Status of Lattice QCD Determination of Nucleon Form Factors and their Relevance for the Few-GeV Neutrino Program

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February 22, 2022

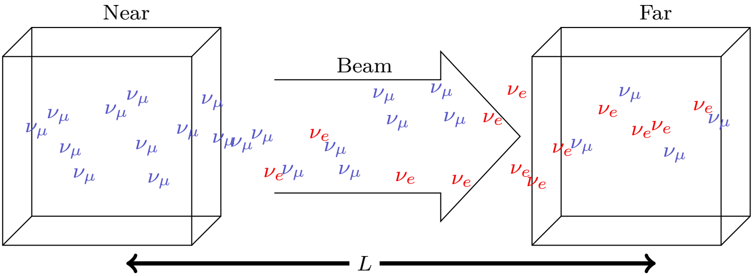
NSD Staff Meeting

Outline

- ν oscillation
- Quasielastic scattering
- LQCD Intro
- $F_A(Q^2)$ from LQCD
  - Axial radius
  - Electro Pion Production
- Free Nucleon XSec
- T2K/DUNE Implications

Introduction

Neutrino Oscillation

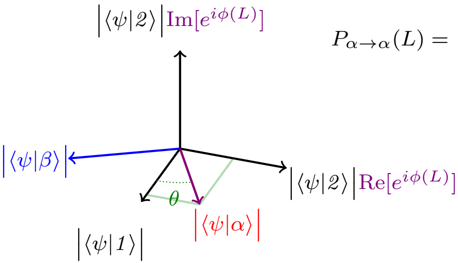


Neutrino oscillation: ν spontaneously change flavor during propagation

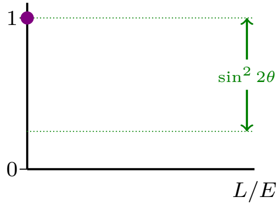
- Unaligned flavor/mass eigenstates (CKM matrix in qk sector)
- Dynamics of $\nu \implies m_\nu > 0$
- *Direct evidence of BSM physics*

Still unmeasured parameters of oscillation $\implies$ precision oscillation expts

Two-Flavor Neutrino Oscillation



$$P_{\alpha\to\alpha}(L) = \left|\langle\alpha|\psi_L\rangle\right|^2$$



$\Delta m^2 \equiv m_2^2 - m_1^2, \quad \phi(L) = \frac{1}{4}\Delta m^2 \frac{L}{E}$

creation → flavor eigenstate (α,β) $\quad |\psi_0\rangle = |\alpha\rangle = \left(\cos\theta|1\rangle + \sin\theta|2\rangle\right)$

propagation → *mass eigenstate* $(1,2)$ $\quad |\psi_L\rangle \propto \left(\cos\theta|1\rangle + e^{-i\phi(L)}\sin\theta|2\rangle\right)$

"Survival probability": $\quad \left|\langle\alpha|\psi_L\rangle\right|^2 = 1 - \sin^2 2\theta \sin^2 \frac{\phi(L)}{2}$

Want precise measurements of θ, Δm^2, CPV in 3-flavor scenario

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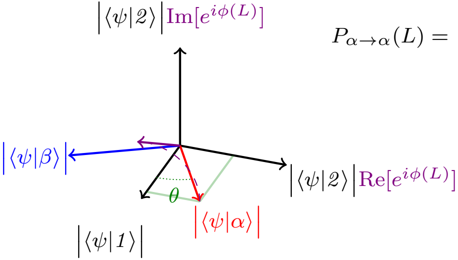

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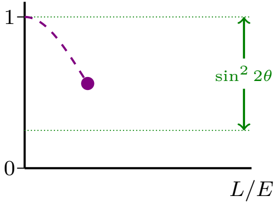

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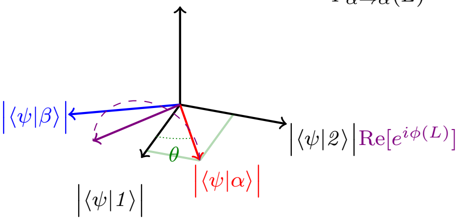

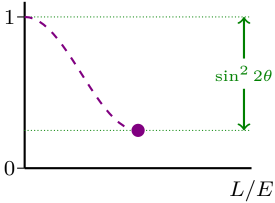

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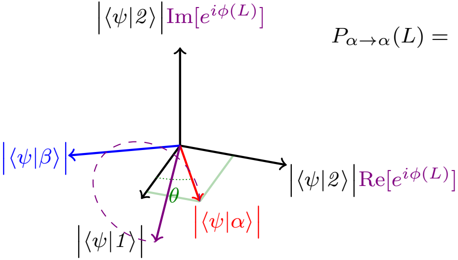

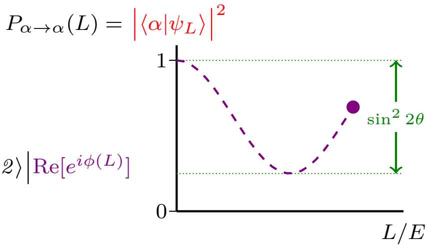

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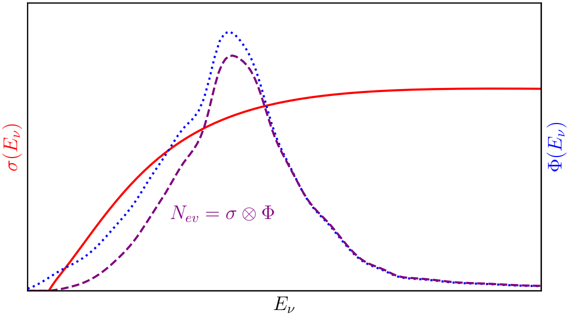
Oscillation Experiments



ν from secondary beam, decay of π mesons: distribution of E_ν
Cross section depends on E_ν...

σ : ν cross section, Φ : ν flux $\implies$ # events: convolution $\sigma \otimes \Phi$

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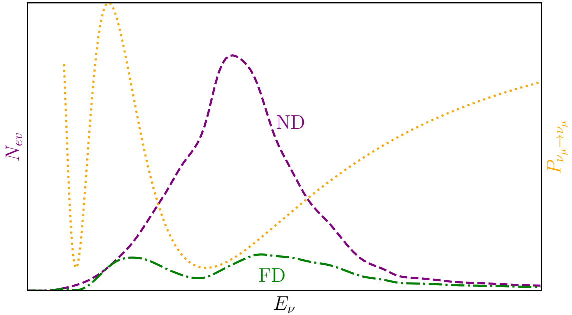


ν from secondary beam, decay of π mesons: distribution of E_ν
Cross section depends on E_ν...

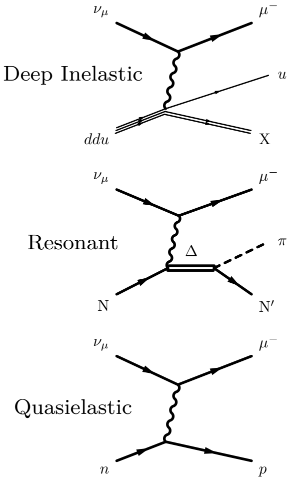
σ : ν cross section, Φ : ν flux $\implies$ # events: convolution $\sigma \otimes \Phi$

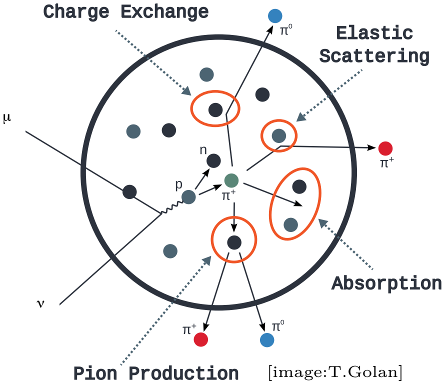
Oscillation as ν propagate, dependent on L/E_ν
ND and FD event distributions different $\implies$ determine oscillation probability

Not so simple in practice...

ν Interactions in Nuclear System

Many interaction topologies: primary & intranuclear rescattering





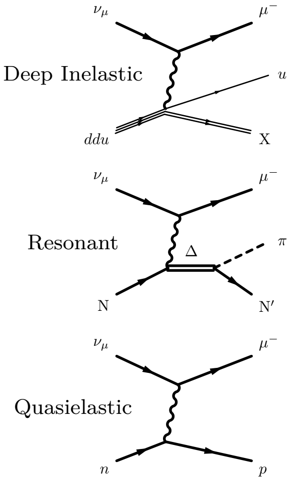
[image:T.Golan]

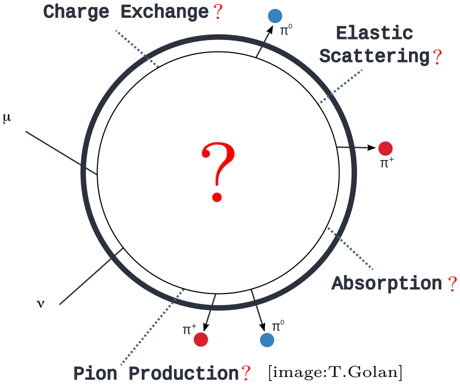
ν Interactions in Nuclear System

Many interaction topologies: primary & intranuclear rescattering

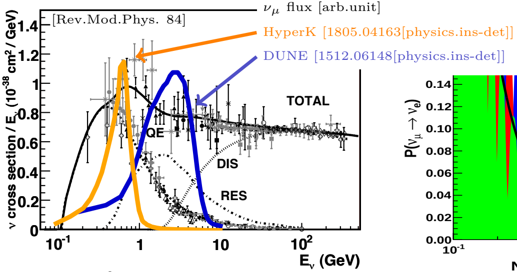
Only particles that leave nucleus are detected

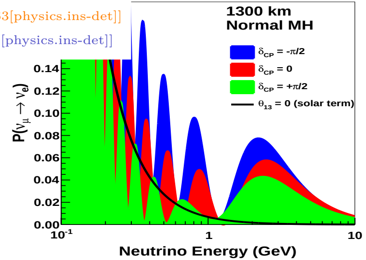
Must infer interaction topology from statistical distribution





Complications





$$N_{\text{event}} \sim \int dE_\nu \underbrace{\Phi}_{\text{flux}} \otimes \underbrace{\sigma}_{\text{xsec}} \left(\otimes \underbrace{\epsilon}_{\text{eff}}\right)$$

- ν beam not monoenergetic $\Longrightarrow$ $\Phi(E_\nu) \neq \delta(E_\nu - E_{\text{beam}})$
- E_ν not known event-by-event $\Longrightarrow$ reconstruct E_ν from distribution
- beam dynamics $\Longrightarrow$ different efficiency @ near, far
- multiple interaction mechanisms $\Longrightarrow$ precise xsecs to disentangle topologies

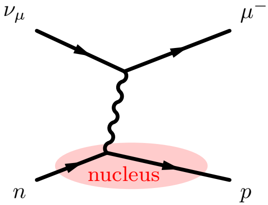
High precision expt needs robust, precise cross sections

Quasielastic Form Factors

Nuclear cross sections complicated – use free nucleon FFs are starting point

QE dominates low E_ν: HyperK mostly QE, DUNE ∼1/3 QE

Assume impulse appx.: interact with "free" nucleon in nucleus



$$\mathcal{M} = \langle \ell | J^\mu | \nu_\ell \rangle \langle N' | J_\mu | N \rangle$$

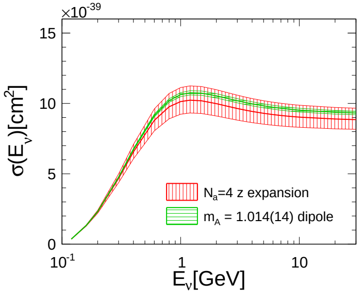
$$\langle N'(p') | J_\mu(q) | N(p) \rangle$$
$$= \bar{u}(p') \Big[\gamma_\mu F_1(q^2) + \frac{i}{2M_N} \sigma_{\mu\nu} q^\nu F_2(q^2) + \gamma_\mu \gamma_5 F_A(q^2) + \frac{1}{2M_N} q_\mu \gamma_5 F_P(q^2) \Big] u(p)$$

- F_1, F_2: constrained by eN scattering
- F_P: subleading in xsec; PCAC $\implies \propto F_A$
- F_A: poorly constrained by expt, obtained from large-A data

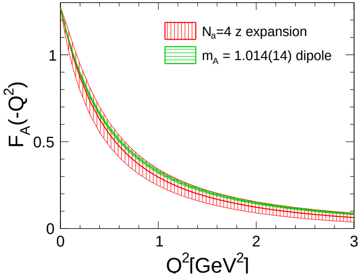
In expt, both μ and p measured

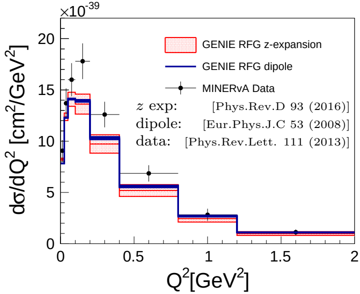
QE F_A from z Expansion

- Dipole $F_A(Q^2) = g_A/(1+Q^2/m_A^2)^2$
  strict Q^2 shape,
  inconsistent w/ QCD
- Dipole ansatz underestimates
  FF uncertainty $\times O(10)$



- Nucl. xsec uncertainty from FF
  same size as data-MC tensions
- Source of tensions unclear
  btw. nucleon/nuclear





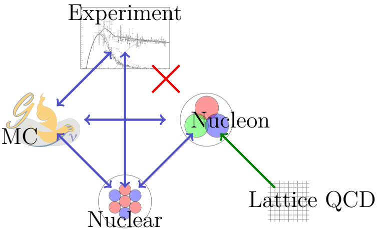
LQCD as Disruptive Technology

Ideal: Modern high stats ν-D_2 scattering bubble chamber expt

Some community push, safety concerns

$\Longrightarrow$ LQCD as a alternative/complement to expt

- ✓ No nuclear effects
- ✓ Realistic uncertainty estimates
- ✓ Systematically improvable
- ✓ Computers are (relatively) inexpensive



Build from the ground up:

Nucleon amplitudes on solid theoretical ground,
more robust inputs to nuclear model/EFT

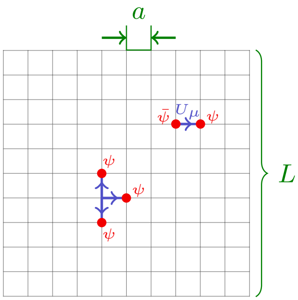
Lattice QCD

Lattice QCD Formalism

Numerical eval of path integral

Quark, gluon DOFs —

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi\, \mathcal{D}\overline{\psi}\, \mathcal{D}U\ \exp(-S)\, \mathcal{O}_{\psi}\,[U]$$



Inputs —

Computational: $am_{(u,d),\text{bare}}$, $am_{s,\text{bare}}$, $\beta = 6/g^2_{\text{bare}}$

Scale setting: e.g. $\frac{M_\pi}{M_\Omega}$, $\frac{M_K}{M_\Omega}$, M_Ω
1-to-1 w/ computational input

Results — first principles predictions from QCD Lagrangian, gluons to all orders

"Complete" error budget $\implies$ extrap in a, L, M_π guided by EFT, FVχPT

- $a \to 0$ (continuum limit)
- $L \to \infty$ (infinite volume limit)
- $M_\pi \to M_\pi^{\text{phys}}$ (chiral limit)

arXiv:2201.01839v1 [hep-lat] 5 Jan 2022

Status of Lattice QCD Determination of Nucleon Form Factors and their Relevance for the Few-GeV Neutrino Program

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Annu. Rev. Nucl. Part. Sci. 2022. AA:1–29

This article's doi:
10.1146/((please add article doi))

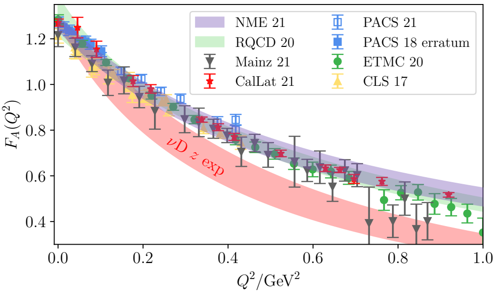
Keywords

Neutrino Oscillations, Nucleon Form Factors, Lattice QCD

Abstract

Neutrino-nucleon interactions provide the dominant contribution to neutrino-nucleus cross sections, which are critical inputs to billion-dollar experimental efforts aimed at measuring neutrino oscillation parameters. The neutrino-nucleon interactions are difficult to measure experimentally and current parameterizations rely on low-statistics measurements from a handful of historic measurements to inform a number of nucleon form factors and other key quantities. Lattice QCD can be used to determine these interactions directly from the Standard Model with fully quantified theoretical uncertainties. Recent lattice QCD results of g_{A} are in excellent agreement with experimental data, offering hope that soon, results for the (quasi-)elastic nucleon form factors will be available. We review the status of the field and lattice QCD results for the nucleon axial form factor, $F_{\mathrm{A}}(Q^2)$, a major source of uncertainty in neutrino-nucleon interaction parameterizations for $E_\nu \lesssim 1$ GeV. Results from different lattice calculations are in good agreement with each

Nucleon Axial Form Factor



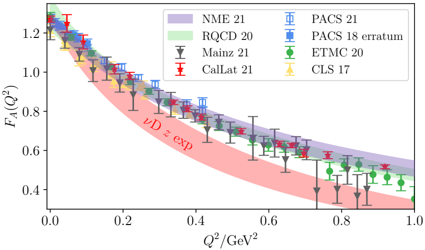
LQCD results maturing:

- Many results: independent data & different methods
- Full error budget (bands) vs. single ensemble (scatter points)
- Agreement w/ single ensemble $\Longrightarrow$ uncontrolled systematics are small
- Extrapolated results (bands) satisfy GGT/PCAC checks
  Lots of recent effort to understand

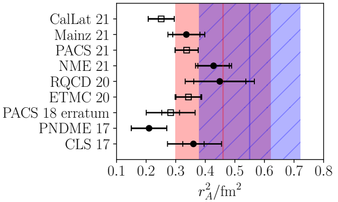
Indication of slow Q^2 falloff

Situation unlikely to change drastically

Axial Radius



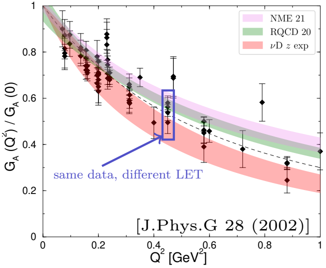
Filled circle: full error budget
Open square: incomplete
elec π prod: [Phys.Rev.D 84 (2011)]
D_2 scatter: [Phys.Rev.D 93 (2016)]

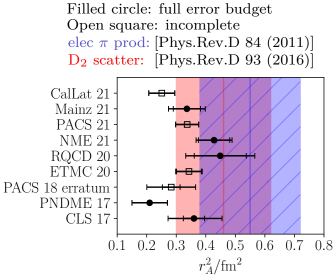


- Dipole shape fixed by $r_A^2 = -\frac{6}{g_A}\frac{dF_A}{dQ^2} \equiv \frac{12}{m^2_{A,\mathrm{dipole}}}$
- LQCD, D_2 agree on r_A^2 (slope) at $Q^2 = 0$
- Dipole tracks with D_2 over Q^2, tension w/ LQCD at large Q^2
- If LQCD prefers dipole, must also track with D_2

LQCD (c.f. NME, RQCD) disfavors dipole ansatz

Electro Pion Production





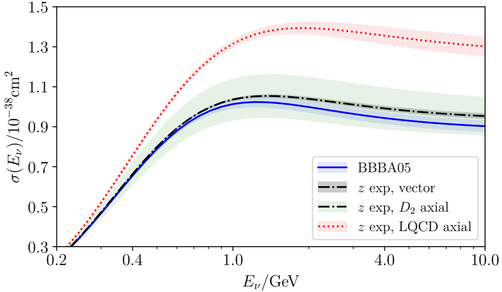
LQCD appropriately consistent with (circa 2002) EPP data:

- Known problems — lacked systematic power counting at time
- EPP strictly valid in $M_\pi \to 0$, $k \sim q \to 0$ limits

Approximate agreement with old data, but much progress since then

Comparison w/ modern literature is work in progress

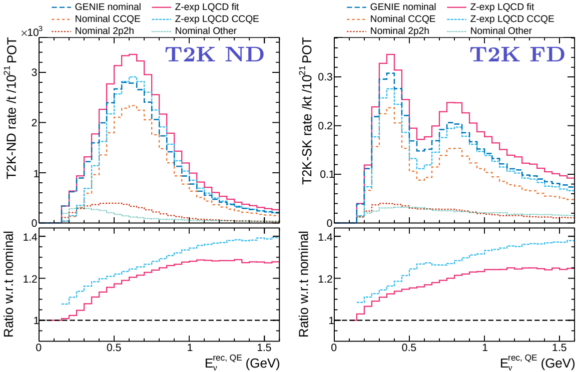
Free Nucleon Cross Section



Slow falloff integrated over Q^2 magnifies tensions:

- Uncertainty reduced $\times 2$ (red vs. green)
  $\Longrightarrow$ better precision than D_2 scattering data
- LQCD prefers 30-40% enchancement of ν_μ QE cross section
- Experimental evidence -
  recent MicroBooNE tune preferred 20% enhancement of QE
  no more indication of low-E_ν excess [2110.14028[hep-ex]]
- With improved precision, sensitive to vector FF tension (black vs blue)
  [Phys.Rev.D 102 (2020)] vs [Nucl.Phys.B Proc.Suppl. 159 (2006)]

T2K/DUNE Implications



- Dashed dark blue (GENIE nominal) vs solid magenta (z exp LQCD fit)
- QE xsec enhancements produce 10-20% ν_μ event rate enhancement
- E_ν-dependent modifications to event rate
- xsec changes at ND $\neq$ *effective* xsec changes at FD:
  insufficient CCQE model freedom $\rightarrow$ bias in FD prediction

Risk of bias in oscilliation predictions

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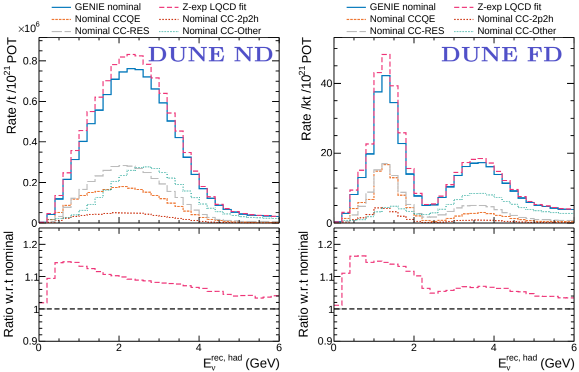

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Risk of bias in oscilliation predictions

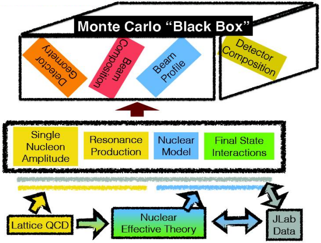
Future — PD+NSD LDRD proposal (Wilkinson)

- LQCD calculations of $N \to N\pi$ transitions
- reweighting pkg for integration into MC generators

The solution 2: fast-MC/reweighting

- Challenge to integrate new theory models into generators
- **Short term, LDRD:** fast-MC focused on pion production. “Reweighting” tool for rapid integration into existing analysis chains

$$w = \frac{\sigma(\vec{x})_{\mathrm{NEW}}}{\sigma(\vec{x})_{\mathrm{OLD}}}$$



- First building block of a neutrino generator rooted in SM
- **Long term potential:** evolve into a full generator with “plug and play” theory interfaces to overcome HEP/NP divide

Outlook

Hints from LQCD suggest large changes to single nucleon amplitudes
will have large impact on ν oscillation —

- LQCD prefers F_A with slow Q^2 falloff relative to D_2 scattering
  $\rightarrow$ consistency btw collabs w/ independent data, different methods
- LQCD disfavors dipole ansatz
- LQCD consistency with EPP fine
- Slower Q^2 falloff translates to 30-40% enhancement of QE xsec
- Implies modifications to xsec model are warranted
  $\Longrightarrow$ caution advised to avoid bias in ν oscillation analyses

Demonstration of utility of LQCD to ν oscillation, other computations to come
$N\pi$, inclusive resonant, DIS, NN, ...

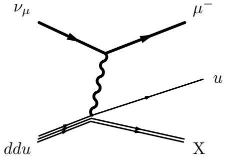
Thanks for your attention!

Backup

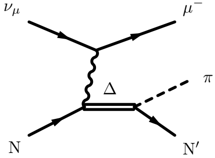
Interaction Modes

(incomplete list!)

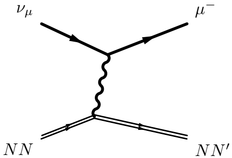
Deep Inelastic



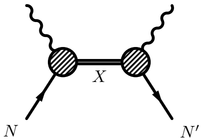
Resonant $N\pi$



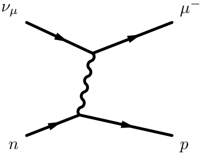
NN Quasielastic

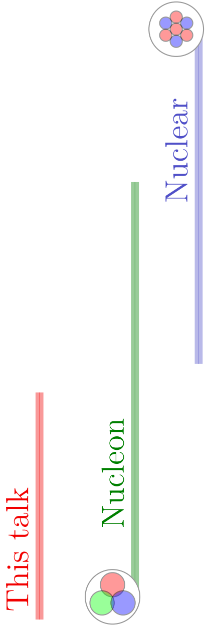


Four-point Inclusive



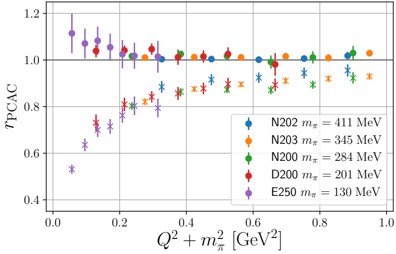
Quasielastic



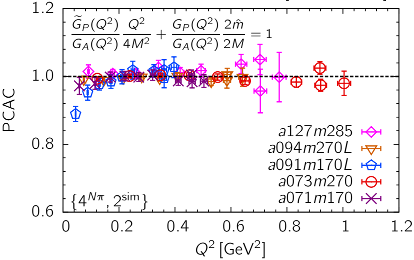


Generalized Goldberger-Triemann

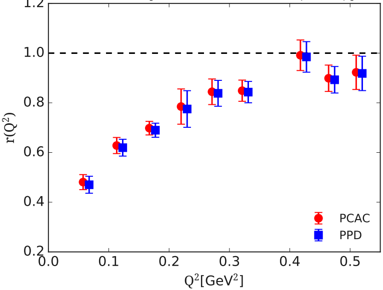
RQCD [JHEP 05 (2020)]



NME [2103.05599]

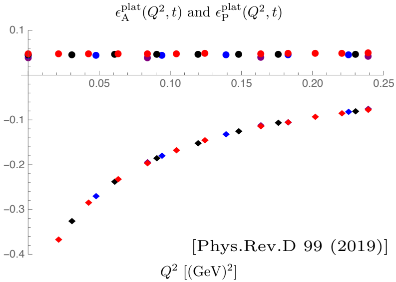


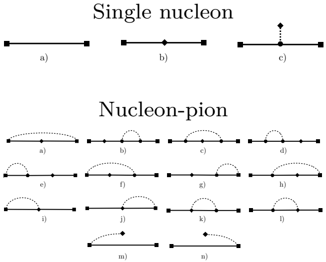
ETMC [Phys.Rev.D 103 (2021)]



- Relation btw F_A, F_P, $\tilde{F}_P$ via PCAC
- $N\pi$ excited states important
  [Phys.Rev.Lett. 124 (2020)]
- Contamination in F_A and $\tilde{F}_P$, F_P
  very different [Phys.Rev.D 99 (2019)]
  $\Longrightarrow$ nontrivial consistency check
- Two approaches:
  χPT-inspired fit function (RQCD)
  Spectrum from A_4 current (NME)

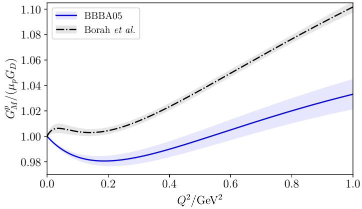
χPT Expectation





- (induced) pseudoscalar $\sim$ tree level (strong Q^2 dep.)
- axial $\sim$ loop level (mild Q^2 dep.)

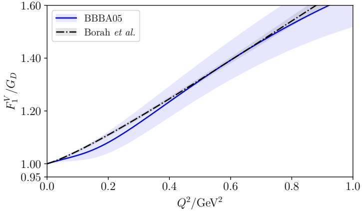
Vector Form Factors



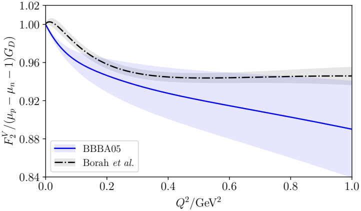
Proton Magnetic

- Tension in $G_{M,p}$ btw BBBA05 [Nucl.Phys.B Proc.Suppl. 159 (2006)] vs Borah *et al.* [Phys.Rev.D 102 (2020)]
- Borah *et al.* uses z expansion, modern data
- Mainz data included, only low Q^2

Isovector F_1



Isovector F_2



Dipole Axial Form Factor

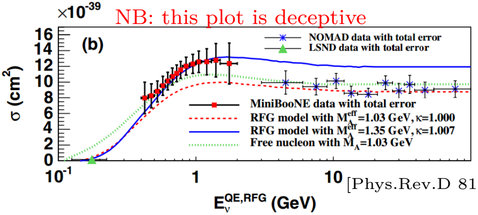
Most widely used: dipole ansatz [Phys.Rept.3 (1972)]

$$F_A^{\text{dipole}}(Q^2) = g_A \left(1 + Q^2/M_A^2\right)^{-2}$$

Large variation in M_A ("axial mass problem"):

- $M_A = 1.026 \pm 0.021$ [J.Phys.G 28 (2002)]
- $M_A^{\text{eff}} = 1.35 \pm 0.17$ [Phys.Rev.D 81 (2010)]

NB: this plot is deceptive



[Phys.Rev.D 81 (2010)]

- M_A^{eff}: nuclear modeling & nucleon FF entangled
- Expts: different selection criteria, sensitivity, MC model, ...

Goal: isolate nucleon FF, then address modeling

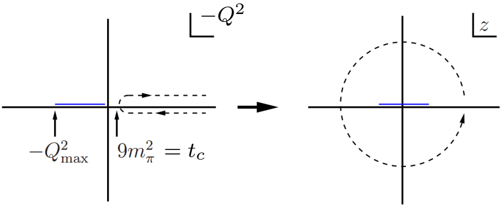
z Expansion

Want model independence, Q^2 expansion only good for $Q^2 \ll 1$

Conformal mapping: [Phys.Rev.D 84 (2011)]

$$z(-Q^2; t_0, t_c) = \frac{\sqrt{t_c + Q^2} - \sqrt{t_c - t_0}}{\sqrt{t_c + Q^2} + \sqrt{t_c - t_0}} \qquad F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad t_c = 9m_\pi^2$$

$-Q^2 \leq 0$ kinematically allowed $\quad \rightarrow \quad |z| < 1$



- Long history w/ flavor physics & CKM determination (≤ 1971)
- Model independent: motivated by analyticity of QCD
- $|z|^k$, $|a_k| \rightarrow 0$ as $k \rightarrow \infty$
- Truncate at finite $k_{\rm max}$, use sum rules to regulate large-Q^2

Axial Charge: $g_A(Q^2 = 0)$

g_A is benchmark for
nucleon matrix elements in LQCD

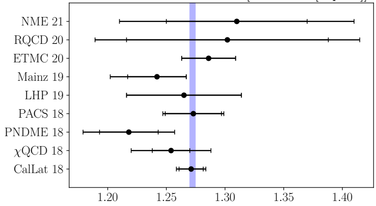
Status circa 2018 summarized by
USQCD white paper
[Eur.Phys.J.A 55 (2019)]

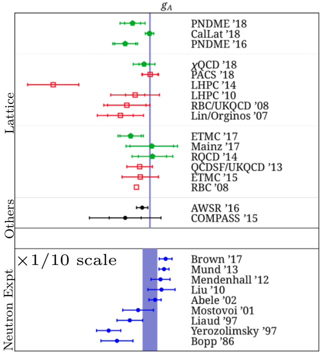
See also: FLAG review
[Eur.Phys.J.C 80 (2020)]

Historically g_A low compared to expt
excited states (+other...)

Lots of activity since 2018,
consistent agreement with PDG
full error budgets available

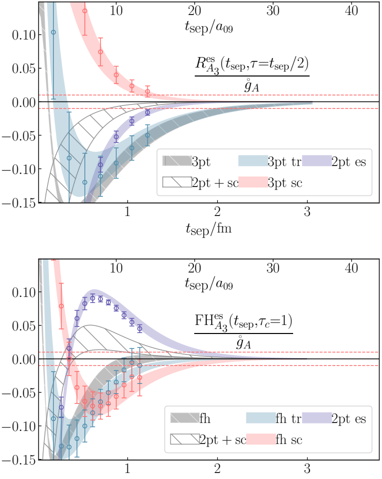
See refs. in: [2103.05599[hep-lat]]





[Eur.Phys.J.A 55 (2019)]

Excited States in g_A



[2104.05226[hep-lat]]

Compare traditional 3pt
vs Feynman-Hellman

$a \approx 0.09$ fm, $M_\pi \approx 310$ MeV

Detailed excited state fit w/ 5 states

2pt excited state (blue) &
3pt "scattering" ($n \to n$, red)
contrib. similar, approx. cancellation

Remnant contamination dominated
by "transition" ($m \to n$, violet)

Fits to many small $t_{\rm sep}$ better than
fits with few large $t_{\rm sep}$

Expect excited states to be problematic