Towards a unified picture for dilute-dense dynamics of QCD

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work in preparation in collaboration with
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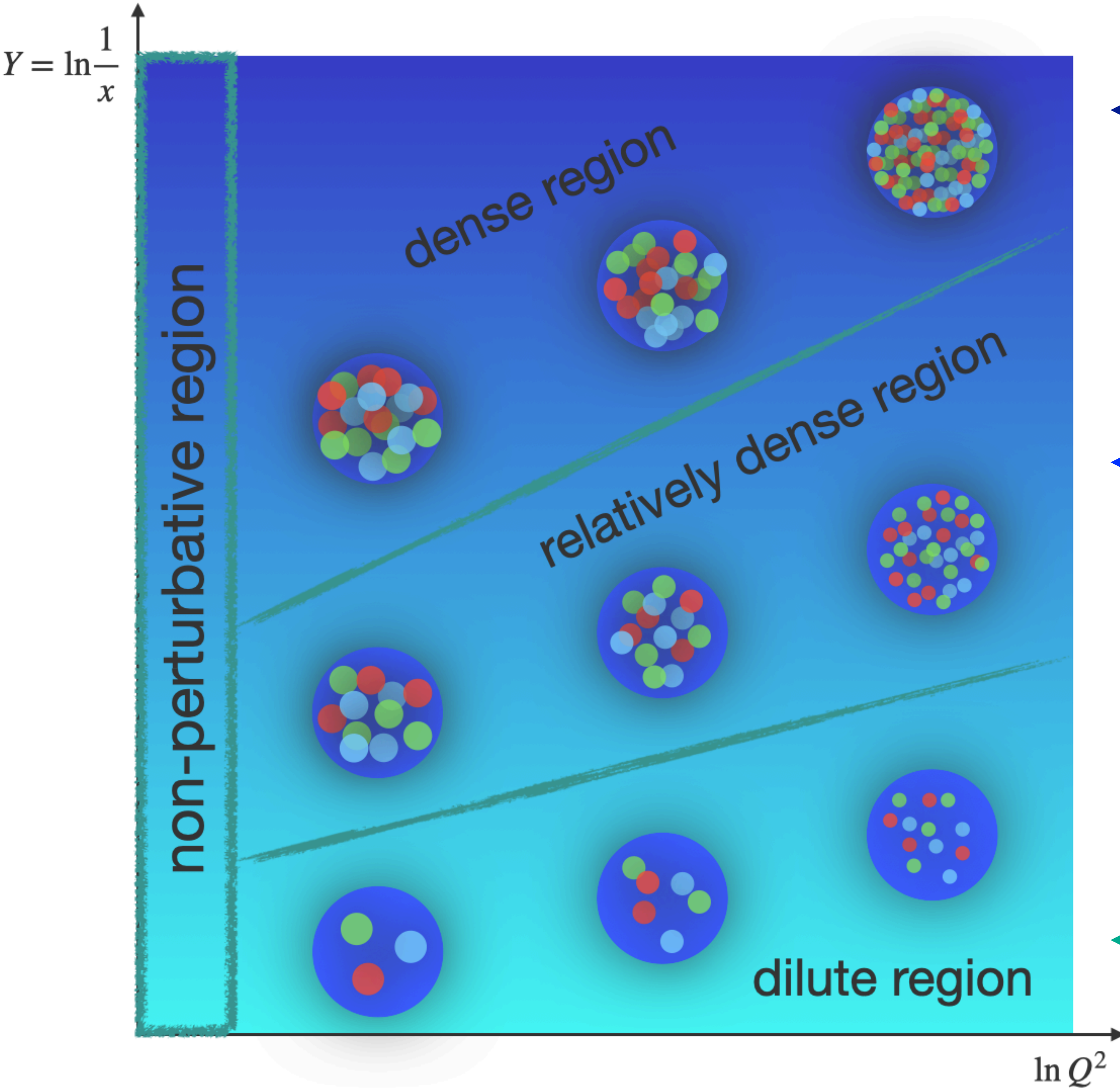
Outline

- The physics of dilute and dense regimes of QCD
- Direct photon production as a probe of QCD matter
- Matching CGC and high twist formalism
- Outlook

Anatomy of QCD matter

The Physics of dilute and dense regimes of QCD



Color Glass Condensate (CGC) saturation formalism

Strong fields, Wilson lines, non-linear evolution (BK/JIMWLK)

Review:
Gelis, Iancu, Venugopalan (2003)

High-twist formalism

Multiparton correlations, non closed evolution equations

Qiu, Sterman (1991)
Guo, Wang (2000)
Qiu, Vitev (2003)
Kang, Wang, Wang, Xing (2013)

Collinear factorization

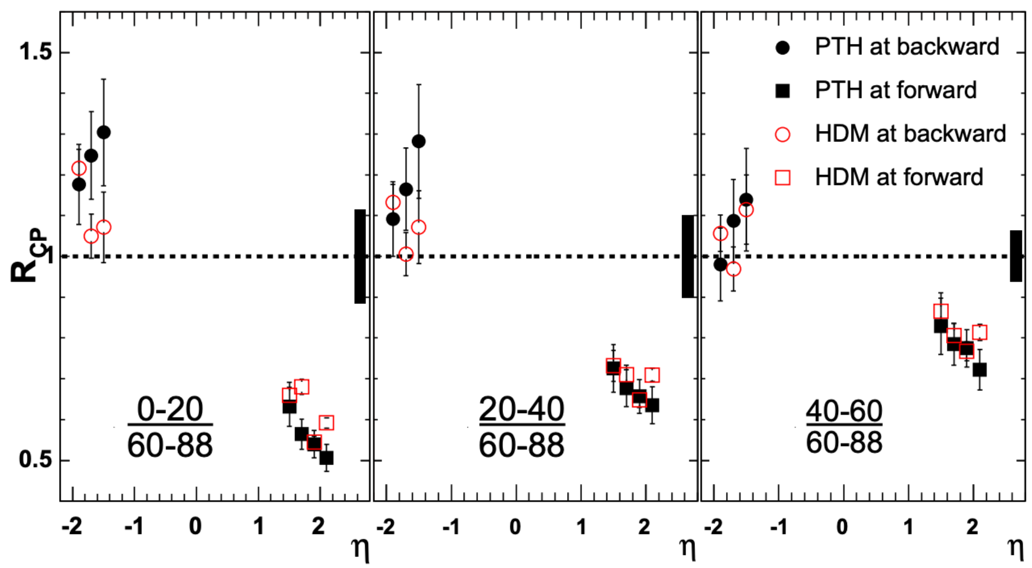
Parton distribution functions, linear evolution (DGLAP)

Collins, Soper (1981)

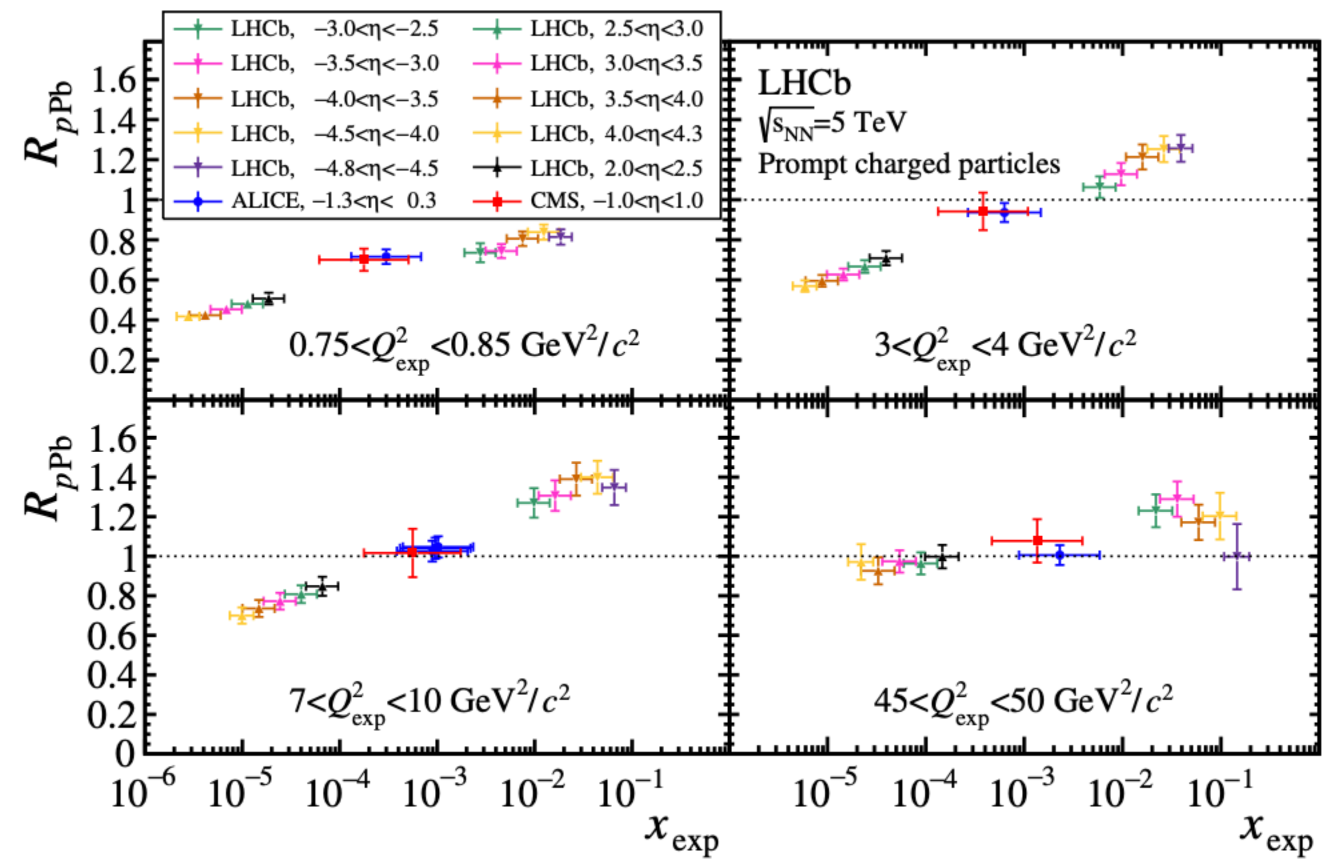
Nuclear modification ratio

Enhancement (backward region) vs suppression (forward region)

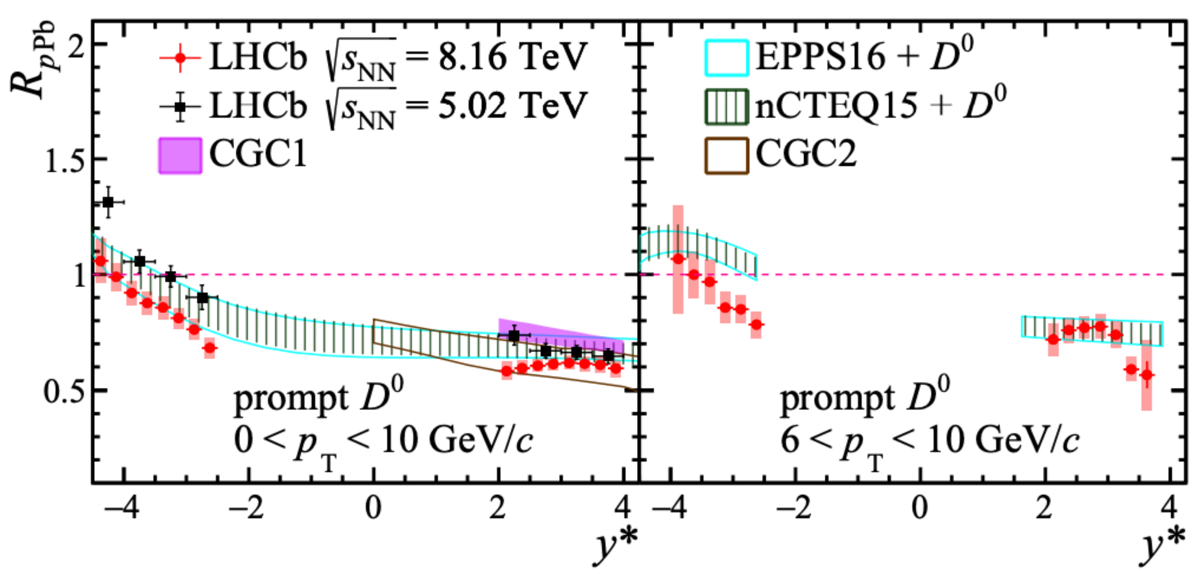
PHENIX (2004)
hadron production in dAu



LHCb (2022)
Charged particle production pPb

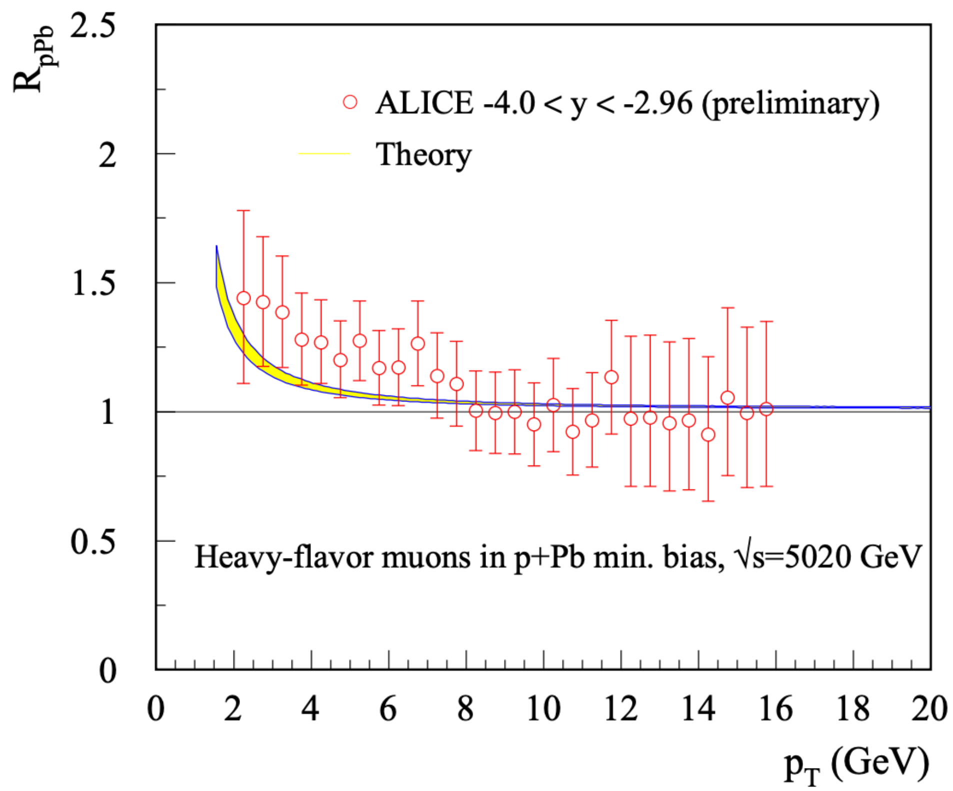


LHCb (2022)
prompt D meson in pPb

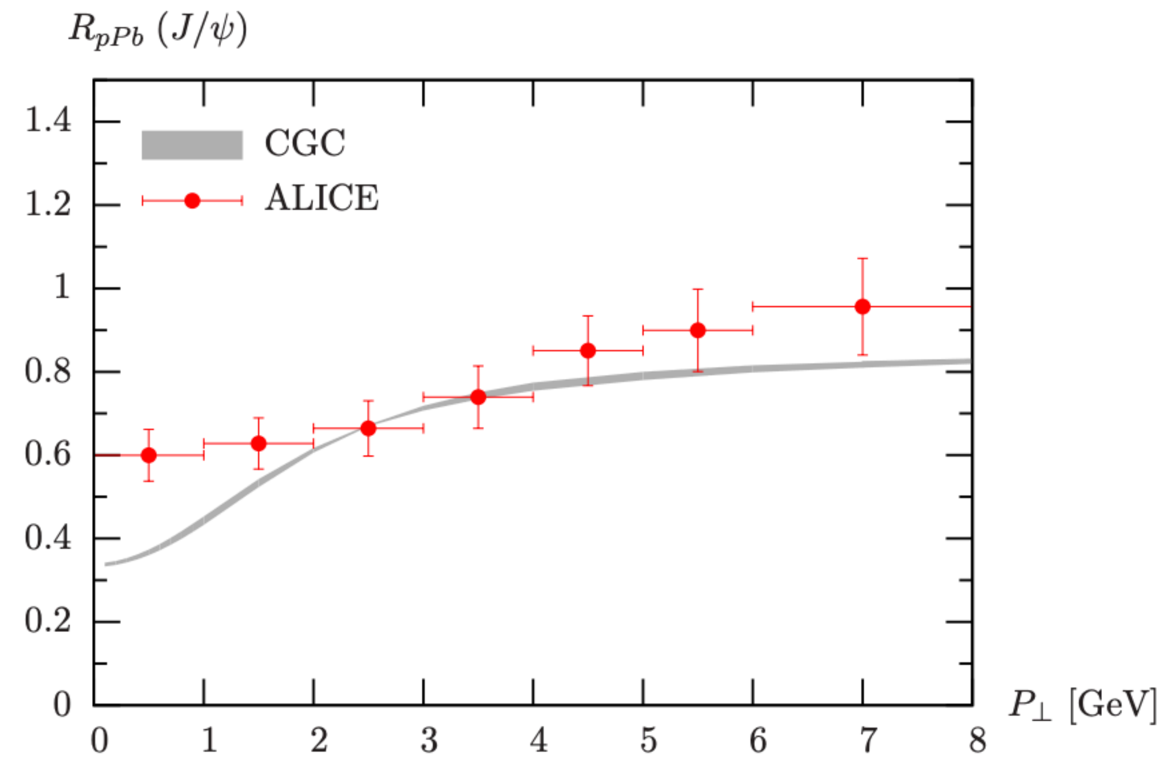


Nuclear modification ratio

Enhancement (backward region) vs suppression (forward region)



Kang, Vitev, E. Wang, Xing, Zhang (2014)



Ducloué, Lappi, Mäntysaari (2015)

High-twist formalism

<u>enhancement in backward production</u> due to nuclear enhancement of incoherent scattering

Luo, Qiu, Sterman (1993)

CGC/saturation

<u>suppression in forward production</u> due to coherence and non-linear evolution

Kharzeev, Kovchegov, Tuchin (2003)

Can we unify both formalisms and provide a simultaneous description of both regimes?

A unified picture of dilute and dense limits

Several efforts in this direction:

Quark jets scattering from a gluon field: From saturation to high p_t

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(Received 18 September 2018; published 30 January 2019)

We continue our studies of possible generalization of the color glass condensate effective theory of high energy QCD to include the high p_t (or equivalently large x) QCD dynamics as proposed in [Phys. Rev. D **96**, 074020 (2017)]. Here, we consider scattering of a quark from both the small and large x gluon degrees of freedom in a proton or nucleus target and derive the full scattering amplitude by including the interactions between the small and large x gluons of the target. We thus generalize the standard eikonal approximation for parton scattering, which can now be deflected by a large angle (and therefore have large p_t) and also lose a significant fraction of its longitudinal momentum (unlike the eikonal approximation). The corresponding production cross section can thus serve as the starting point toward the derivation of a general evolution equation that would contain the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equation at large Q^2 and the Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner evolution equation at small x. This amplitude can also be used to construct the quark Feynman propagator, which is the first ingredient needed to generalize the color glass condensate effective theory of high energy QCD to include the high p_t dynamics. We outline how it can be used to compute observables in the large x (high p_t) kinematic region where the standard color glass condensate formalism breaks down.

DOI: 10.1103/PhysRevD.99.014043

Next-to-eikonal corrections in the CGC: gluon production and spin asymmetries in pA collisions

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ABSTRACT: We present a new method to systematically include corrections to the eikonal approximation in the background field formalism. Specifically, we calculate the subleading, power-suppressed corrections due to the finite width of the target or the finite energy of the projectile. Such power-suppressed corrections involve Wilson lines decorated by gradients of the background field — thus related to the density - of the target. The method is of generic applicability. As a first example, we study single inclusive gluon production in pA collisions, and various related spin asymmetries, beyond the eikonal accuracy.

KEYWORDS: QCD Phenomenology, Hadronic Colliders

ARXIV EPRINT: 1404.2219

Gluon TMD in particle production from low to moderate x

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[b] *Theory Group, Jefferson Lab, 12000 Jefferson Ave, Newport News, VA 23606, U.S.A.*
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ABSTRACT: We study the rapidity evolution of gluon transverse momentum dependent distributions appearing in processes of particle production and show how this evolution changes from small to moderate Bjorken x.

KEYWORDS: Deep Inelastic Scattering (Phenomenology), QCD Phenomenology

ARXIV EPRINT: 1603.06548

Gluon-mediated inclusive Deep Inelastic Scattering from Regge to Bjorken kinematics

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ABSTRACT: We revisit high energy factorization for gluon mediated inclusive Deep Inelastic Scattering (DIS) for which we propose a new semi-classical approach that accounts systematically for the longitudinal extent of the target in contrast with the shockwave limit. In this framework, based on a partial twist expansion, we derive a factorization formula that involves a new gauge invariant unintegrated gluon distribution which depends explicitly on the Feynman x variable. It is shown that both the Regge and Bjorken limits are recovered in this approach. We reproduce in particular the full one loop inclusive DIS cross-section in the leading twist approximation and the all-twist dipole factorization formula in the strict $x = 0$ limit. Although quantum evolution is not discussed explicitly in this work, we argue that the proper treatment of the x dependence of the gluon distribution encompasses the kinematic constraint that must be imposed on the phase-space of gluon fluctuations in the target to ensure stability of small-x evolution.

KEYWORDS: Deep Inelastic Scattering or Small-X Physics, Parton Distributions

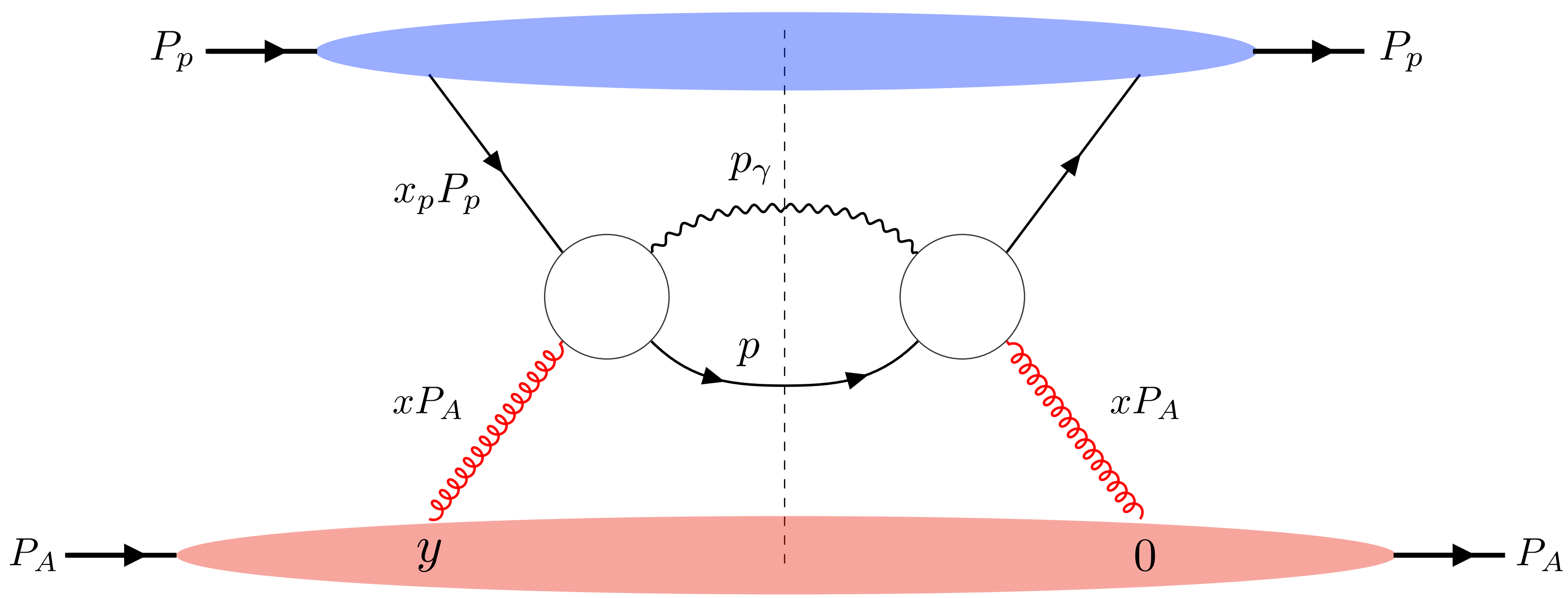
ARXIV EPRINT: 2112.01412

+ many more!

Probing QCD matter with photons

Direct photon production in collinear factorization

- Consider quark-gluon initiated channel



$$p_\gamma^- \frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}p_\gamma^-\,\mathrm{d}^2\boldsymbol{p_\gamma}_\perp} = \frac{\alpha_{\mathrm{em}}\alpha_s}{N_c}\int \mathrm{d}x_p f(x_p)\frac{\xi^2\left[1+(1-\xi)^2\right]}{\boldsymbol{p_\gamma}_\perp^4} xg(x)$$

$$\boxed{xg(x) = \frac{1}{P_A^+}\int\frac{\mathrm{d}y^-}{2\pi}e^{ixP_A^+y^-}\left\langle P_A\middle|F_a^{\alpha+}(y^-)F_a^{\beta+}(0^-)\middle|P_A\right\rangle\delta_{\perp\alpha\beta}}$$

Probing QCD matter with photons

Direct photon production beyond twist-2

$$\mathrm{d}\sigma \sim \frac{1}{p_{\gamma\perp}^4}\left[A + B\frac{\langle k_\perp^2\rangle}{p_{\gamma\perp}^2} + C\frac{\langle k_\perp^2\rangle^2}{p_{\gamma\perp}^4}\ldots\right]$$

leading twist
(twist-2)

Higher twist
(twist-4 and twist-6)

- Higher twist become important at moderate $p_{\gamma\perp}^2$
- What is the intrinsic momentum $\langle k_\perp^2\rangle$ of the nucleus ?

Λ_{QCD}^2? Q_s^2?

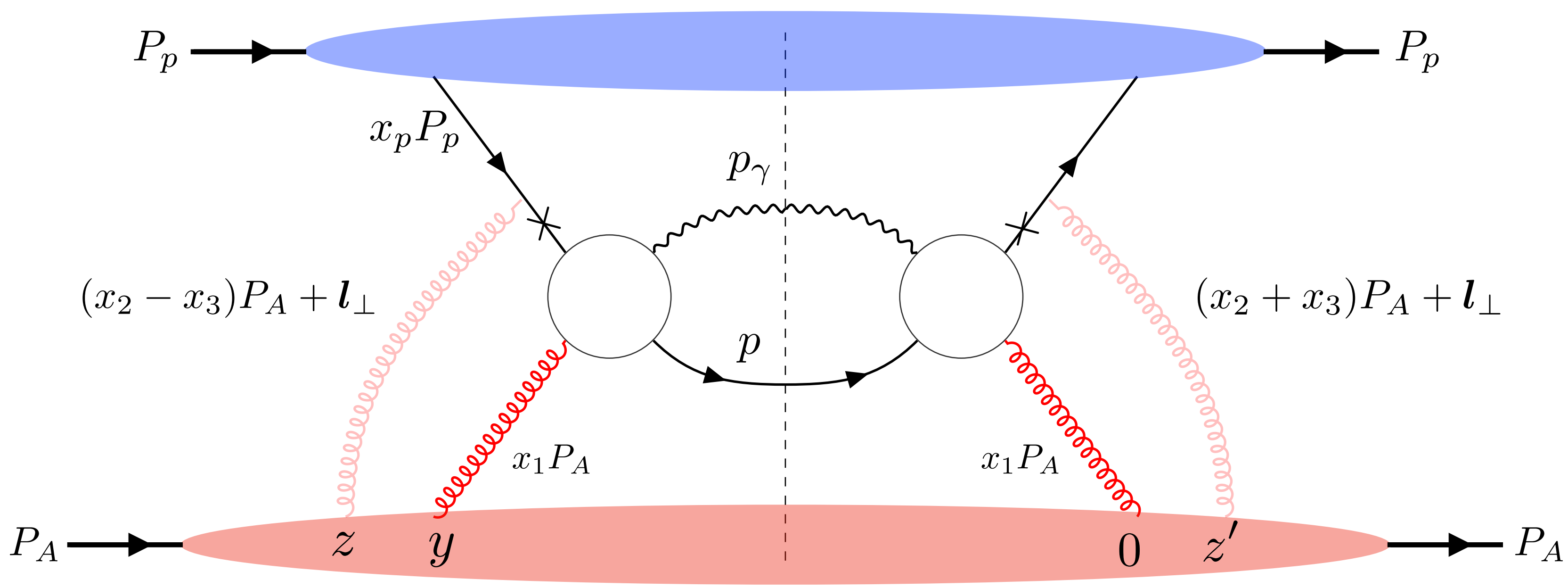
CGC: Saturation scale grows with energy and nuclear number

$$Q_s^2 \propto A^{1/3}x^{-\lambda}$$

Probing QCD matter with photons

Direct photon production at twist-4

- Consider initial state scattering central cut*



Guo, Qiu (1995)

$$p_\gamma^- \frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}p_\gamma^- \mathrm{d}^2\boldsymbol{p}_{\gamma\perp}} = \frac{(2\pi)^2\alpha_{\mathrm{em}}\alpha_{\mathrm{s}}^2}{N_c^2}\int \mathrm{d}x_p f(x_p)\xi^2\left[1+(1-\xi)^2\right]\int \mathrm{d}^2\boldsymbol{z}_\perp \int \mathrm{d}^2\boldsymbol{z}'_\perp \int \frac{\mathrm{d}^2\boldsymbol{l}_\perp}{(2\pi)^2}\frac{\mathcal{T}_{\mathrm{C,I}}(\boldsymbol{l}_\perp,\boldsymbol{p}_{\gamma\perp};\boldsymbol{z}_\perp,\boldsymbol{z}'_\perp)e^{-i\boldsymbol{l}_\perp\cdot(\boldsymbol{z}_\perp-\boldsymbol{z}'_\perp)}}{\left(\xi\boldsymbol{l}_\perp-\boldsymbol{p}_{\gamma\perp}\right)^4}$$

$$\boxed{\begin{aligned}\mathcal{T}_{\mathrm{C,I}}(\boldsymbol{l}_\perp,\boldsymbol{p}_{\gamma\perp};\boldsymbol{z}_\perp,\boldsymbol{z}'_\perp) &= \frac{1}{P^+}\int\frac{\mathrm{d}y^-}{2\pi}\int\frac{\mathrm{d}z^-\mathrm{d}z'^-}{2\pi}e^{ix_1P_A^+y^-}e^{ix_2P_A^+(z^--z'^-)}e^{ix_3P_A^+(z^-+z'^-)}\\ &\times\Theta(y^--z^-)\Theta(-z'^-)\left\langle P_A^+\middle|F_a^{\alpha+}(y^-,\boldsymbol{z}_\perp)A_b^+(z^-,\boldsymbol{z}_\perp)A_b^+(z'^-,\boldsymbol{z}'_\perp)F_a^{\beta+}(0^-,\boldsymbol{z}'_\perp)\middle|P_A^+\right\rangle\delta_{\perp\alpha\beta}\end{aligned}}$$

$$x_1 = x\left[\frac{\xi(\boldsymbol{l}_\perp-\boldsymbol{p}_{\gamma\perp})^2-\xi(1-\xi)\boldsymbol{l}_\perp^2}{\boldsymbol{p}_{\gamma\perp}^2}+(1-\xi)\right] \qquad x_2 = x\frac{\xi(1-\xi)\boldsymbol{l}_\perp^2}{\boldsymbol{p}_{\gamma\perp}^2} \qquad x_3 = 0$$

*Similarly, consider final state, interference, and asymmetric cuts

Probing QCD matter with photons

Direct photon production at twist-4 & collinear expansion

Initial state scattering central cut

- Collinear expansion = $\boldsymbol{l}_\perp$ expansion

$$p_\gamma^- \frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}p_\gamma^-\,\mathrm{d}^2\boldsymbol{p}_{\boldsymbol{\gamma}\perp}}\bigg|_{\mathrm{C,I}} = \int \mathrm{d}x_p f(x_p)\frac{(2\pi)^2\alpha_{\mathrm{em}}\alpha_{\mathrm{s}}^2}{N_c^2}\xi^2\left[1+(1-\xi)^2\right]\delta_{\perp\alpha\beta}\boxed{\frac{1}{2}\frac{\partial}{\partial \boldsymbol{l}_\perp^\rho}\frac{\partial}{\partial \boldsymbol{l}_\perp^\delta}\frac{T_{\mathrm{C,I}}^{\alpha\beta\rho\delta}(x_1,x_2,x_3)}{\left(\xi\boldsymbol{l}_\perp-\boldsymbol{p}_{\boldsymbol{\gamma}\perp}\right)^4}\bigg|_{\boldsymbol{l}_\perp=\boldsymbol{0}_\perp}}$$

- $\boldsymbol{l}_\perp$ dependence on the hard factor and <u>also on the distribution through x_i's</u>

Guo, Qiu (1995)

$$p_\gamma^- \frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}p_\gamma^-\,\mathrm{d}^2\boldsymbol{p}_{\boldsymbol{\gamma}\perp}}\bigg|_{\mathrm{C,I}} = \frac{(2\pi)^2\alpha_{\mathrm{em}}\alpha_{\mathrm{s}}^2}{N_c^2}\int \mathrm{d}x_p f(x_p)\frac{\left[1+(1-\xi)^2\right]}{\boldsymbol{p}_{\boldsymbol{\gamma}\perp}^6}$$
$$\times\left[4\xi^4 T_{\mathrm{C,I}}(x,0,0)+\xi^3(1-\xi)x\frac{\partial(T_{\mathrm{C,I}}(x,x_2,0))}{\partial x_2}\bigg|_{x_2=0}-3\xi^4 x\frac{\partial(T_{\mathrm{C,I}}(x_1,0,0))}{\partial x_1}\bigg|_{x_1=x}+\xi^4x^2\frac{\partial^2(T_{\mathrm{C,I}}(x_1,0,0))}{\partial x_1^2}\bigg|_{x_1=x}\right]$$

derivatives of twist-4 distribution!

- Corresponding twist-4 gluon distribution

$$\boxed{\begin{aligned}T_{\mathrm{C,I}}(x_1,x_2,x_3) = &\frac{1}{P_A^+}\int\frac{\mathrm{d}y^-}{2\pi}\int\frac{\mathrm{d}z^-\mathrm{d}z'^-}{2\pi}\int e^{ix_1P_A^+y^-}e^{ix_2P_A^+(z^--z'^-)}e^{ix_3P_A^+(z^-+z'^-)}\\ &\times\Theta(y^--z^-)\Theta(-z'^-)\left\langle P_A\middle|F_a^{\alpha+}(y^-)F_b^{\rho+}(z^-)F_b^{\delta+}(z'^-)F_a^{\beta+}(0^-)\middle|P_A\right\rangle\delta_{\perp\alpha\beta}\delta_{\perp\rho\delta}\end{aligned}}$$

Final state, interference, and asymmetric cuts involve other twist-4 distribution

Probing QCD matter with photons

CGC/saturation framework in a nutshell

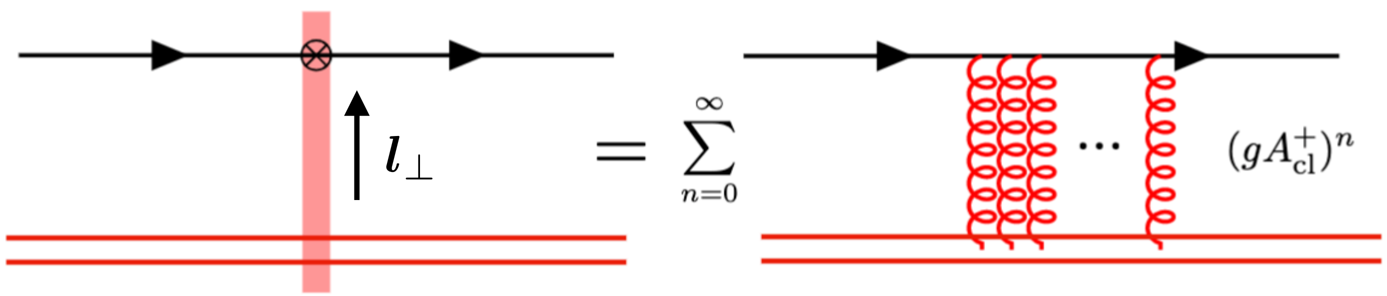
McLerran, Venugopalan (1993,1994)

- Color Glass Condensate is an effective theory of sources and fields

  Large-x partons = localized and static current J^+ (drawn from gauge invariant distribution)

  Small-x partons = background field $A^+_{\rm cl}$ sourced by current (large-x)

- Partons propagate in the small-x background field via Wilson lines



Ayala, Jalilian-Marian, McLerran, Venugopalan (1995)

e.g. for quark propagation:

$$\mathcal{T}^q_{\sigma\sigma',ij}(l) = 2\pi\delta(l^-)\gamma^-_{\sigma\sigma'}\int \mathrm{d}^2\boldsymbol{y}_\perp e^{-i\boldsymbol{l}_\perp\cdot\boldsymbol{y}_\perp} V_{ij}(\boldsymbol{y}_\perp)$$

$$V_{ij}(\boldsymbol{y}_\perp) = \mathcal{P}\exp\left(ig\int \mathrm{d}y^- A^{+,c}_{\rm cl}(y^-,\boldsymbol{y}_\perp)t^c_{ij}\right)$$

- Observables (e.g. cross-section) are convolutions of Wilson lines with perturbative factors

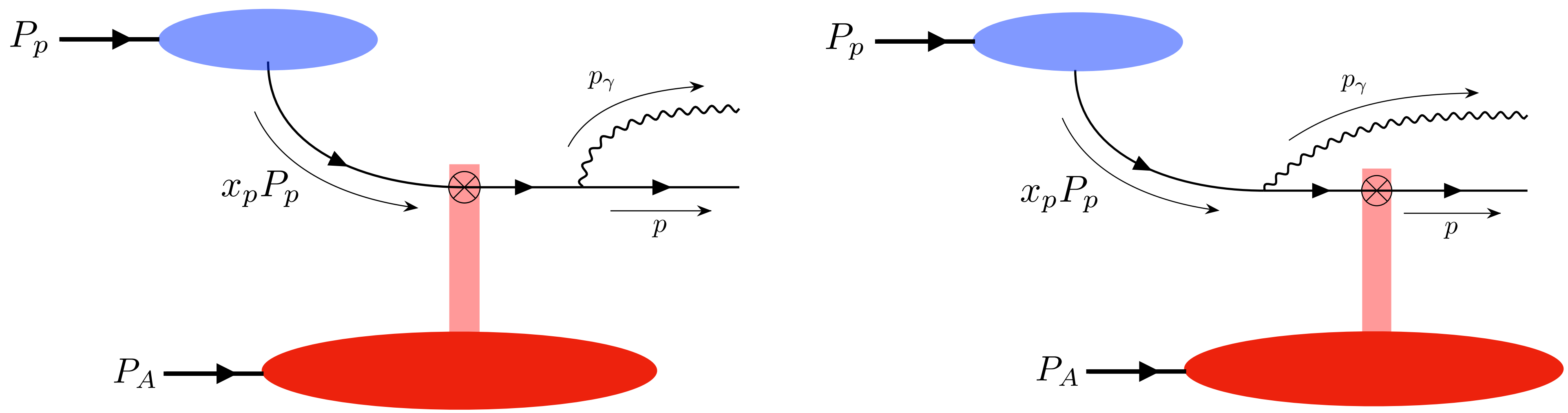
- Wilson lines (correlators) obey non-linear evolution equations (BK/JIMWLK)

Balitsky (1996)
Kovchegov (1999)
Jalilian-Marian, Leonidov, Weigert (1997)
Iancu, Leonidov, McLerran (2001)

Probing QCD matter with photons

Direct photon production with the CGC/saturation framework

- Amplitudes



- Differential cross-section CGC

Gelis, Jalilian-Marian (2002)

$$p_\gamma^- \frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}p_\gamma^- \mathrm{d}^2\boldsymbol{p_\gamma}_\perp} = \frac{\alpha_{\mathrm{em}}}{2\pi^2}\int \mathrm{d}x_p f(x_p)\xi^2\left[1+(1-\xi)^2\right]$$
$$\times \int \mathrm{d}^2\boldsymbol{l}_\perp \int \frac{\mathrm{d}^2\boldsymbol{y}_\perp}{2\pi}\int \frac{\mathrm{d}^2\boldsymbol{y}'_\perp}{2\pi} e^{-i\boldsymbol{l}_\perp\cdot(\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)} \frac{\boldsymbol{l}_\perp^2 D(x_A;\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}{\left(\xi\boldsymbol{l}_\perp-\boldsymbol{p_\gamma}_\perp\right)^2 \boldsymbol{p_\gamma}_\perp^2}$$

- Dipole correlator

$$\boxed{D(x_A,\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp) = \left\langle \frac{1}{N_c}\mathrm{Tr}\left[V(\boldsymbol{y}_\perp)V^\dagger(\boldsymbol{y}'_\perp)\right]\right\rangle_{x_A}}$$

Matching CGC and high twist formalism

Collinear expansion in the CGC

$$\frac{D(x_A;\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}{(\xi\boldsymbol{l}_\perp-\boldsymbol{p}_{\gamma\perp})^2\,\boldsymbol{p}_{\gamma\perp}^2}=\frac{D(x;\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}{\boldsymbol{p}_{\gamma\perp}^4}+\frac{1}{2}\boldsymbol{l}_\perp^\alpha\boldsymbol{l}_\perp^\beta\frac{\partial}{\partial\boldsymbol{l}_\perp^\alpha}\frac{\partial}{\partial\boldsymbol{l}_\perp^\beta}\left[\frac{D(x_A;\boldsymbol{y}_\perp-\boldsymbol{y}'_\perp)}{(\xi\boldsymbol{l}_\perp-\boldsymbol{p}_{\gamma\perp})^2}\right]\Bigg|_{\boldsymbol{l}_\perp=\boldsymbol{0}_\perp}+\ldots$$

Twist-2 Twist-4

- Twist-2

$$p_\gamma^-\frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}p_\gamma^-\mathrm{d}^2\boldsymbol{p}_{\gamma\perp}}=\frac{\alpha_{\mathrm{em}}\alpha_s}{N_c}\int\mathrm{d}x_p f(x_p)\frac{\xi^2\left[1+(1-\xi)^2\right]}{\boldsymbol{p}_{\gamma\perp}^4}xg(x)\Big|_{x\to0}$$

Twist-2 gluon PDF = second moment dipole correlator

$$xg(x)\overbrace{=}^{x\to0}\frac{N_c}{2\pi^2\alpha_s}\int\boldsymbol{l}_\perp^2\mathrm{d}^2\boldsymbol{l}_\perp C(x,\boldsymbol{l}_\perp)$$

Dipole correlator in momentum space

Phase $e^{ixP^+y^-}$ dropped out ("sub-eikonal")

$$\boxed{xg(x)\Big|_{x\to0}=\frac{1}{P_A^+}\int\frac{\mathrm{d}y^-}{2\pi}\left\langle P_A\right|\mathrm{Tr}\left[F_a^{\alpha+}(y^-)F_a^{\beta+}(0^-)\right]\left|P_A\right\rangle\delta_{\perp\alpha\beta}}$$

Matching CGC and high twist formalism

Collinear expansion in the CGC

- Twist-4

$$p_\gamma^- \frac{\mathrm{d}\sigma^{p+A\to\gamma+X}}{\mathrm{d}p_\gamma^- \mathrm{d}^2\boldsymbol{p}_{\boldsymbol{\gamma}\perp}} = \frac{(2\pi)^2\alpha_{\mathrm{em}}\alpha_s^2}{N_c^2}\int \mathrm{d}x_p f(x_p)\frac{\left[1+(1-\xi)^2\right]}{\boldsymbol{p}_{\boldsymbol{\gamma}\perp}^{\;6}}\xi^4 T_{\mathrm{HT}}(x)$$

Missing terms with derivatives of twist-4 distribution

Twist-4 gluon distribution = fourth moment dipole correlator

$$T_{\mathrm{HT}}(x) = \frac{2N_c^2}{(2\pi)^4\alpha_s^2}\int \boldsymbol{l}_\perp^4 \mathrm{d}^2\boldsymbol{l}_\perp F(x,\boldsymbol{l}_\perp)$$

$$T_{\mathrm{HT}}(x_1,x_2,x_3) = \frac{1}{P_A^+}\int\frac{\mathrm{d}y^-}{2\pi}\left\langle P_A\right|F_a^{\alpha+}(y^-)F_b^{\rho+}(z^-)F_b^{\delta+}(z'^-)F_a^{\beta+}(0^-)\left|P_A\right\rangle\delta_{\perp\alpha\beta}\delta_{\perp\rho\delta}$$
$$\times\left[\Theta(y^- - z^-)\Theta(-z'^-) + \Theta(z^- - y^-)\Theta(-z'^-) + \Theta(y^- - z^-)\Theta(z'^-) + \Theta(z^- - y^-)\Theta(z'^-)\right]$$

Contains all orderings (central cut)

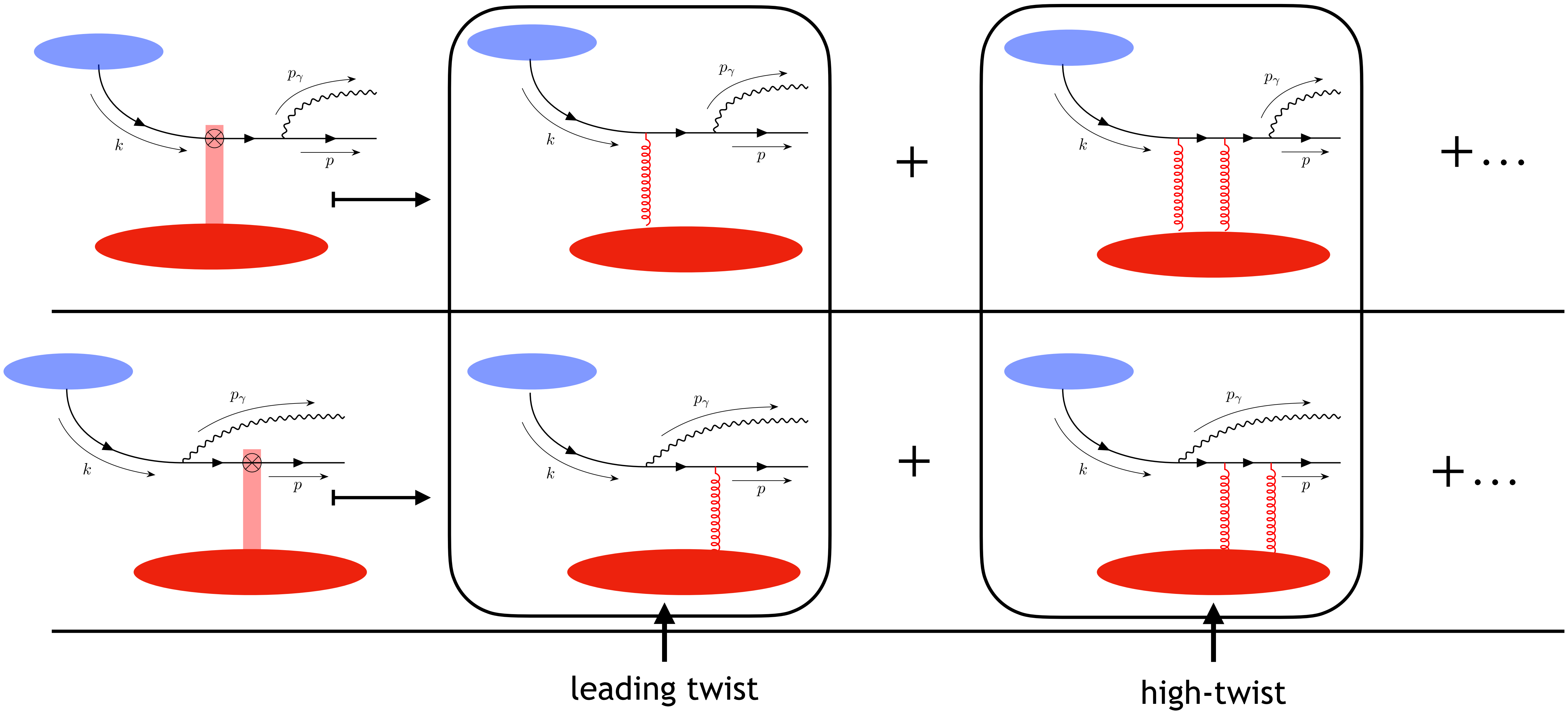
At small-x all twist-4 distributions collapse into a single distribution – no distinction between soft and hard gluons!

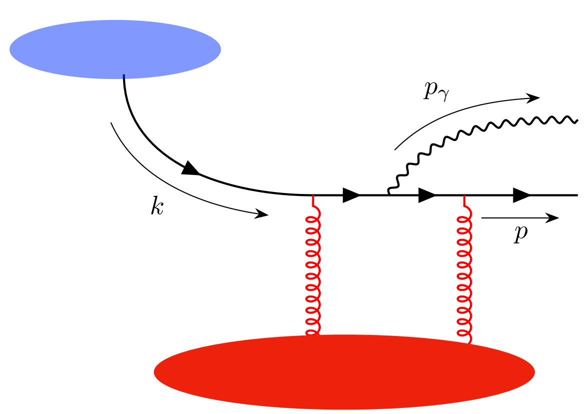
$$\lim_{x_1,x_2,x_3\to 0} T_{\mathrm{C,I}}(x_1,x_2,x_3) = \lim_{x_1,x_2,x_3\to 0} T_{\mathrm{C,F}}(x_1,x_2,x_3) = \lim_{x_1,x_2,x_3\to 0} T_{\mathrm{C,FI}}(x_1,x_2,x_3)$$

Initial state | final state | Interference

Matching CGC and high twist formalism

Opening up shock-wave (expand Wilson lines)





Emission between scatterings suppressed in the CGC → Recovered by keeping "sub-eikonal" phase

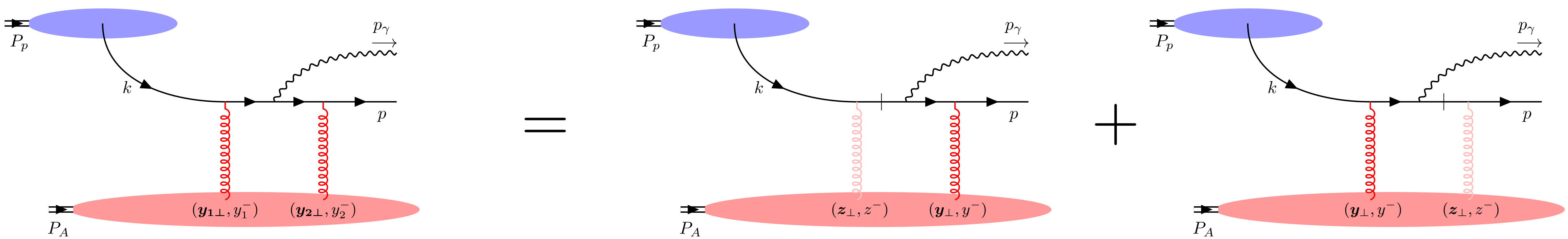
Gelis, Jalilian-Marian (2002)

Matching CGC and high twist formalism

Putting back “sub-eikonal "the phase

$$(2\pi)\delta(\ell^-)\gamma^- \int \mathrm{d}^2\boldsymbol{y}_\perp e^{-i\boldsymbol{\ell}_\perp\cdot\boldsymbol{y}_\perp} \int \mathrm{d}y^- e^{i\ell^+ y^-} igA_a^+(y^-,\boldsymbol{y}_\perp)(t^a)_{ij}$$

Contour integration puts on-shell intermediate propagators

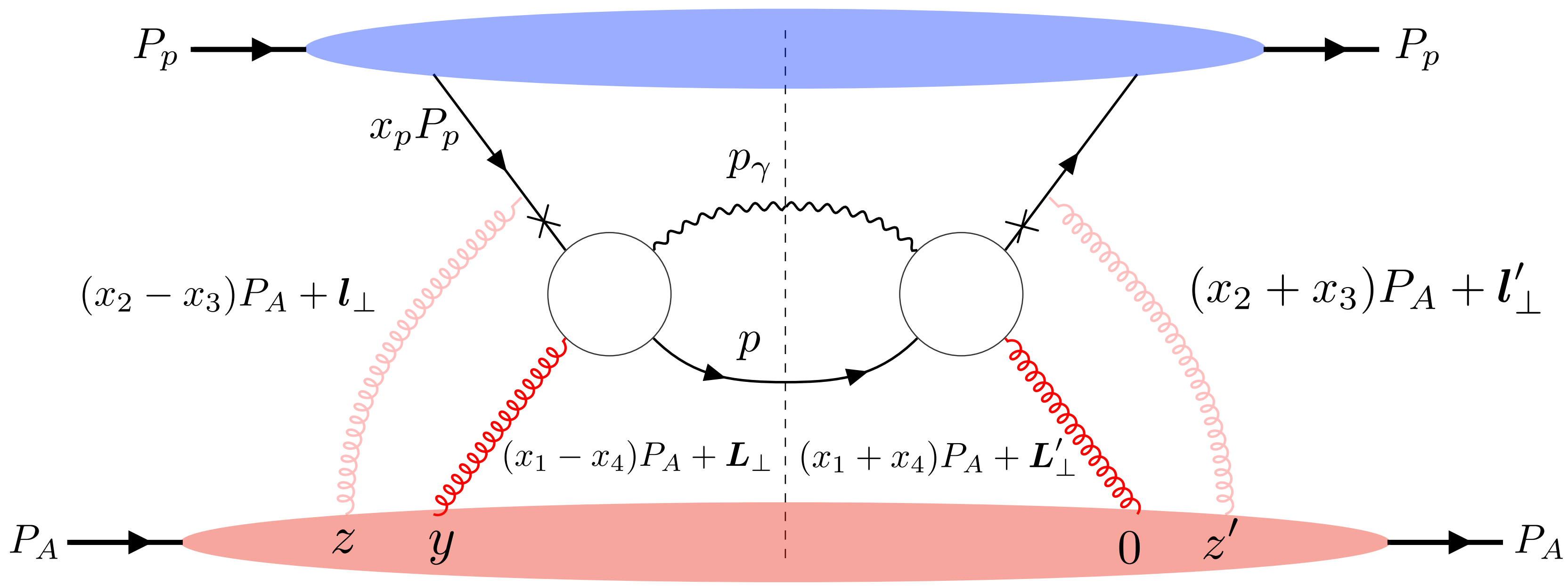


$$\propto \left[1 - e^{-\frac{iP_A^+}{2x_p s}\frac{(\xi \boldsymbol{l}_\perp - \boldsymbol{p}_{\boldsymbol{\gamma}\perp})^2}{\xi(1-\xi)}(y^- - z^-)}\right] \overbrace{\longrightarrow}^{s\to\infty} 0$$

Landau-Pomeranchuk-Migdal effect at play – coherence vs incoherence

Matching CGC and high twist formalism

In the CGC all gluons are in the covariant gauge



Before collinear expansion all gluons carry different momenta

$$\mathcal{H}_{\mathrm{cov}}(\boldsymbol{L}_\perp, \boldsymbol{L}'_\perp, \boldsymbol{l}_\perp, \boldsymbol{l}'_\perp) \otimes \langle A^+ A^+ A^+ A^+ \rangle$$

After expansion on hard gluon momentum

$$\mathcal{H}_{\mathrm{LC}}(\boldsymbol{l}_\perp) \otimes \langle F^{+\alpha} A^+ A^+ F^{+\beta} \rangle \delta_{\perp\alpha\beta}$$

recover result from high-twist formalism

Summary

- Using direct photon production as an example we establish the consistency between CGC and high-twist formalism in the collinear limit

$$\mathrm{d}\sigma \sim \frac{1}{p_{\gamma\perp}^4}\left[\underbrace{A + B\frac{\langle k_\perp^2\rangle}{p_{\gamma\perp}^2}}_{\text{CGC-high twist formalism consistency up to twist-4}} + C\frac{\langle k_\perp^2\rangle^2}{p_{\gamma\perp}^4}\dots\right]$$

- Identify twist-4 gluon distributions in terms of fourth moment of the dipole of Wilson lines

$$\int \mathrm{d}^2\boldsymbol{l}_\perp \boldsymbol{l}_\perp^4 C(\boldsymbol{l}_\perp) \leftrightarrow \langle P_A|FFFF|P_A\rangle$$

- Highlight the importance of "sub-eikonal" phases in the collinear limit

$$e^{ix_i P^+ y^-}$$

Outlook

- Does the consistency between CGC and high-twist formalism persist at NLO?

- Matching between CGC and twist-4 TMDs

- Establish a framework that allows to resum all twists (modify Wilson lines to keep track of phases?)

- Phenomenology: describe low-x and large-x data with a single framework

Salud for 60 more years of physics

Hope for many collaborations with you!

Happy birthday Xin-Nian!