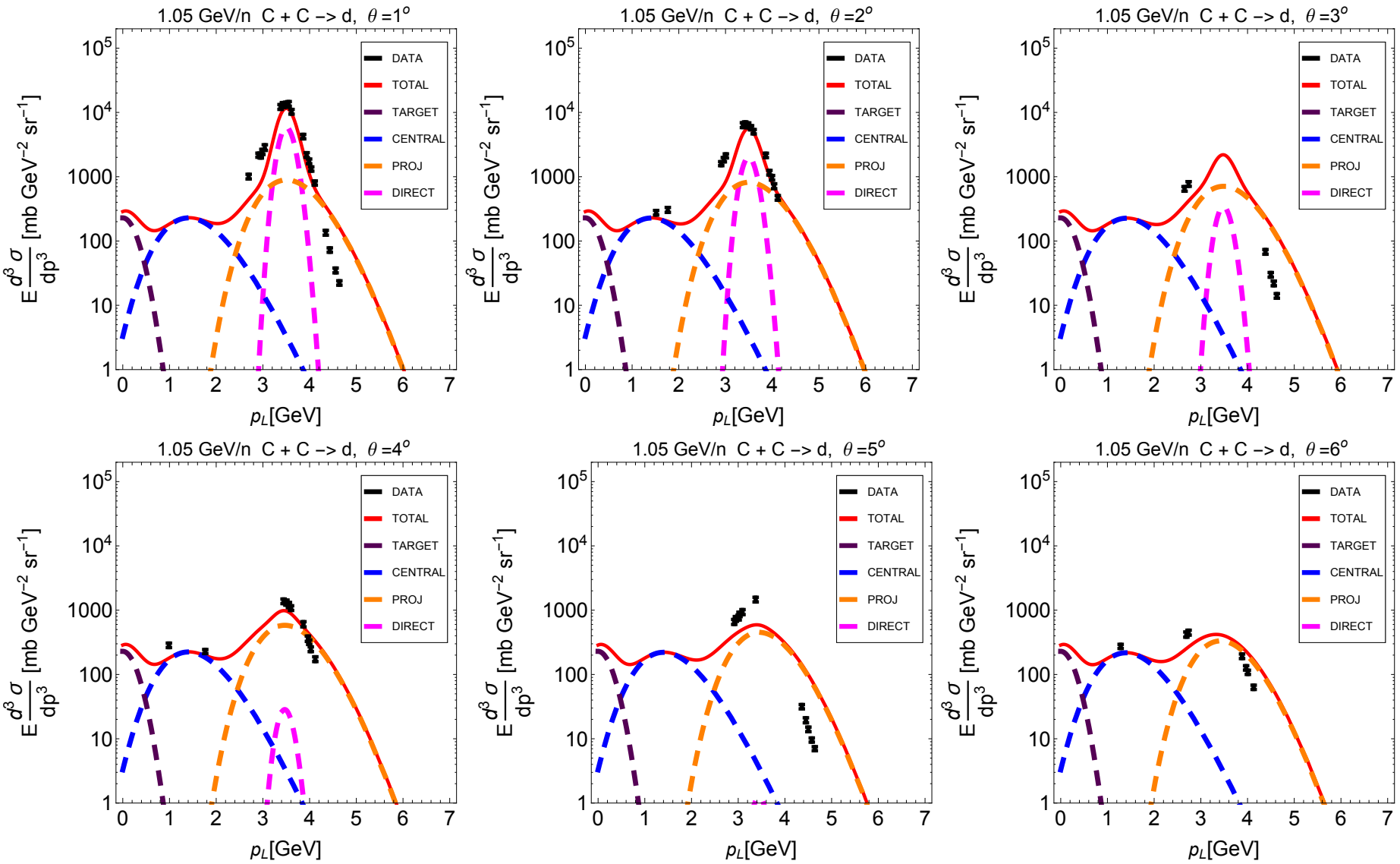
Small Angle DDFRG 1.05 GeV/n C + C → d



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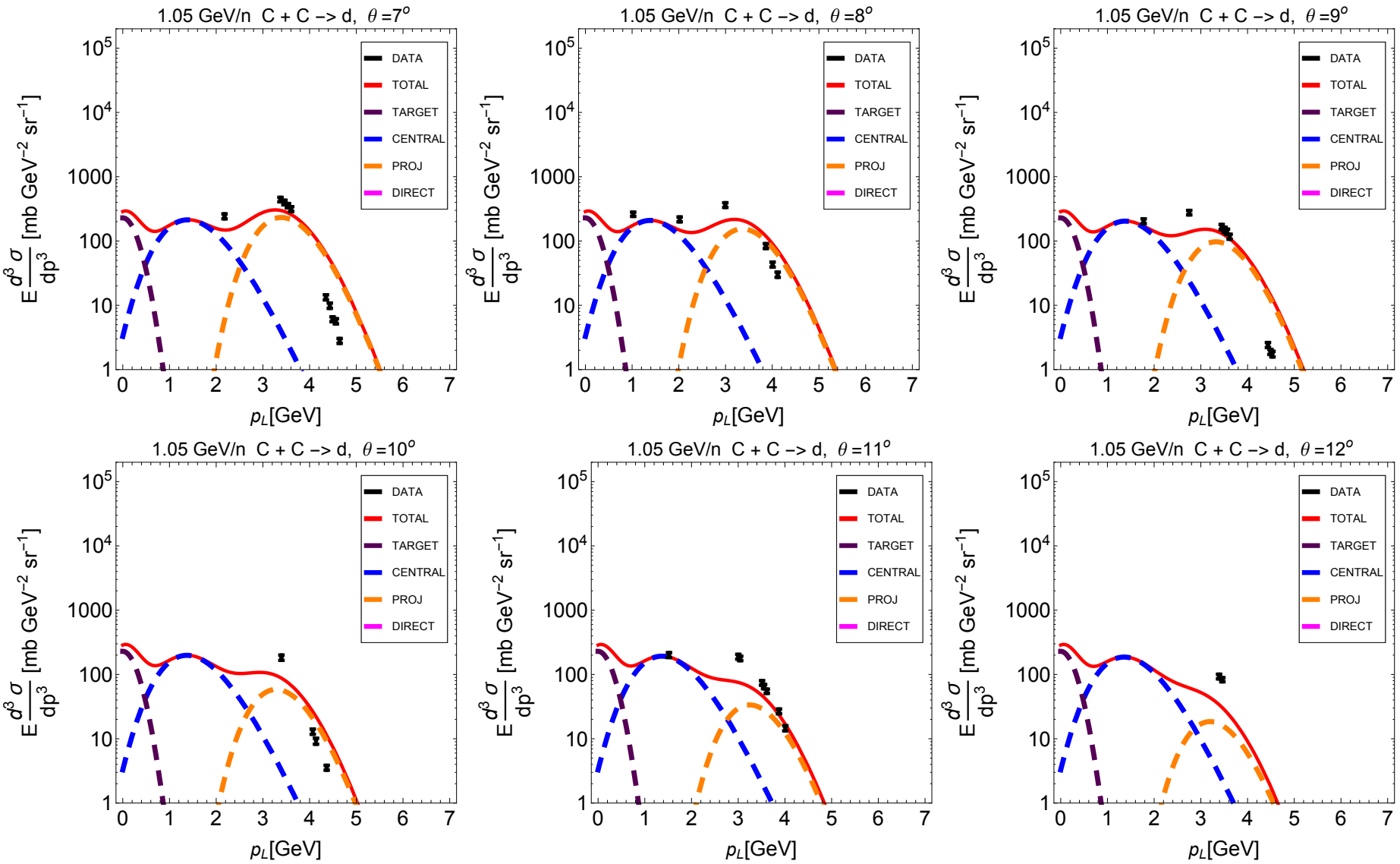
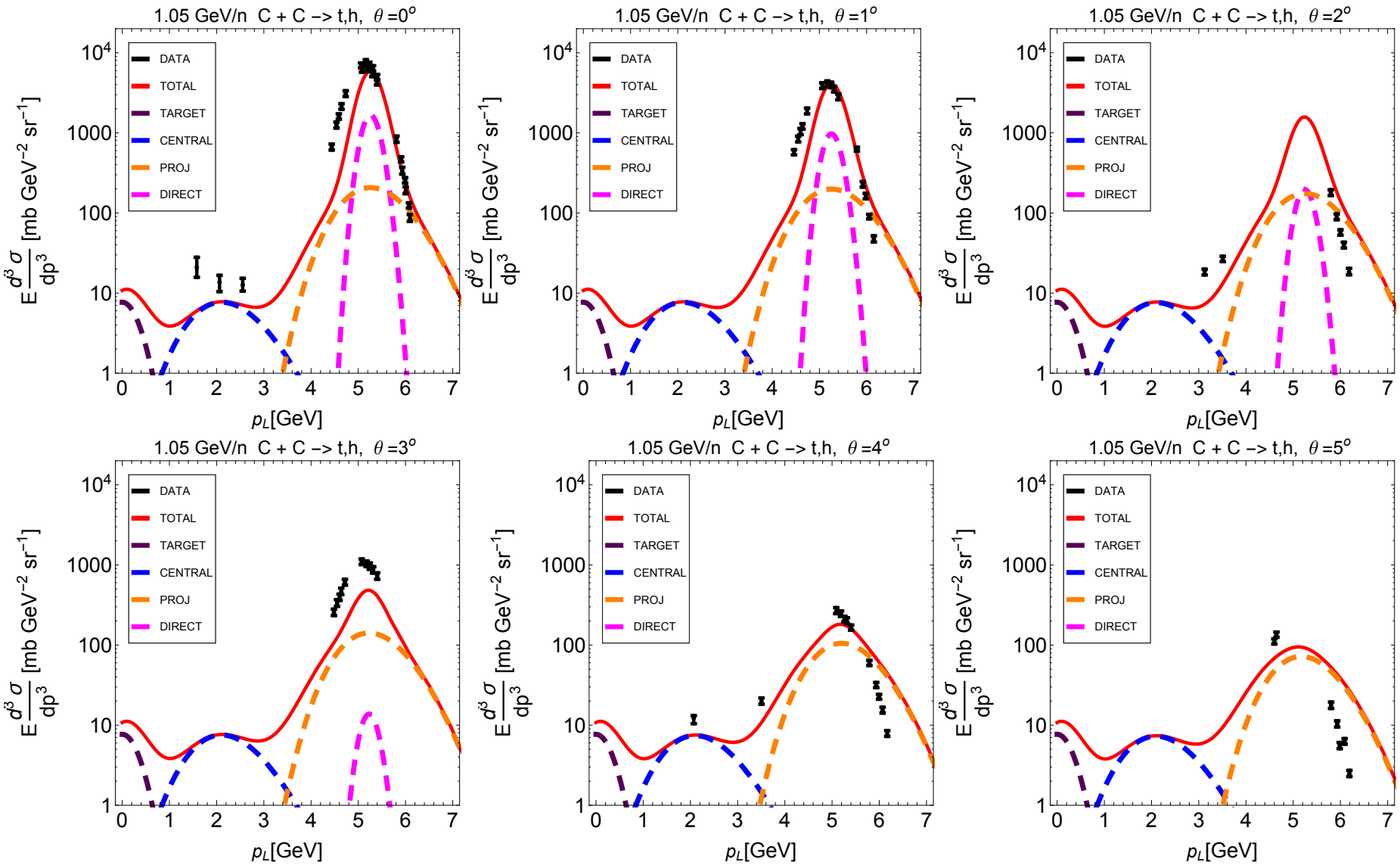
Small Angle DDFRG 1.05 GeV/n C + C → d



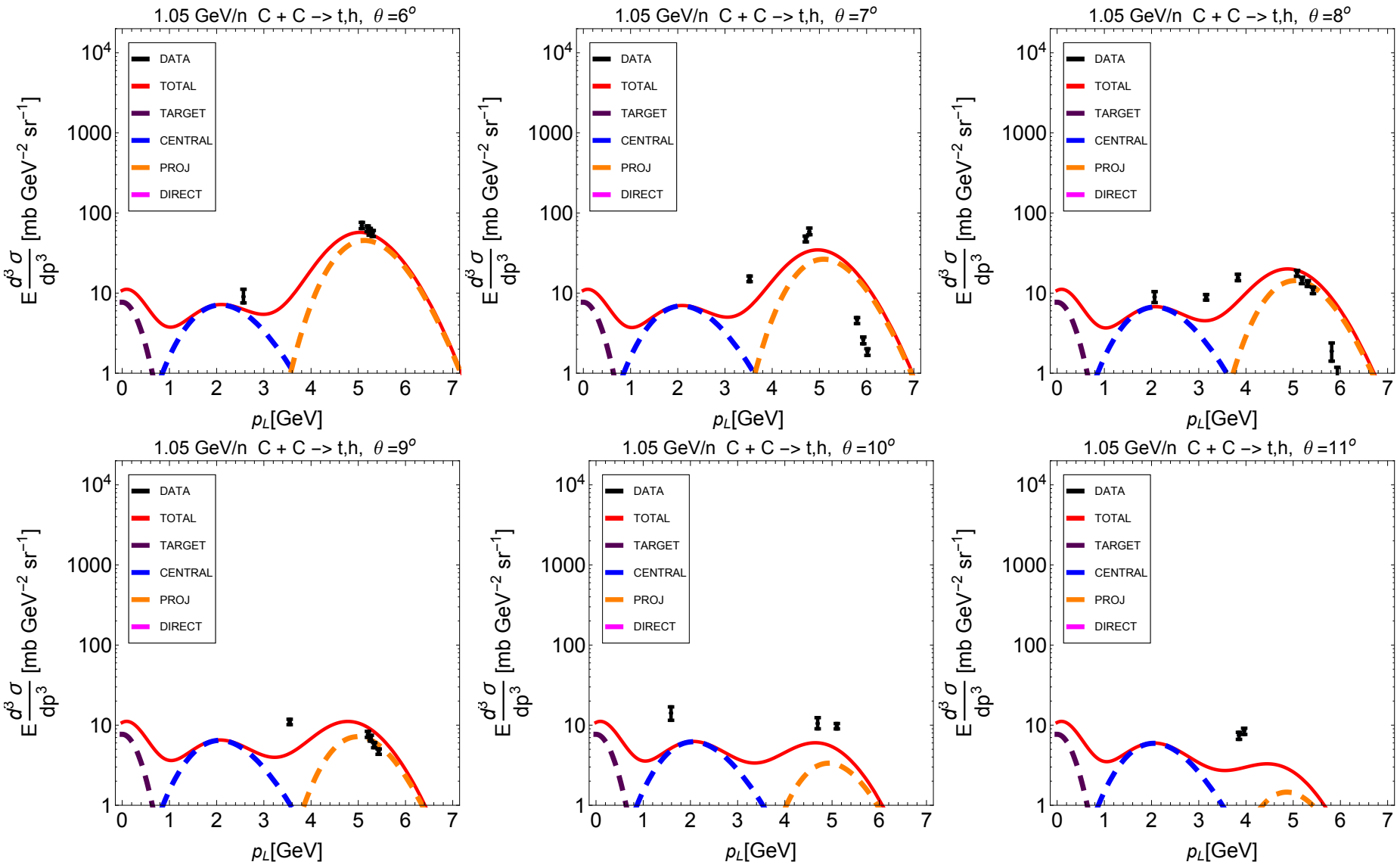
DDFRG model agrees well with data, but some differences

Small Angle DDFRG 1.05 GeV/n C + C → t, h



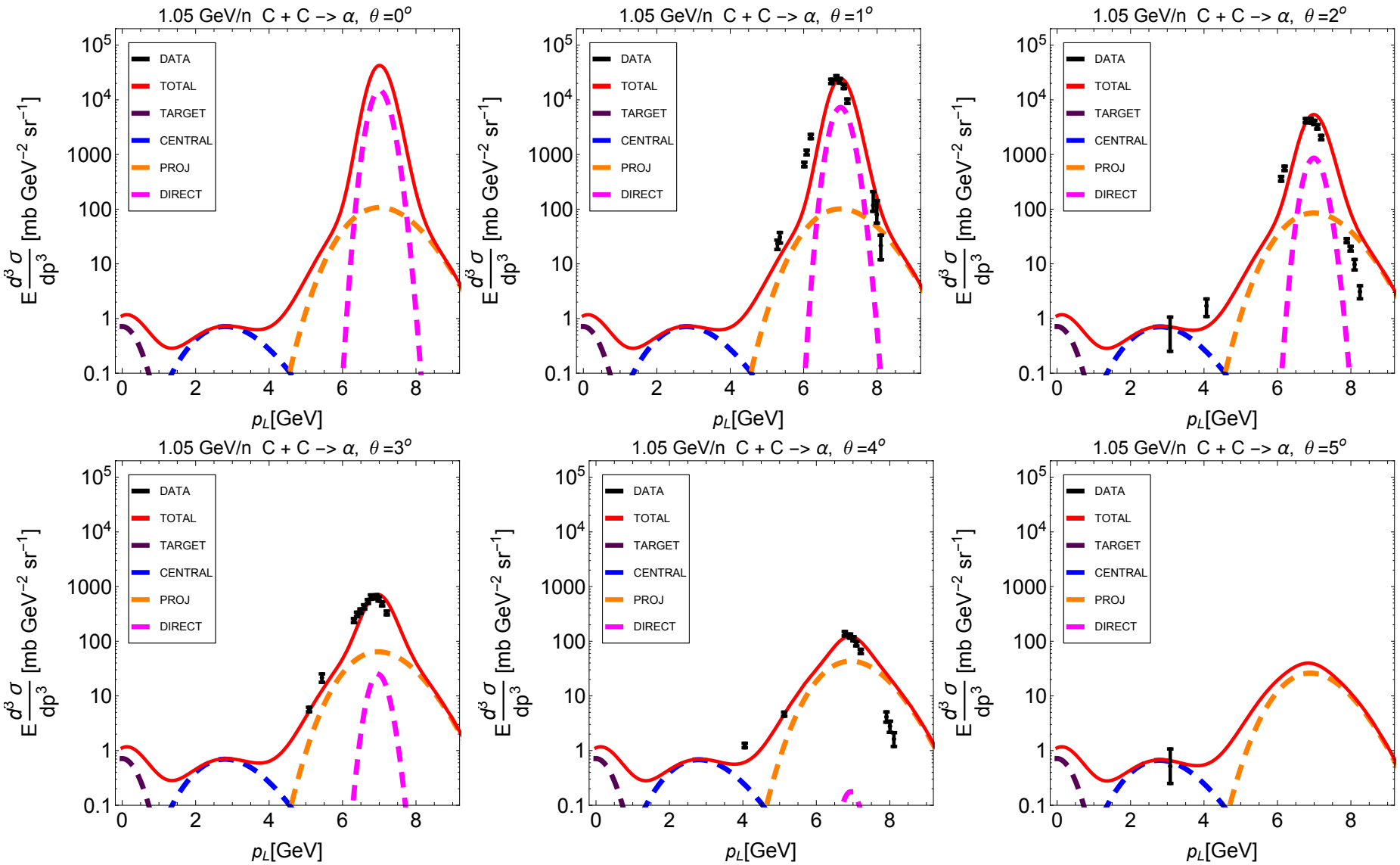
DDFRG model agrees well with data, but some differences

Small Angle DDFRG 1.05 GeV/n C + C → t, h



DDFRG model agrees well with data, but some differences

Small Angle DDFRG 1.05 GeV/n C + C → α



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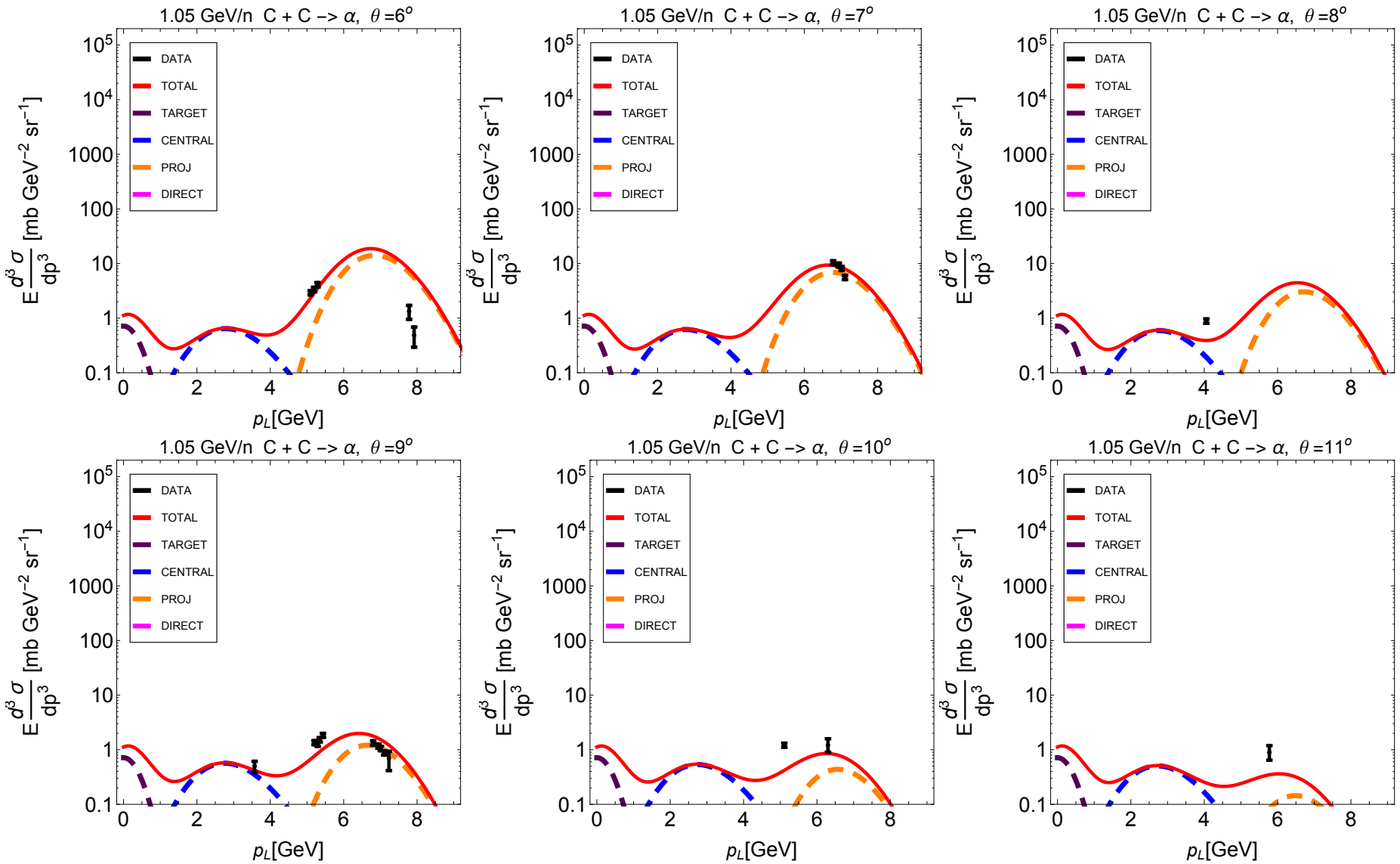

DDFRG model agrees well with data, but some differences

Summary, Conclusions & Future work

- Proton production
  - 3 thermal sources (Projectile, Target, Central fireball)
  - 1 direct source - accounts for quasielastic peak at beam rapidity
- Light ion production
  - Modified coalescence model
- DDFRG
  - Proton & Light ion double differential nuclear fragmentation model
  - 2000 data points
  - Large & small angles
- Issues
  - 15 parameters
  - Limited data comparisons - need more light ion projectile data
- Future work
  - Pions
  - Neutrons
  - Electromagnetic dissociation

THE END

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