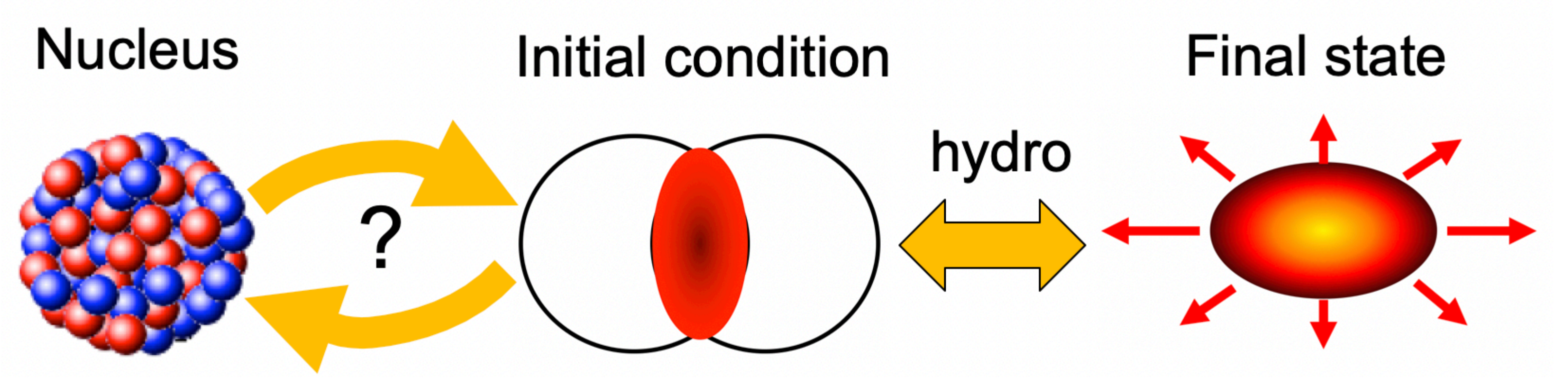
Berkeley Symposium on Hard Probes and Beyond

Imaging nuclear structure and heavy-ion initial condition

Jiangyong Jia

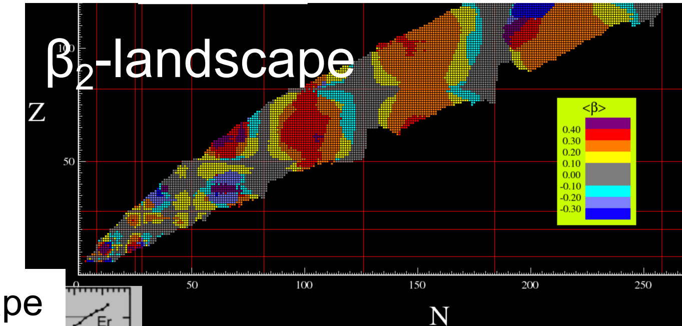
Nuclear Structure

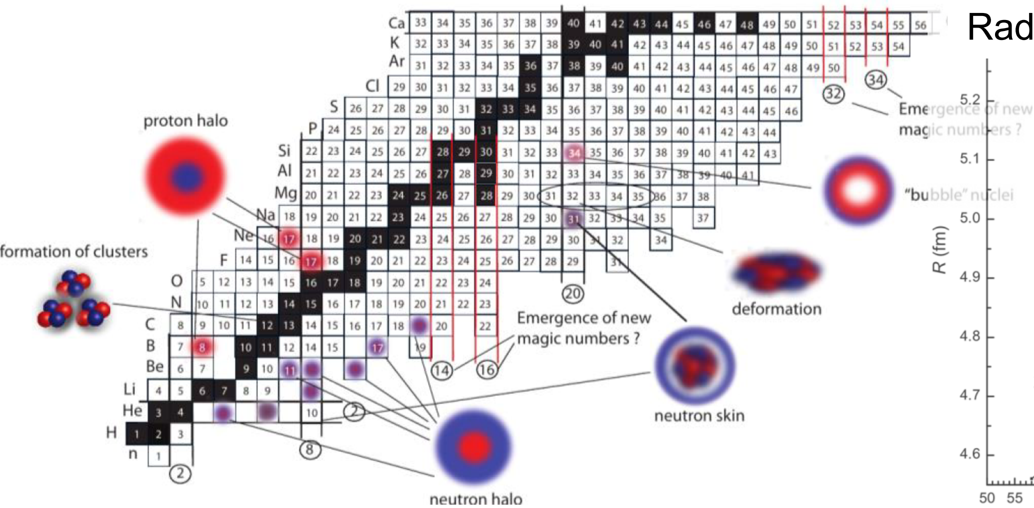
High-energy heavy-ion collisions

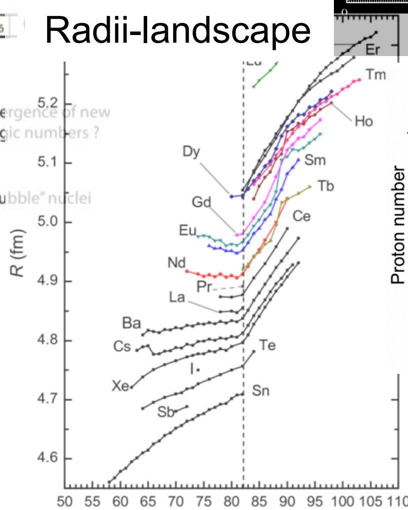


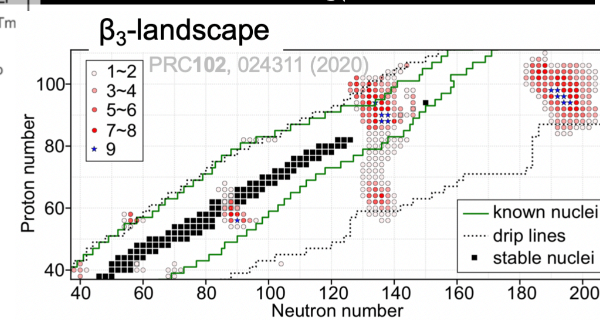
Rich structure of atomic nuclei

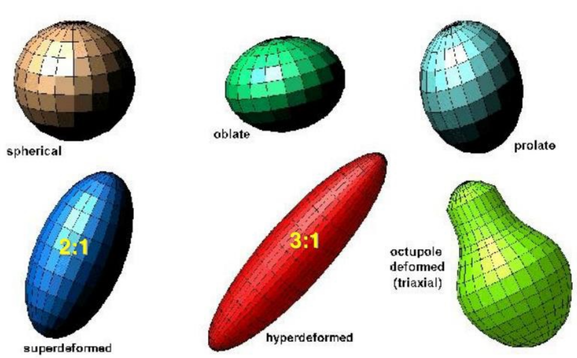
- Collective phenomena of many-body quantum system
  - clustering, halo, skin, bubble…
  - quadrupole/octupole/hexdecopole deformations
  - Nontrivial evaluation with N and Z.





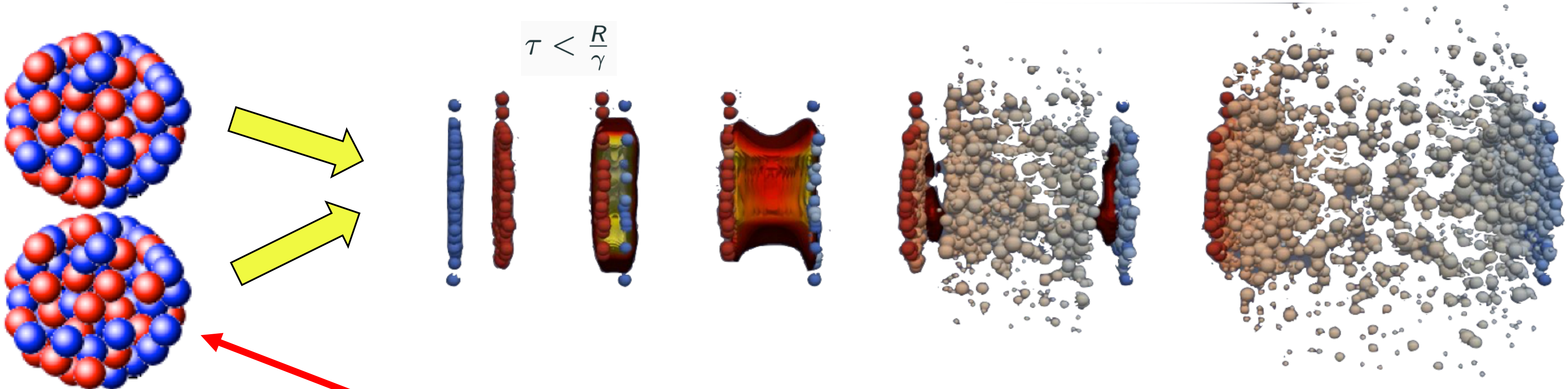


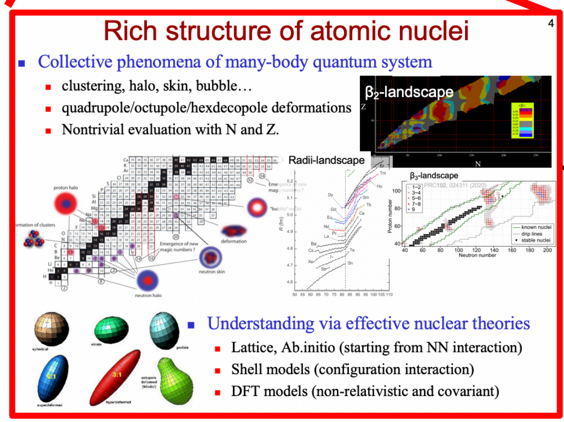




- Understanding via effective nuclear theories
  - Lattice, Ab.initio (starting from NN interaction)
  - Shell models (configuration interaction)
  - DFT models (non-relativistic and covariant)

High-energy heavy ion collision





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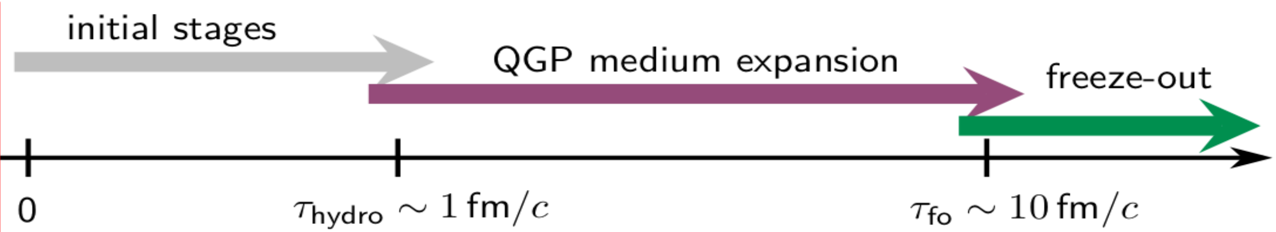


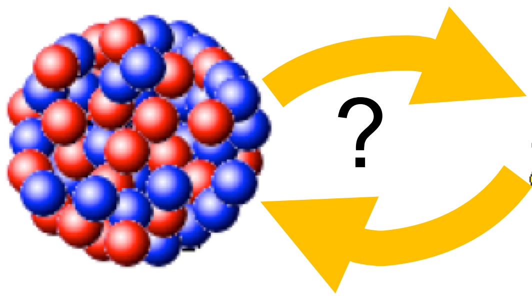
1) Are nuclear structures important for HI initial condition and final state evolution?

2) What HI experimental observables can be used to infer structure information?

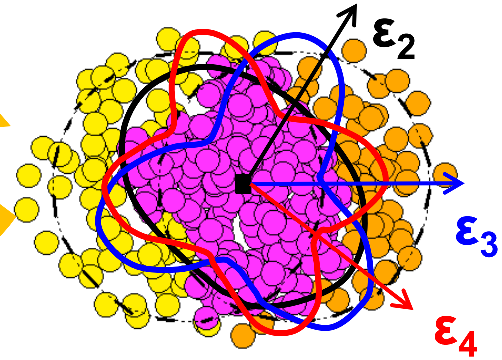
3) Can HI provides competitive constraints on nuclear shape and radial profile? can consideration of nuclear structure improves understanding of HI initial condition?

Collective flow in fluctuating events

Nucleus

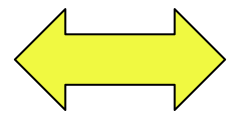


Initial condition

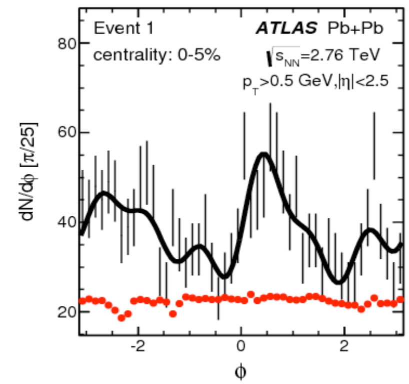


Fixed impact parameter

hydro



Final state



Nuclear structure	Initial volume	Initial Size	Initial Shape	Multiplicity	Radial Flow	Harmonic Flow

$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a_0}}$$

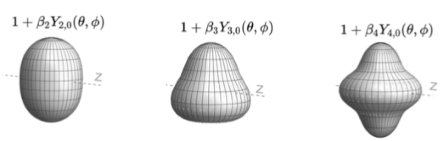
$$R(\theta,\phi) = R_0\Big(1+\beta_2[\cos\gamma Y_{2,0}+\sin\gamma Y_{2,2}] + \beta_3\sum_{m=-3}^{3}\alpha_{3,m}Y_{3,m} + \beta_4\sum_{m=-4}^{4}\alpha_{4,m}Y_{4,m}\Big)$$

N_{part}

$$R_\perp^2 \propto \langle r_\perp^2\rangle, \quad \mathcal{E}_2 \propto \left\langle r_\perp^2 e^{i2\phi}\right\rangle$$
$$\mathcal{E}_3 \propto \left\langle r_\perp^3 e^{i3\phi}\right\rangle$$
$$\mathcal{E}_4 \propto \left\langle r_\perp^4 e^{i4\phi}\right\rangle$$
$$\cdots$$

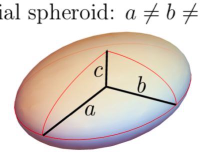
N_{ch}

$$\frac{d^2N}{d\phi dp_T} = N(p_T)\left(\sum_n V_n\, e^{-in\phi}\right)$$



High energy: approx. linear response in each event: $\tau < \frac{R}{\gamma}$

Triaxial spheroid: $a \neq b \neq c$.



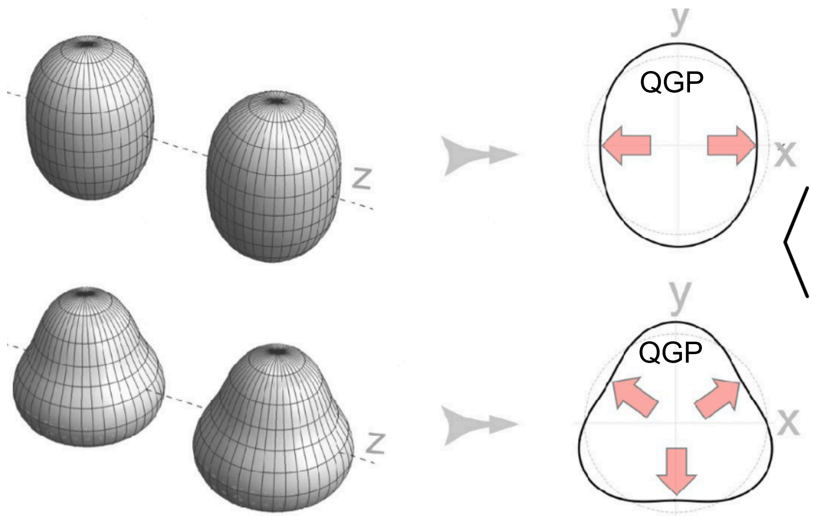
$0 \leq \gamma \leq \pi/3$

$$N_{ch} \propto N_{part} \qquad \frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp} \qquad V_n \propto \mathcal{E}_n$$

Expected structure dependencies

arXiv:2106.08768

Central collisions

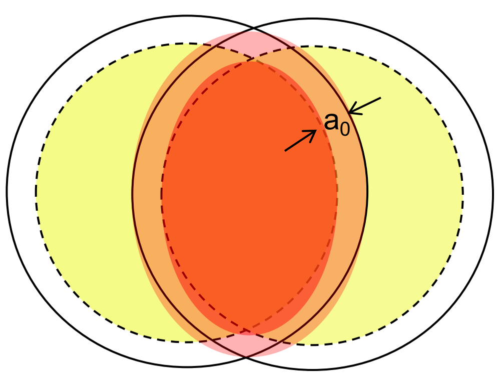


$$\langle v_2^2 \rangle \approx a_2 + b_2 \beta_2^2 + c_3 \beta_3^2$$

$$\left\langle \left(\delta p_{\mathrm{T}} / p_{\mathrm{T}}\right)^2 \right\rangle \approx a_0 + b_0 \beta_2^2 + c_0 \beta_3^2$$

$$\langle v_3^2 \rangle \approx a_3 + b_3 \beta_3^2$$

Non-Central collisions



The shape and size the overlap, therefore v_2 and p_T, also depend on diffuseness a_0 and radius R_0

At fixed N_{part} $a_0 \searrow$ ⇨ $v_2 \nearrow$ $p_T \nearrow$

$R_0 \searrow$ ⇨ $p_T \nearrow$

Application in ^{197}Au+^{197}Au vs ^{238}U+^{238}U

arXiv:2105.01638

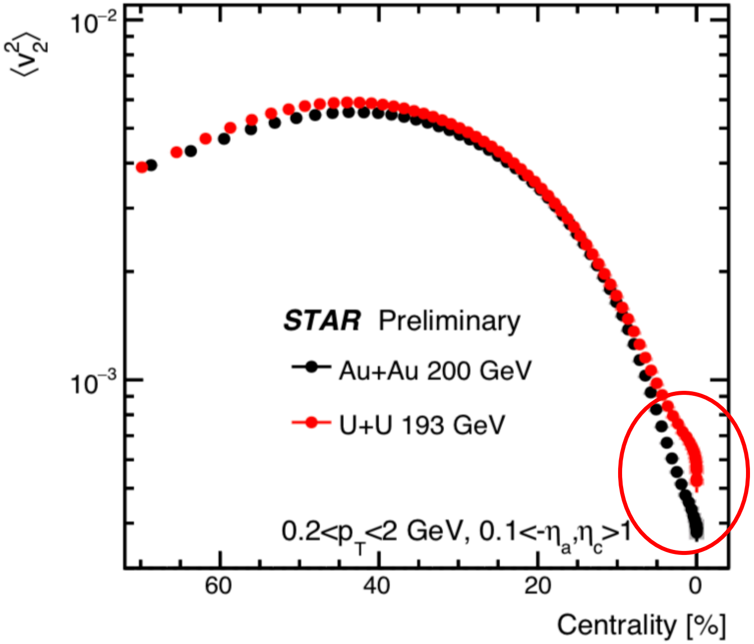
Ultra-central Collisions at $\sqrt{s_{NN}}$=193-200 GeV

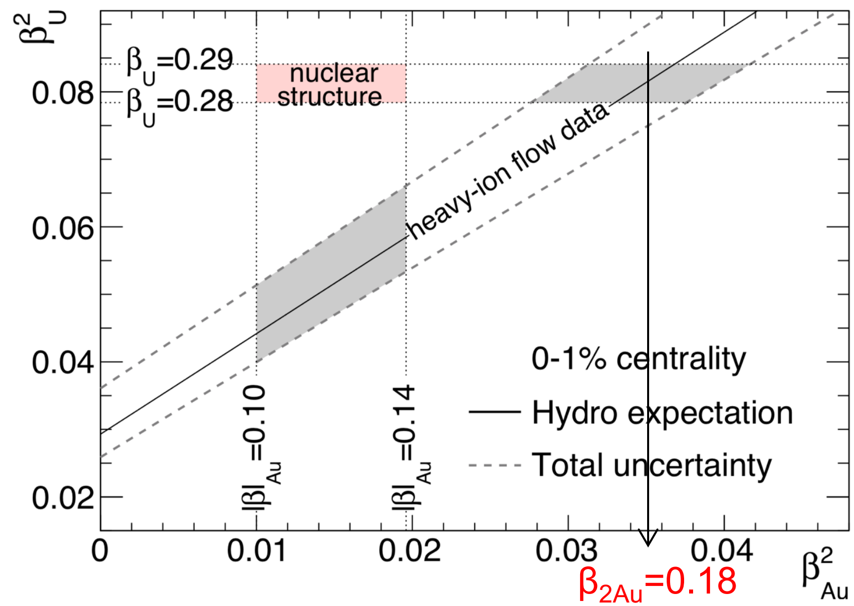
$$\begin{cases} v_{2,\mathrm{Au}}^2 = a_{\mathrm{Au}} + b\beta_{2,\mathrm{Au}}^2 \\ v_{2,\mathrm{U}}^2 = a_{\mathrm{U}} + b\beta_{2,\mathrm{U}}^2 \end{cases}$$

b~0.014

Need to correct for slightly different size: $a \propto 1/A\,,\ \ r_a = \frac{a_{\mathrm{Au}}}{a_{\mathrm{U}}} = \frac{238}{197} = 1.21$

A linear relation for β_{2U} and β_{2Au} : $\beta_{\mathrm{U}}^2 = \frac{r_{v_2^2} r_a - 1}{b/a_{\mathrm{U}}} + r_{v_2^2}\beta_{\mathrm{Au}}^2 \qquad r_{v_2^2} = \frac{v_{2,\mathrm{U}}^2}{v_{2,\mathrm{Au}}^2}$

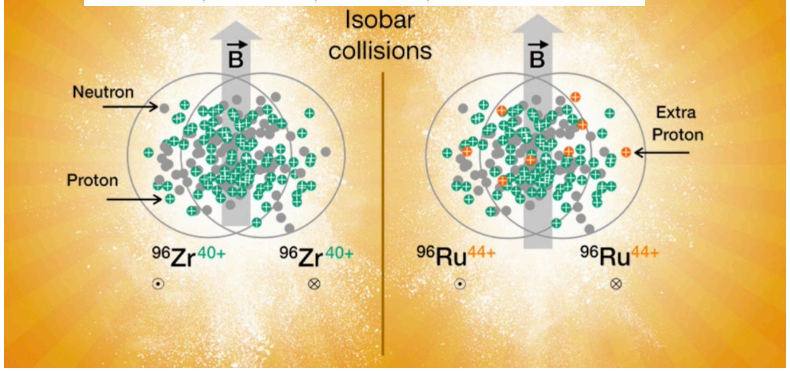




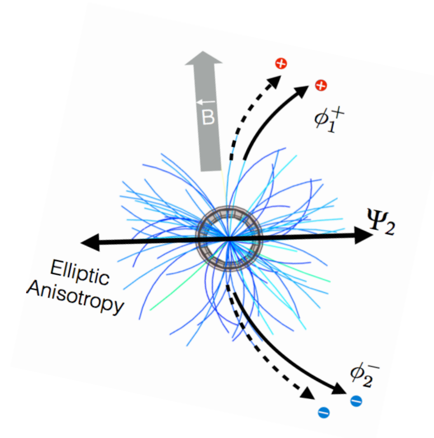
Suggests $|\beta_2|_{Au}$~0.18+-0.02, larger than NS model of 0.13+-0.02

Isobar collisions at RHIC

	Z	N	mass
96Ru	44	52	95.907598(8)
96Zr	40	56	95.908273(3)



arXiv:2109.00131



Voloshin, hep-ph/0406311

- Designed to search for the chiral magnetic effect: strong P & CP violation in the presence of EM field. Turns out the CME signal is small, and isobar-differences are dominated by the nuclear structure differences.

- <0.4% precision is achieved in ratio of many observables between ^{96}Ru+^{96}Ru and ^{96}Zr+^{96}Zr systems→ precision imaging tool

Isobar collisions as precision tool

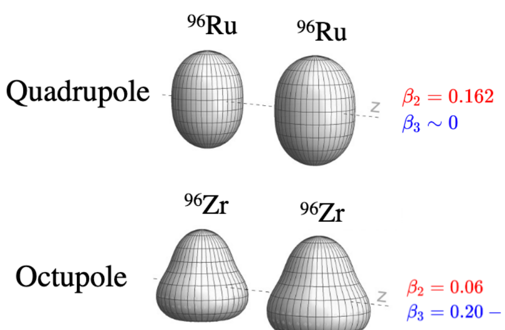
- A key question for any HI observable O:

$$\frac{O_{^{96}\mathrm{Ru}+^{96}\mathrm{Ru}}}{O_{^{96}\mathrm{Zr}+^{96}\mathrm{Zr}}} \stackrel{?}{=} 1$$

Deviation from 1 must has origin in the nuclear structure, which impacts the initial state and then survives to the final state.

- Expectation

$$\rho(r,\theta,\phi) \propto \frac{1}{1+e^{[r-R_0(1+\beta_2 Y_2^0(\theta,\phi)+\beta_3 Y_3^0(\theta,\phi))]/a_0}}$$



J. Jia, aXiv:2106.08768

$$\mathcal{O} \approx b_0 + b_1\beta_2^2 + b_2\beta_3^2 + b_3(R_0 - R_{0,\mathrm{ref}}) + b_4(a - a_{\mathrm{ref}})$$

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

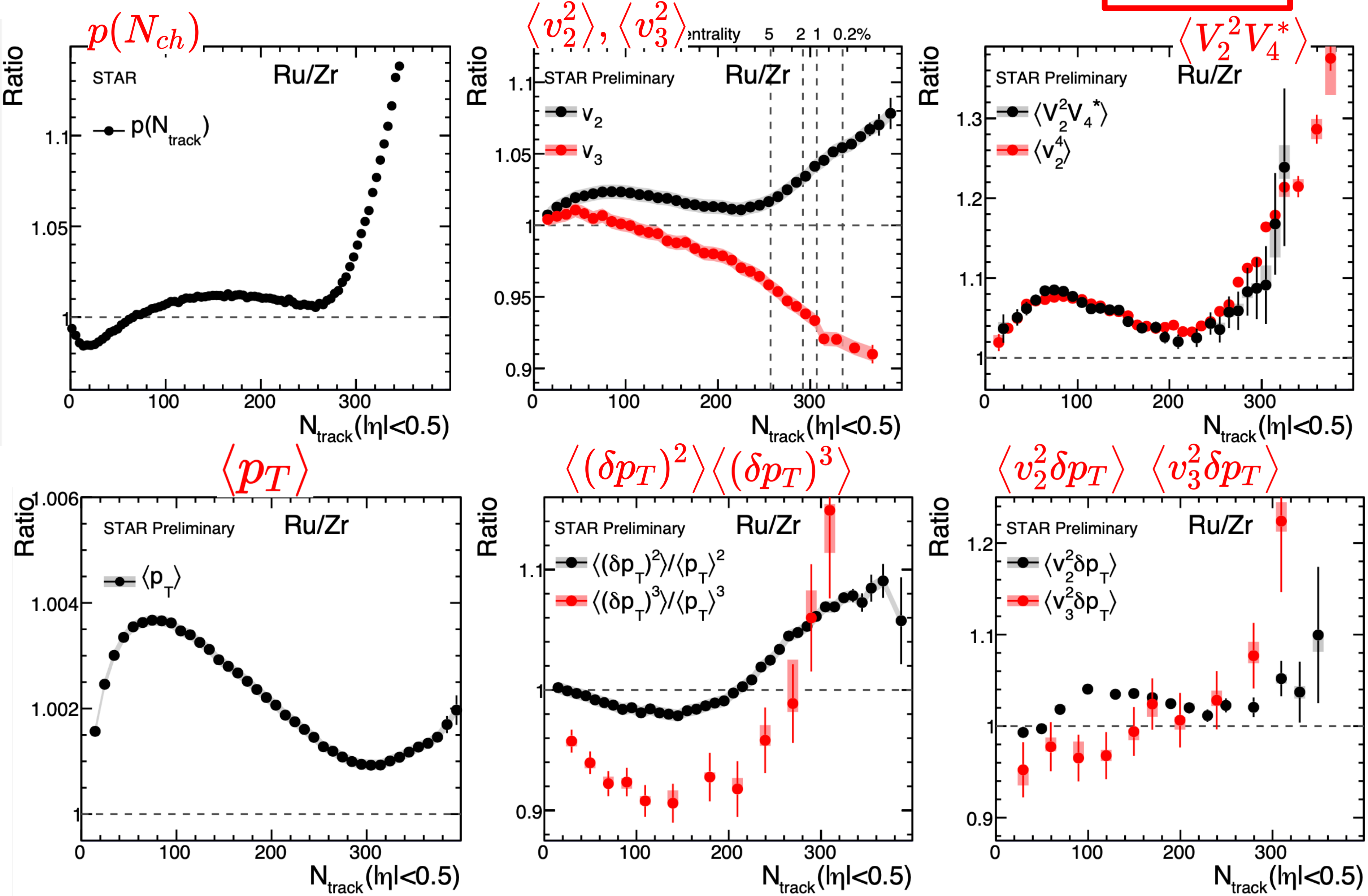
Species	β_2	β_3	a_0	R_0
Ru	0.162	0	0.46 fm	5.09 fm
Zr	0.06	0.20	0.52 fm	5.02 fm
difference	$\Delta\beta_2^2$	$\Delta\beta_3^2$	Δa_0	ΔR_0
	0.0226	-0.04	-0.06 fm	0.07 fm

Only probes isobar differences

2109.00131

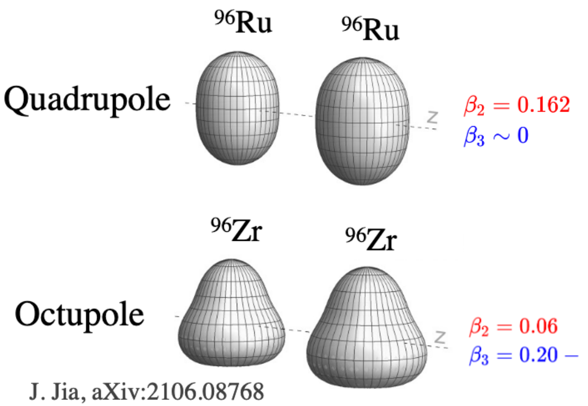
Structure influences everywhere

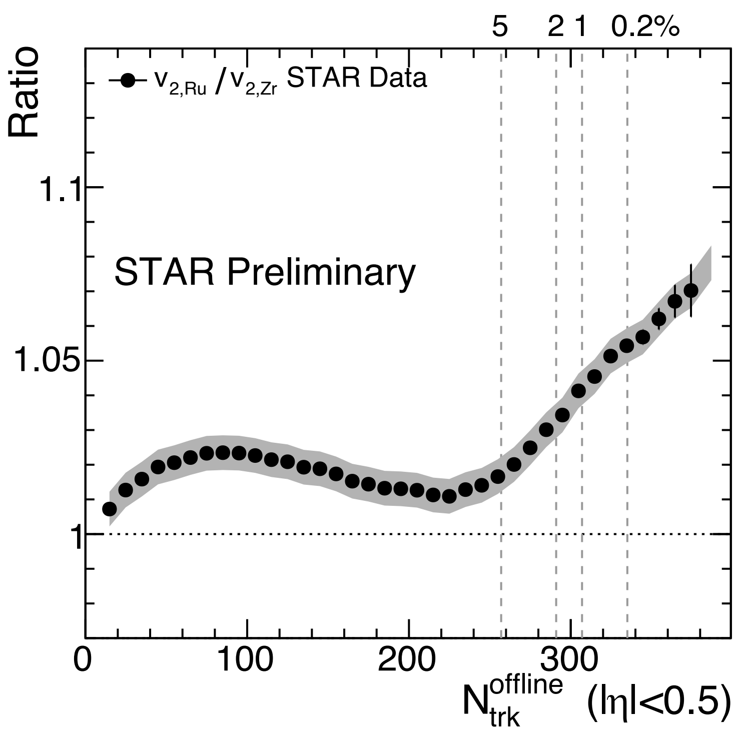
$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}}$$

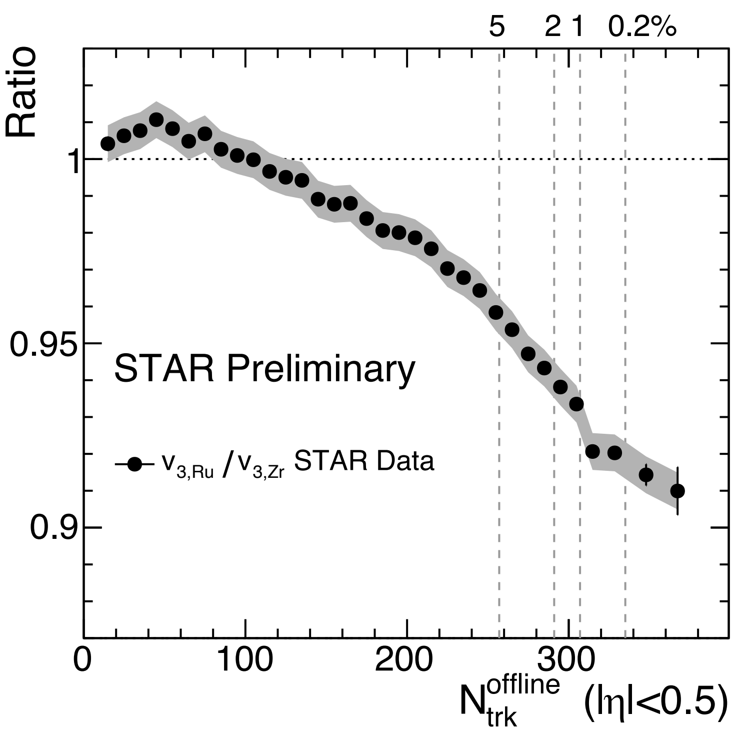


What about 197Au and 208Pb?

Nuclear structure via v_n-ratio



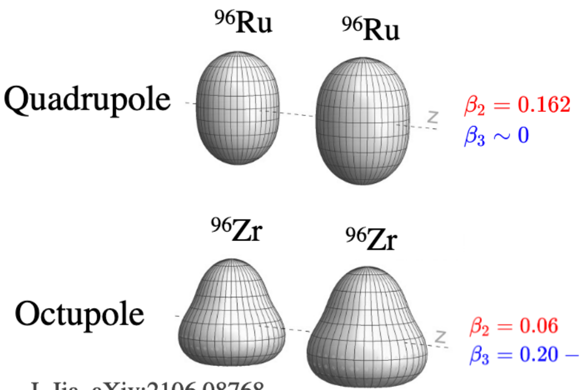




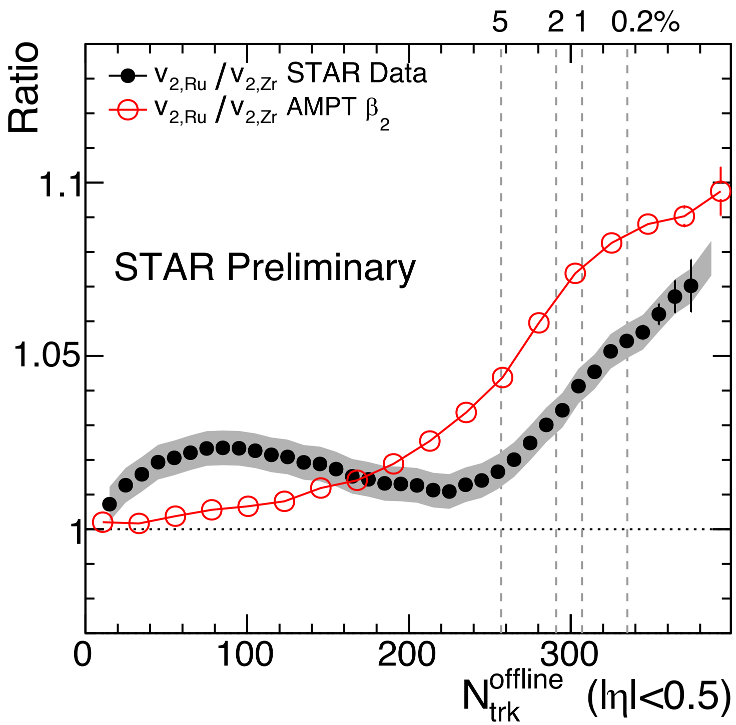
Simultaneously constrain these parameters using different N_{ch} regions

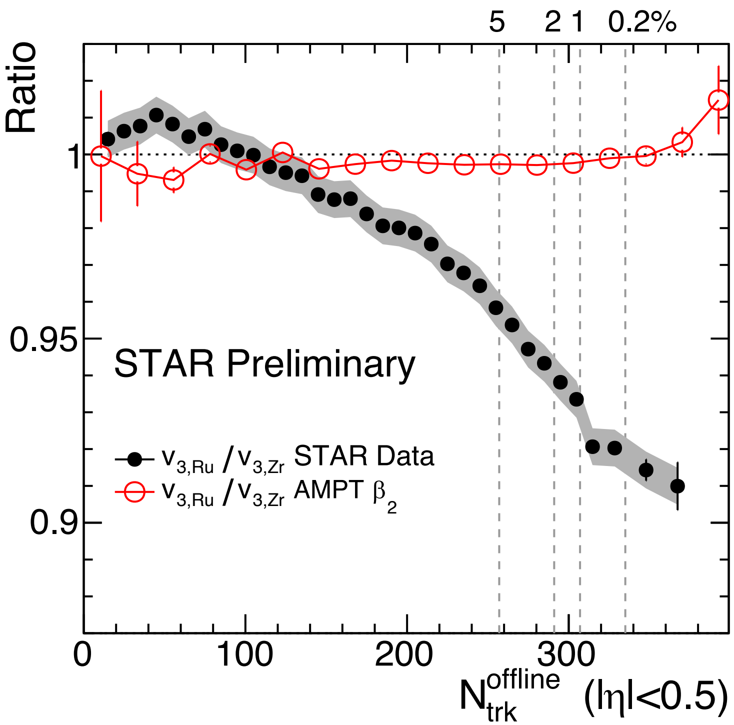
Nuclear structure via v_n-ratio

- β_{2Ru}~0.16 increase v_2, no influence on v_3 ratio



J. Jia, aXiv:2106.08768





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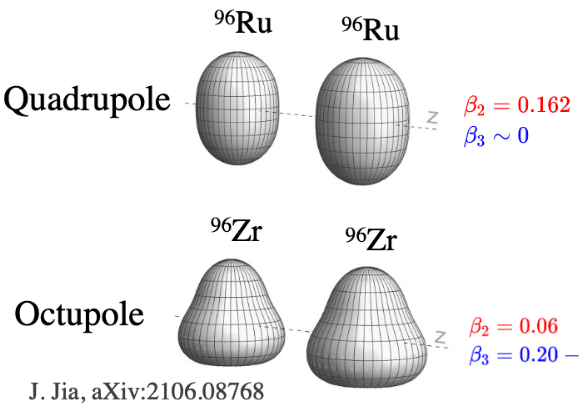
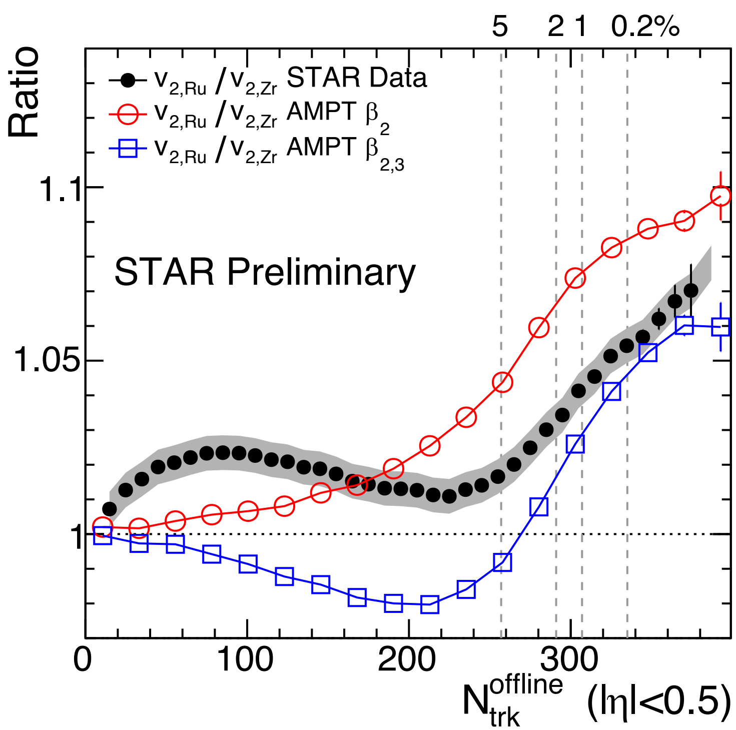
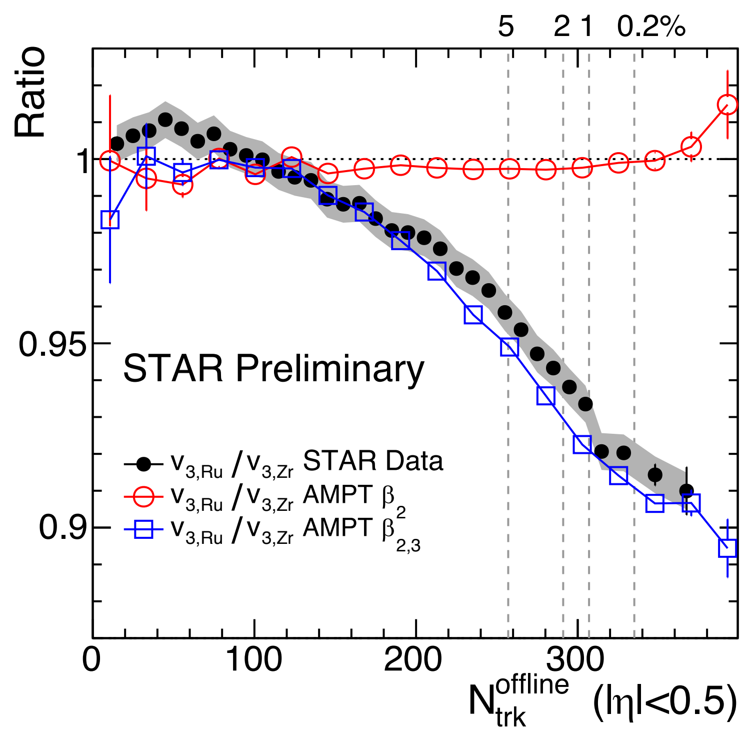


- β_{2Ru}~0.16 increase v_2, no influence on v_3 ratio
- β_{3Zr}~0.2 decrease v_2 in mid-central, decrease v_3 ratio





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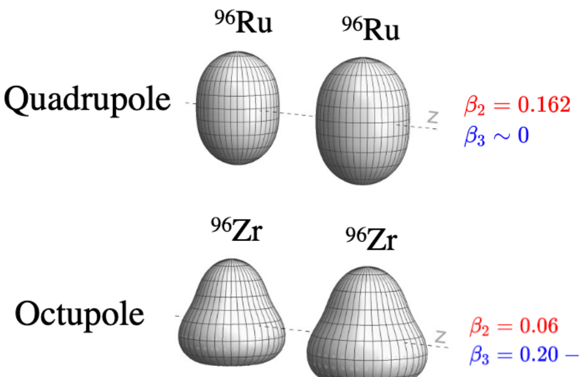

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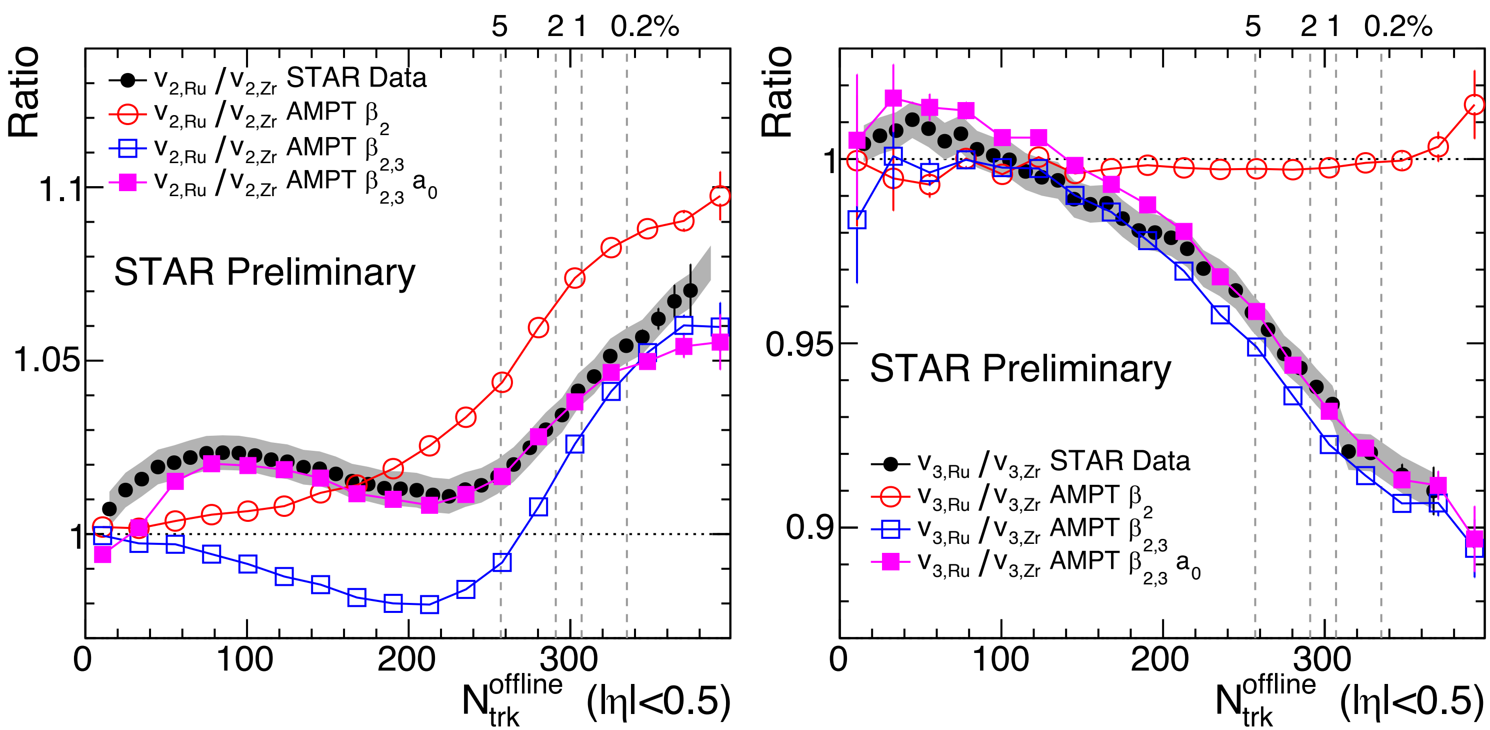

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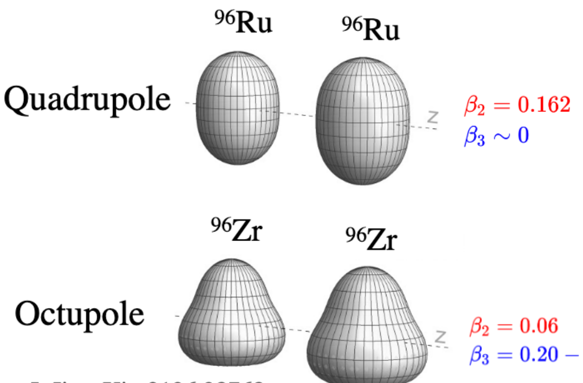

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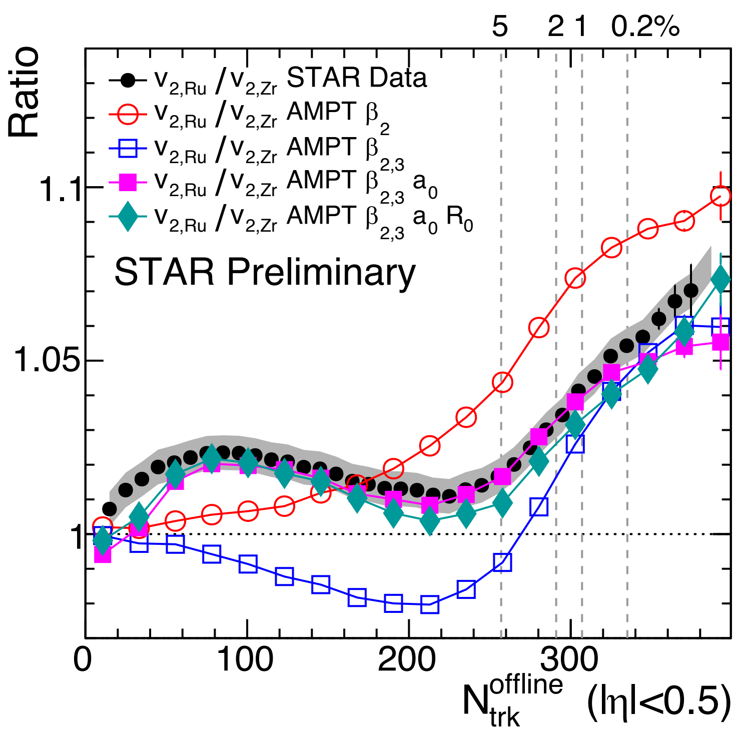
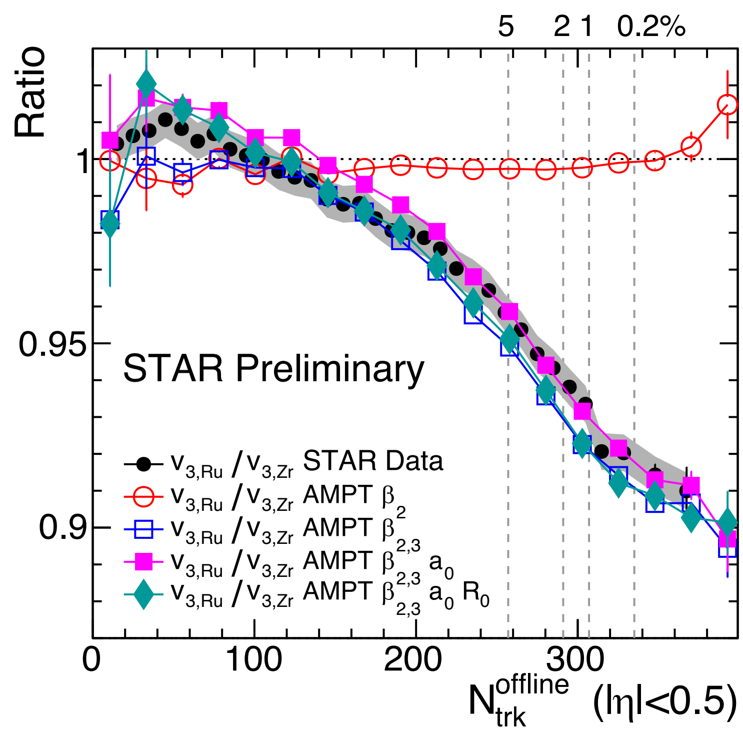
- β_{2Ru}~0.16 increase v_2, no influence on v_3 ratio
- β_{3Zr}~0.2 decrease v_2 in mid-central, decrease v_3 ratio
- Δa_0= -0.06fm increase v_2 mid-central, small influ. on v_3.
- Radius ΔR_0=0.07fm only slightly affects v_2 and v_3 ratio.

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\rm Ru}}{\mathcal{O}_{\rm Zr}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

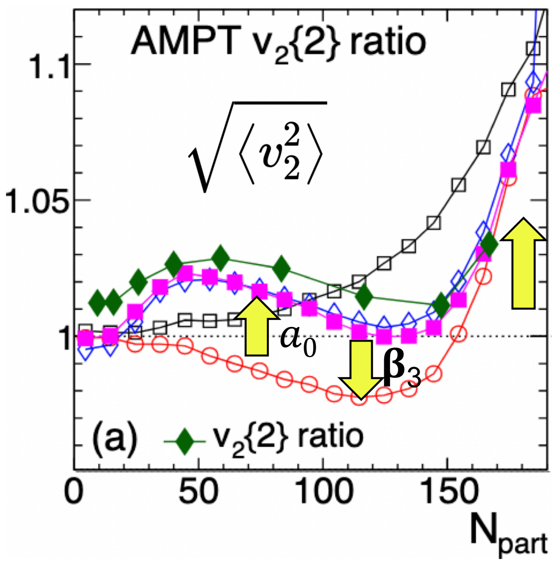




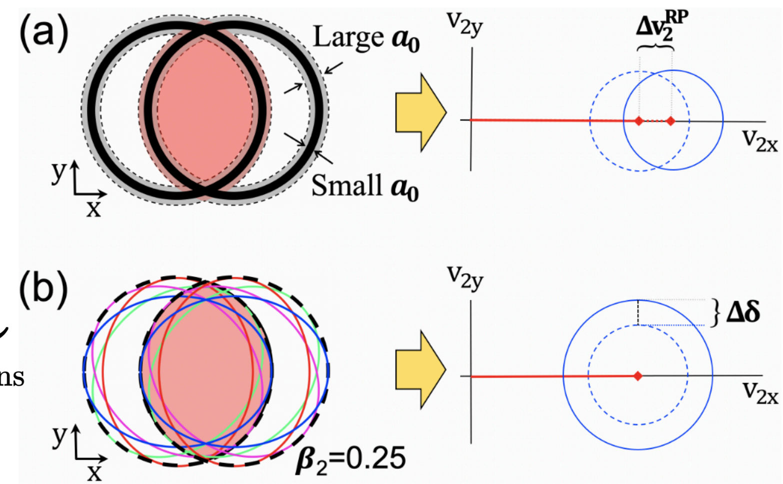
Simultaneously constrain these parameters using different N_{ch} regions

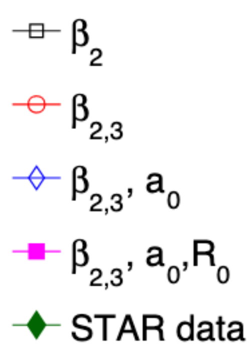
Separating shape and size effects

Nuclear skin contributes to $v_2^{rp} \sim v_2\{4\}$,
deformation contribute to fluctuations



$$\langle v_2^2\rangle = \underbrace{\left(v_2^{rp}\right)^2}_{\text{mean}} + \underbrace{\delta^2}_{\text{fluctuations}}$$





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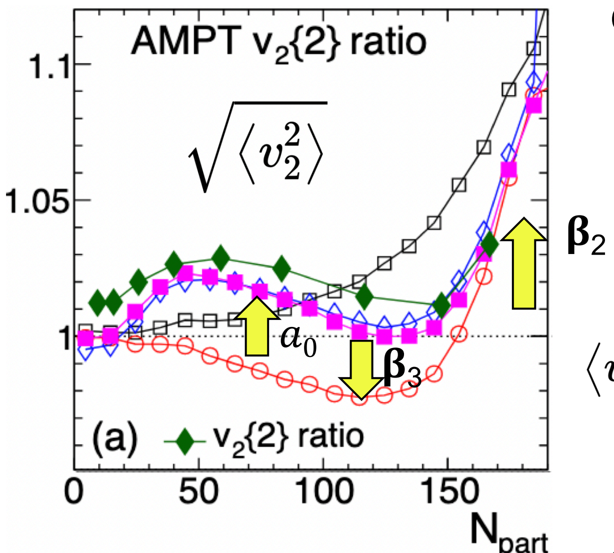

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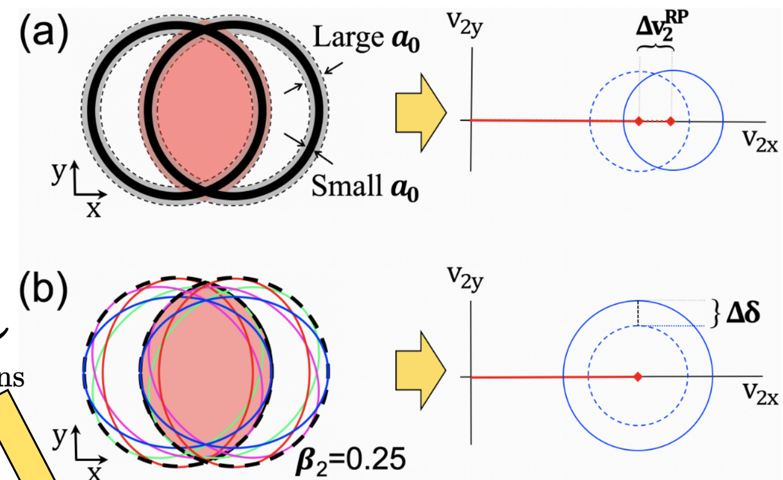

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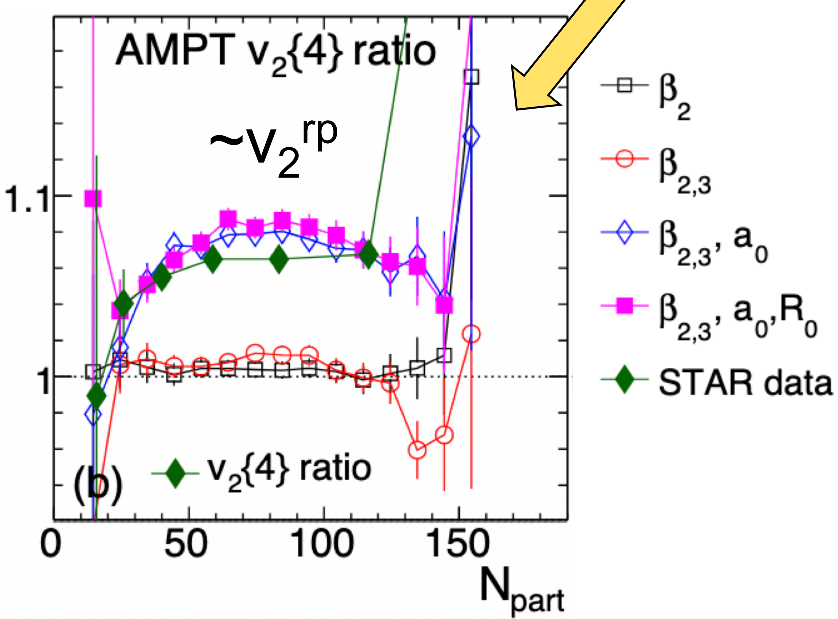
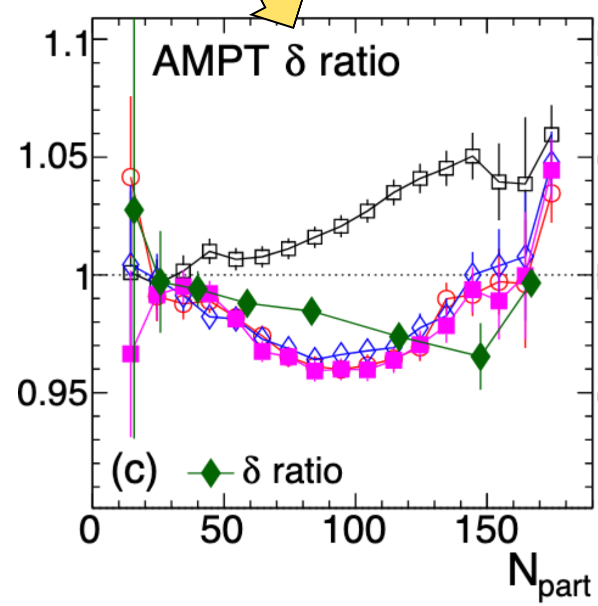
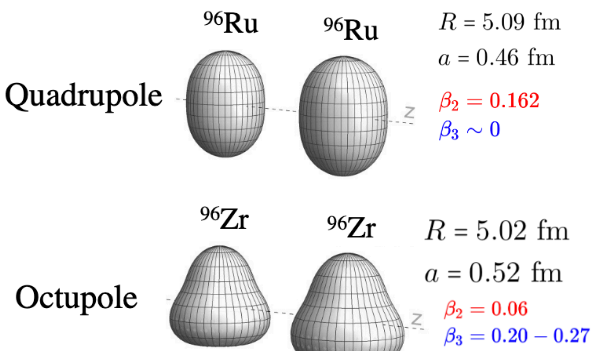




Nuclear structure via $p(N_{ch})$, <pT>-ratio

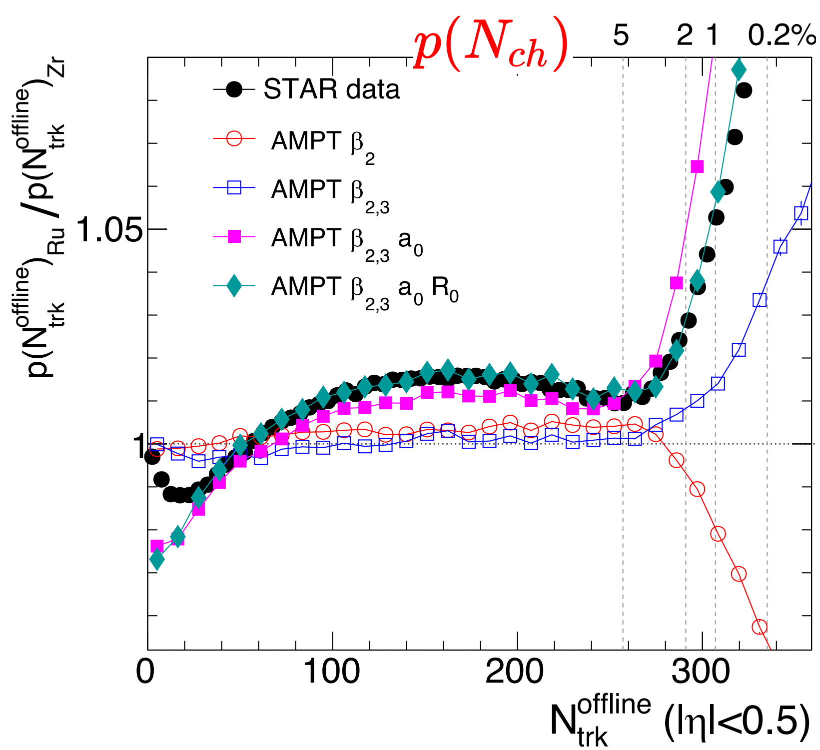
Earlier studies on this from H.Li, H.J Xu, PRL125, 222301 (2020) arXiv:2111.14812



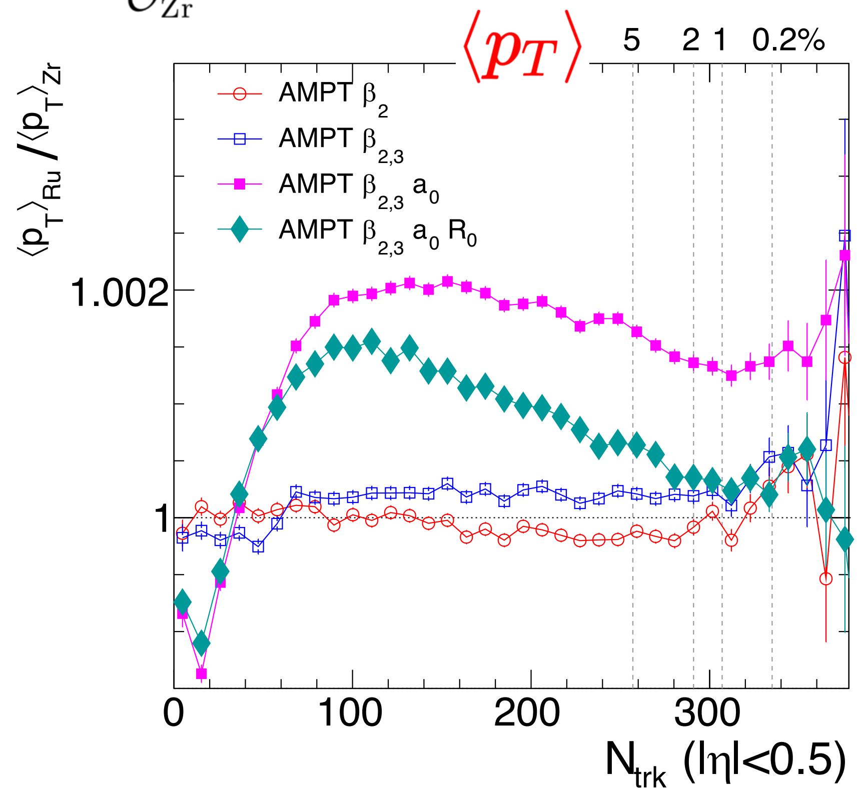
J. Jia, aXiv:2106.08768

- For Nch ratio:
  - β_{2Ru}~0.16 decrease ratio, increase after considering β_{3Zr}~0.2
  - The bump structure in non-central region from Δa_0 and ΔR_0
- For <pT> ratio:
  - Strong influence from Δa_0 and ΔR_0

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\rm Ru}}{\mathcal{O}_{\rm Zr}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$



Δa_0 and ΔR_0 influences add up

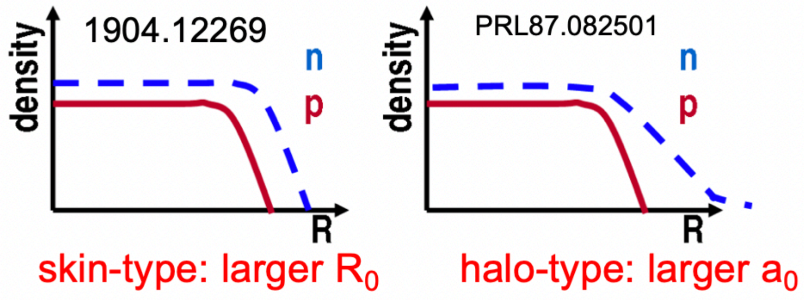


Δa_0 and ΔR_0 influences cancel

Relating to neutron skin: $\Delta r_{\mathrm{np}} = \langle r_n \rangle^{1/2} - \langle r_p \rangle^{1/2}$

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1 \Delta\beta_2^2 + c_2 \Delta\beta_3^2 + c_3 \Delta R_0 + c_4 \Delta a$$

arXiv:2111.15559



$$\Delta r_{np} \approx \frac{\langle r^2 \rangle - \langle r_p^2 \rangle}{\sqrt{\langle r^2 \rangle}(\delta + 1)} \qquad \delta = (N - Z)/A$$

Neutron skin Δ_{np} expressed by R_0 and a_0 for nucleons and protons:

For Woods-Saxon:

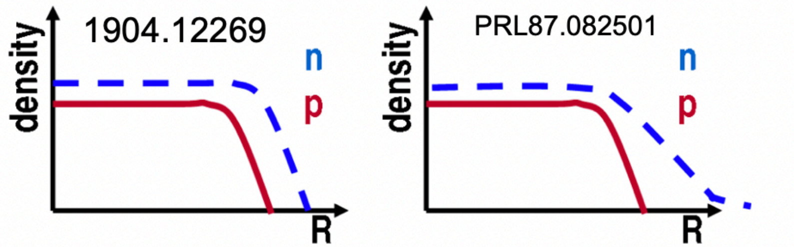
$$\langle r^2 \rangle \approx \left(\frac{3}{5} R_0^2 + \frac{7}{5} \pi^2 a^2 \right)$$

$$\langle r_p^2 \rangle \approx \left(\frac{3}{5} R_{0,p}^2 + \frac{7}{5} \pi^2 a_p^2 \right)$$

Relating to neutron skin: $\Delta r_{\rm np} = \langle r_n \rangle^{1/2} - \langle r_p \rangle^{1/2}$

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\rm Ru}}{\mathcal{O}_{\rm Zr}} \approx 1 + c_1 \Delta\beta_2^2 + c_2 \Delta\beta_3^2 + c_3 \Delta R_0 + c_4 \Delta a$$

arXiv:2111.15559



skin-type: larger R_0 halo-type: larger a_0

$$\Delta r_{np} \approx \frac{\langle r^2 \rangle - \langle r_p^2 \rangle}{\sqrt{\langle r^2 \rangle}(\delta + 1)} \qquad \delta = (N - Z)/A$$

Neutron skin Δ_{np} expressed by R_0 and a_0 for nucleons and protons:

For Woods-Saxon:

$$\langle r^2 \rangle \approx \left(\frac{3}{5} R_0^2 + \frac{7}{5} \pi^2 a^2 \right)$$

$$\langle r_p^2 \rangle \approx \left(\frac{3}{5} R_{0,p}^2 + \frac{7}{5} \pi^2 a_p^2 \right)$$

Isobar collision measure "difference of neutron skin" from ΔR_0 Δa for nucleons, and known ΔR_0 Δa for protons:

$$\Delta(\Delta r_{np}) = \Delta r_{np,1} - \Delta r_{np,2} \approx \frac{\Delta Y - \frac{7\pi^2}{3}\frac{\bar{a}^2}{\bar{R}_0^2}\left(\frac{\Delta Y}{2} + \bar{Y}\left(\frac{\Delta a}{\bar{a}} - \frac{\Delta R_0}{\bar{R}_0}\right)\right)}{\sqrt{15}\bar{R}_0\left(1 + \bar{\delta}\right)}$$

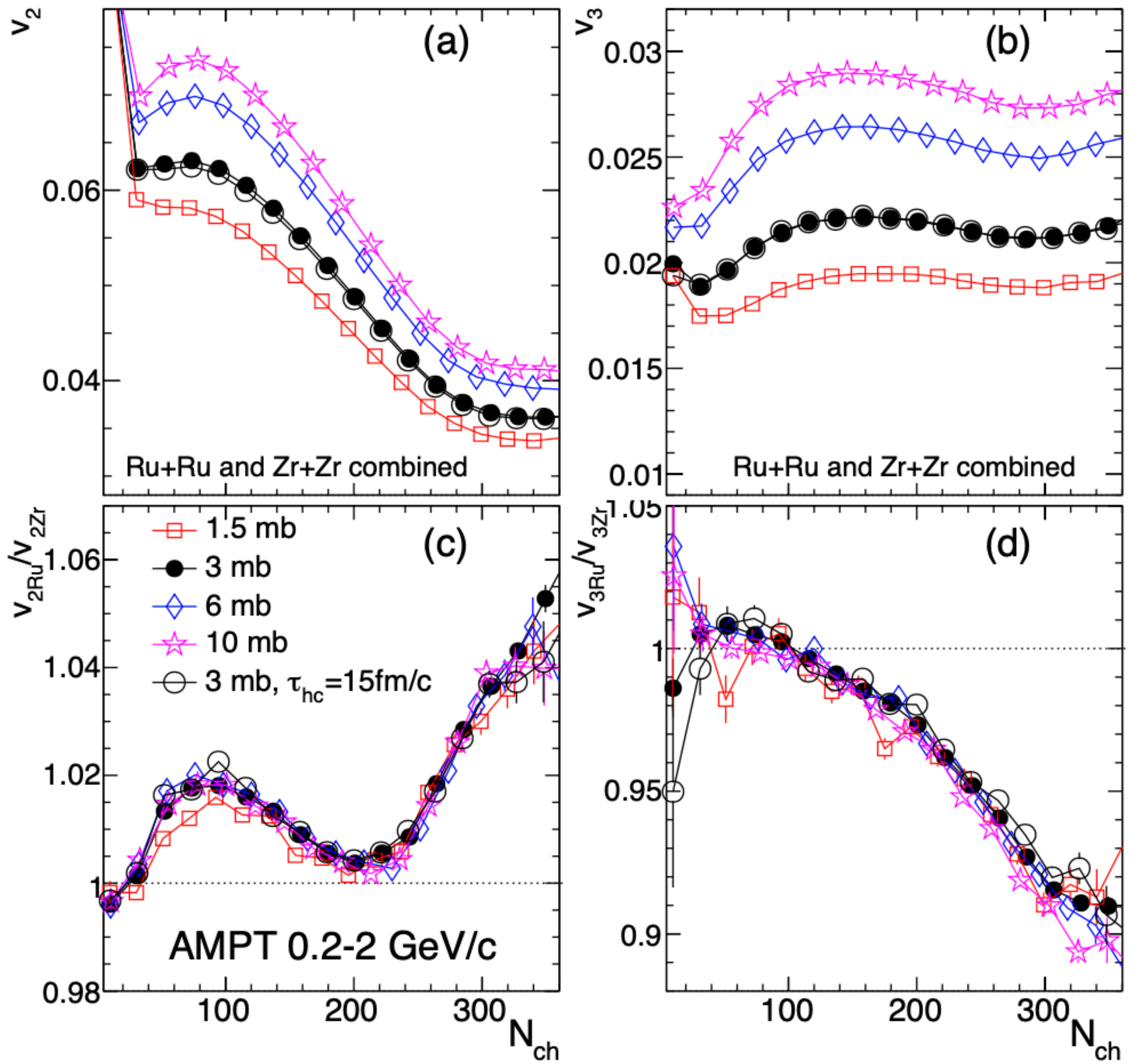
$$\Delta x = x_1 - x_2 \qquad \bar{x} = (x_1 + x_2)/2 \qquad Y \equiv 3(R_0^2 - R_{0,p}^2) + 7\pi^2(a^2 - a_p^2)$$

Isobar ratios not affected by final state

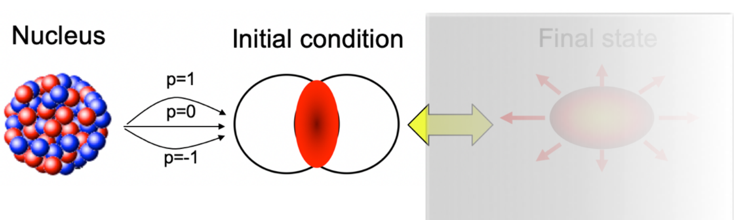
- Vary the shear viscosity via partonic cross-section
  - Flow signal change by 30-50%, the v_n ratio unchanged.

$$v_n = k_n \varepsilon_n$$

$$\frac{v_{n,\mathrm{Ru}}}{v_{n,\mathrm{Zr}}} \approx \frac{\varepsilon_{n,\mathrm{Ru}}}{\varepsilon_{n,\mathrm{Zr}}}$$

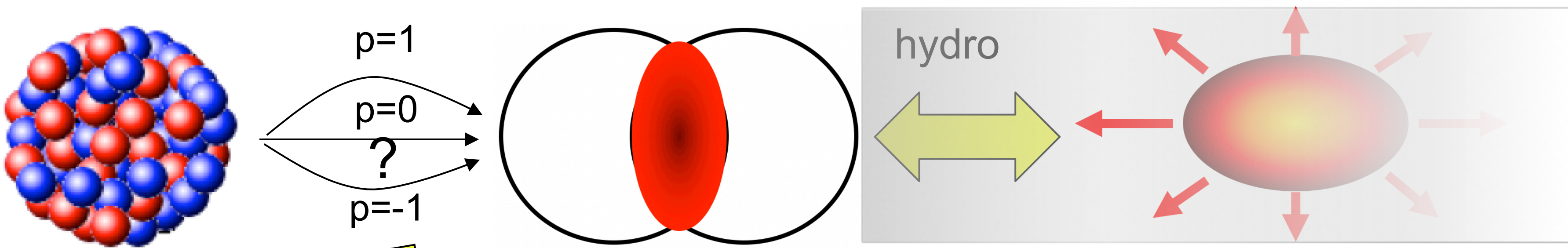


Robust probe of initial state!

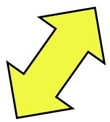


Isobar to constrain initial condition

Nucleus Initial condition Final state



$$T_A(x,y) = \int \rho(x,y,z)dz$$



$$\frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

c_n relates nuclear structure and initial condition

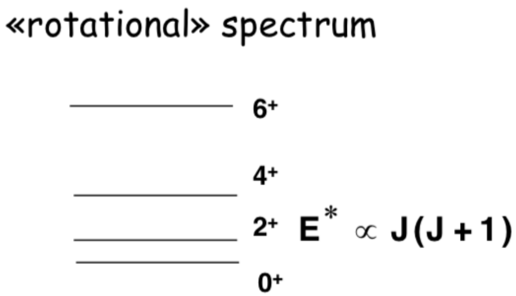
- Different ways of depositing energy $T \propto \left(\frac{T_A^p + T_B^p}{2}\right)^{q/p}$

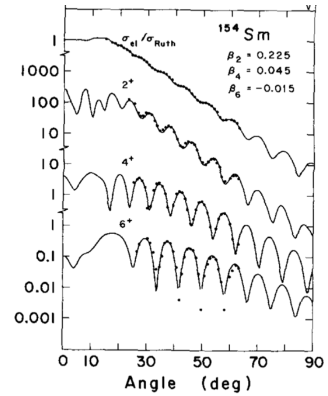
$$e(x,y) \sim \begin{cases} T_A + T_B & N_{\mathrm{part}} - \text{scaling}, p = 1 \\ T_A T_B & N_{\mathrm{coll}} - \text{scaling}, p = 0, q = 2 \\ \sqrt{T_A T_B} & \text{Trento default}, p = 0 \\ \min\{T_A, T_B\} & \text{KLN model}, p \sim -2/3 \\ T_A + T_B + \alpha T_A T_B & \text{two-component model,} \\ & \text{similar to quark-glauber model} \end{cases}$$

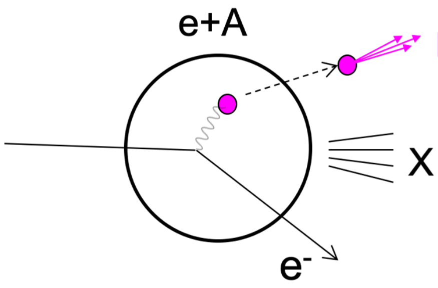
Use nuclear structure as extra lever-arm for initial condition

Low-energy vs high-energy HI method

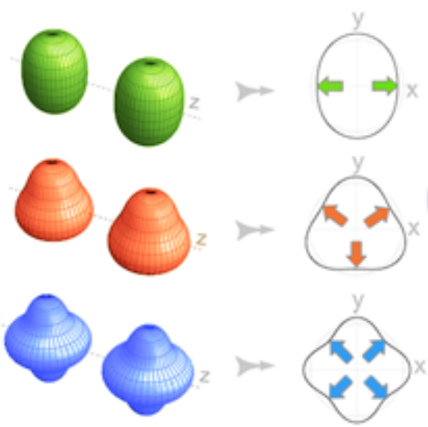
- Shape from B(En), radial profile from e+A or ion-A scattering

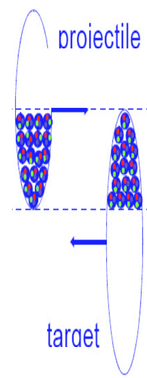


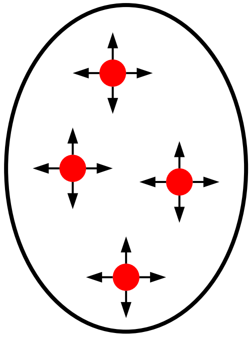




- Shape frozen in crossing time ($<10^{-24}$s), probe entire mass distribution via multi-point correlations.







Collective flow response to nuclear structure

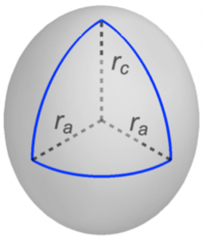
$$S(\mathbf{s}_1, \mathbf{s}_2) \equiv \langle \delta\rho(\mathbf{s}_1)\delta\rho(\mathbf{s}_2)\rangle$$
$$= \langle \rho(\mathbf{s}_1)\rho(\mathbf{s}_2)\rangle - \langle \rho(\mathbf{s}_1)\rangle\langle \rho(\mathbf{s}_2)\rangle.$$

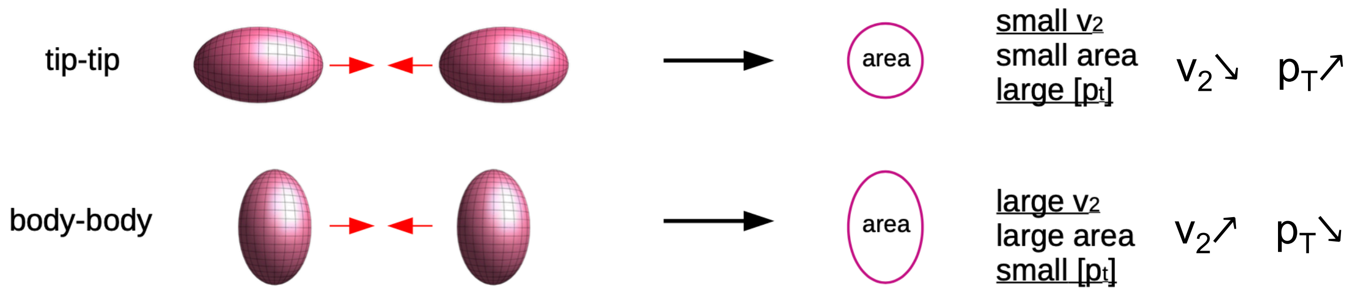
Triaxility $R(\theta,\phi) = R_0\left(1 + \beta_2[\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}]\right.$

1910.04673, 2004.14463

Prolate

$\beta_2 = 0.25, \cos(3\gamma) = 1$





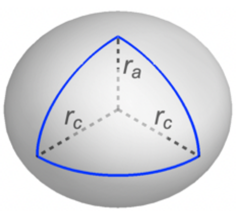
Need 3-point correlators to probe the 3 axes

$$\langle v_2^2 \delta p_{\mathrm{T}} \rangle \sim -\beta_2^3 \cos(3\gamma) \qquad \langle (\delta p_{\mathrm{T}})^3 \rangle \sim \beta_2^3 \cos(3\gamma)$$

2109.00604

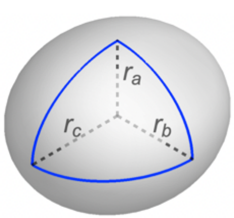
Triaxial

$\beta_2 = 0.25, \cos(3\gamma) = 0$



Oblate

$\beta_2 = 0.25, \cos(3\gamma) = -1$



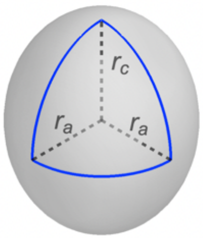
Triaxility

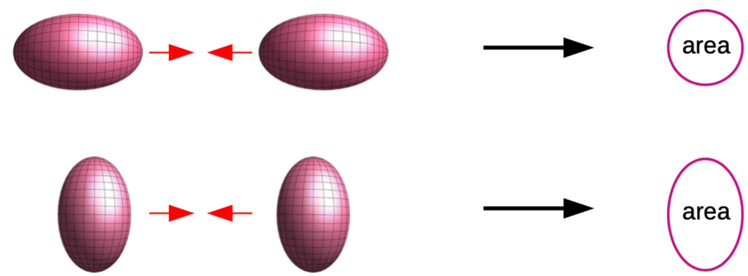
$$R(\theta,\phi) = R_0\Big(1 + \beta_2[\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}]$$

1910.04673, 2004.14463

Prolate

$\beta_2 = 0.25, \cos(3\gamma) = 1$





tip-tip	small v_2 small area large [p_t]	v_2↘	p_T↗
body-body	large v_2 large area small [p_t]	v_2↗	p_T↘

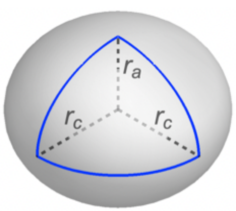
Need 3-point correlators to probe the 3 axes

$$\left\langle v_2^2 \delta p_{\mathrm{T}} \right\rangle \sim -\beta_2^3\cos(3\gamma) \qquad \left\langle (\delta p_{\mathrm{T}})^3 \right\rangle \sim \beta_2^3\cos(3\gamma)$$

2109.00604

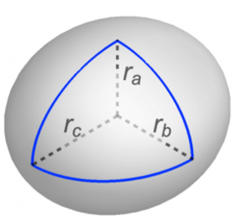
Triaxial

$\beta_2 = 0.25, \cos(3\gamma) = 0$



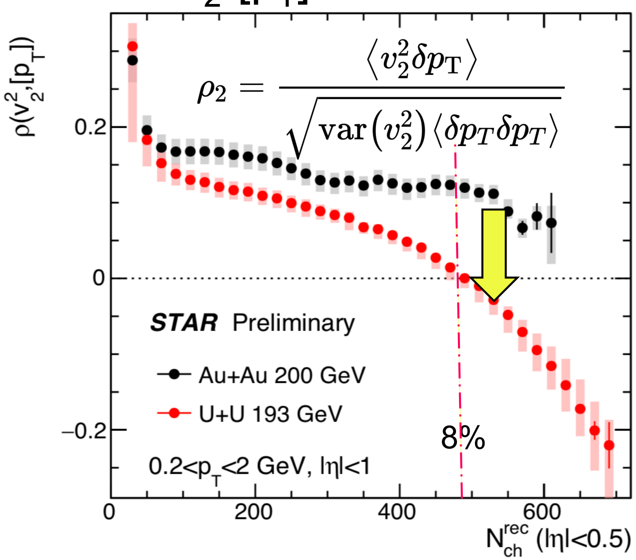
Oblate

$\beta_2 = 0.25, \cos(3\gamma) = -1$

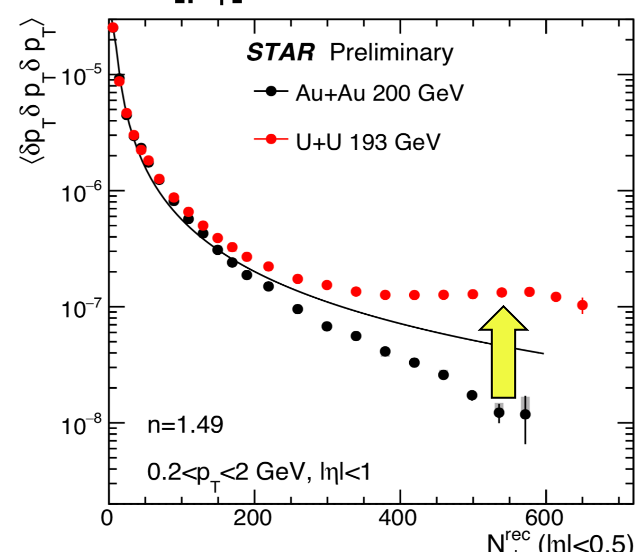


Compare U+U vs Au+Au: $\beta_{2U} \sim 0.28$, $\beta_{2Au} \sim 0.13$:

v_2-[p_T] covariance



[p_T] skewness

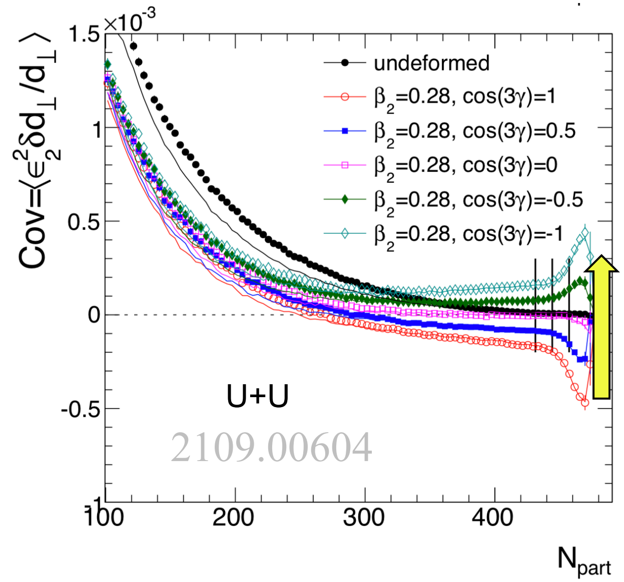


Influence of triaxiality: Glauber model

Skewness super sensitive

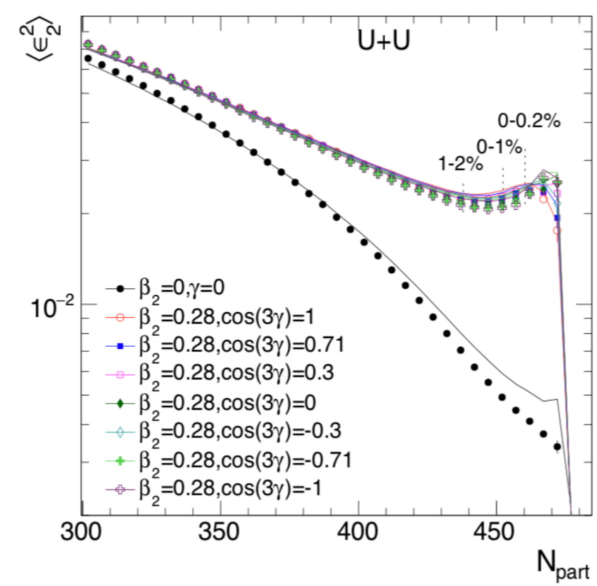
Described by

$$\left\langle \varepsilon_2^2 \frac{\delta d_\perp}{d_\perp} \right\rangle \propto \langle v_2^2 \delta p_{\mathrm{T}} \rangle \propto a + b\cos(3\gamma)\beta_2^3$$



variances insensitive to γ

$$\langle \varepsilon_2^2 \rangle \propto \langle v_2^2 \rangle \propto a + b\beta_2^2$$



Use variance to constrain β_2, use skewness to constrain γ

(β_2,γ) diagram in heavy-ion collisions

The (β_2,γ) dependence in 0-1% U+U Glauber model can be approximated by:

$$d_\perp \propto 1/R_\perp$$

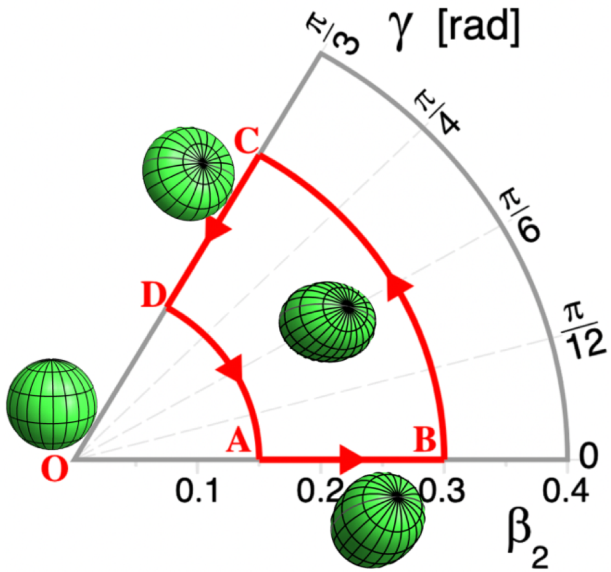
$$\langle \varepsilon_2^2 \rangle \approx [0.02 + \beta_2^2] \times 0.235$$

$$\langle (\delta d_\perp / d_\perp)^2 \rangle \approx [0.035 + \beta_2^2] \times 0.0093$$

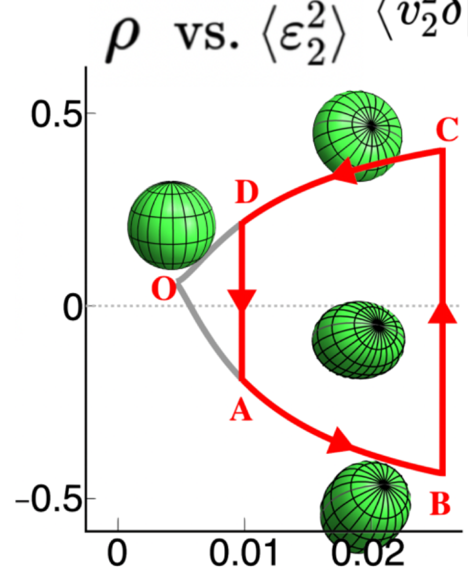
$$\langle \varepsilon_2^2 \delta d_\perp / d_\perp \rangle \approx [0.0005 - (0.07 + 1.36 \cos(3\gamma))\beta_2^3] \times 10^{-2}$$

$$\rho = \frac{\langle \varepsilon_2^2 \delta d_\perp \rangle}{\langle \varepsilon_2^2 \rangle \sqrt{\langle (\delta d_\perp)^2 \rangle}}$$

Map from (β_2,γ) plane to HI observables



How about $\langle v_2^2 \delta[p_T] \rangle$ vs. $\langle v_2^2 \rangle$



Collision system scan to map out this trajectory: calibrate coefficients with species with known β,γ, then predict for species of interest.

Summarizing questions

- How the nuclear shape and radial profile extracted from HI collisions related to those measured in nuclear structure experiments?
- How the uncertainties in nuclear structure impact the initial state of HI collisions and extraction of QGP transport properties?
- What are the most interesting stable isobar species to collide?

PROGRAM

JANUARY 23 - FEBRUARY 24, 2023

Intersection of nuclear structure and high-energy nuclear collisions (23-1a)

INSTITUTE for NUCLEAR THEORY

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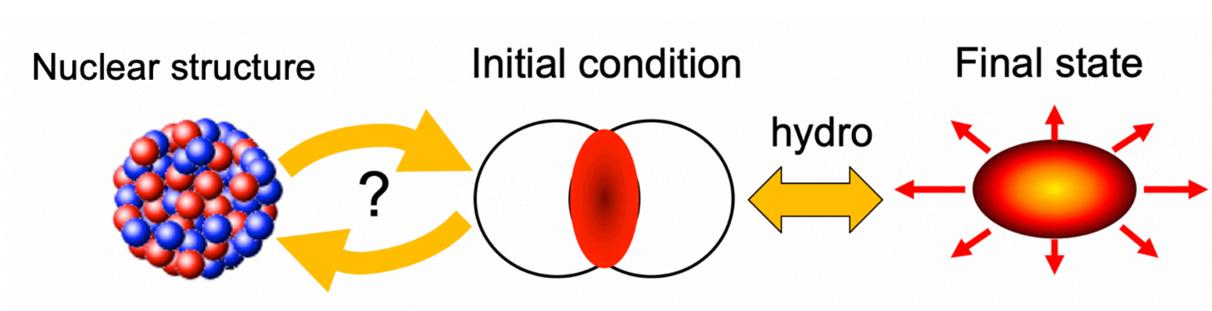


arXiv:2102.08158

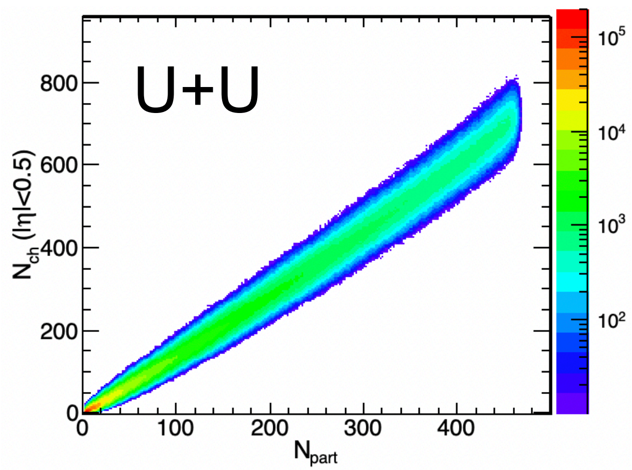
A	isobars	A	isobars	A	isobars
36	Ar, S	106	Pd, Cd	148	Nd, Sm
40	Ca, Ar	108	Pd, Cd	150	Nd, Sm
46	Ca, Ti	110	Pd, Cd	152	Sm, Gd
48	Ca, Ti	112	Cd, Sn	154	Sm, Gd
50	Ti, V, Cr	113	Cd, In	156	Gd, Dy
54	Cr, Fe	114	Cd, Sn	158	Gd, Dy
64	Ni, Zn	115	In, Sn	160	Gd, Dy
70	Zn, Ge	116	Cd, Sn	162	Dy, Er
74	Ge, Se	120	Sn, Te	164	Dy, Er
76	Ge, Se	122	Sn, Te	168	Er, Yb
78	Se, Kr	123	Sb, Te	170	Er, Yb
80	Se, Kr	124	Sn, Te, Xe	174	Yb, Hf
84	Kr, Sr, Mo	126	Te, Xe	176	Yb, Lu, Hf
86	Kr, Sr	128	Te, Xe	180	Hf, W
87	Rb, Sr	130	Te, Xe, Ba	184	W, Os
92	Zr, Nb, Mo	132	Xe, Ba	186	W, Os
94	Zr, Mo	134	Xe, Ba	187	Re, Os
96	Zr, Mo, Ru	136	Xe, Ba, Ce	190	Os, Pt
98	Mo, Ru	138	Ba, La, Ce	192	Os, Pt
100	Mo, Ru	142	Ce, Nd	198	Pt, Hg
102	Ru, Pd	144	Nd, Sm	204	Hg, Pb
104	Ru, Pd	146	Nd, Sm		

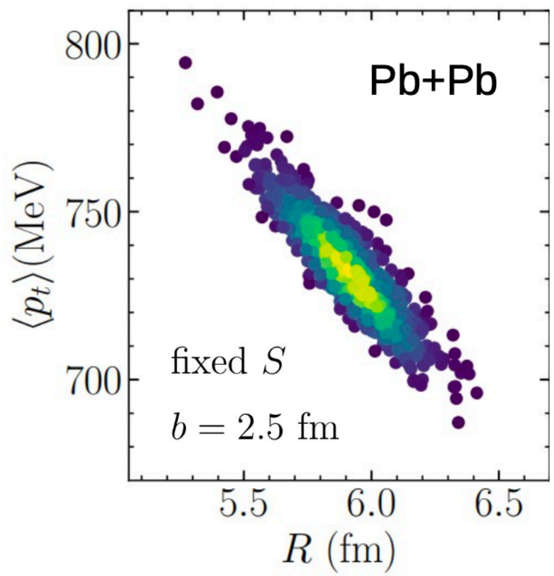
Linear corr. between initial & final state

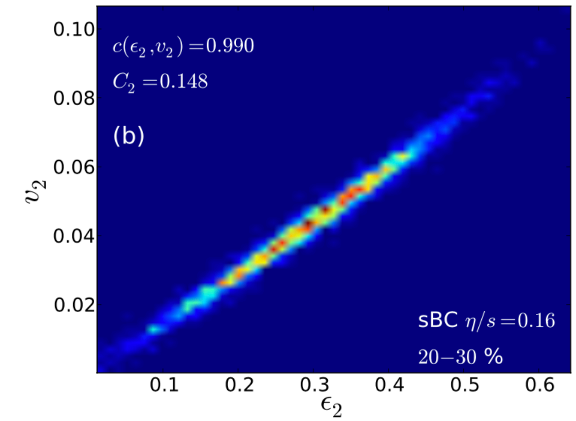
$$N_{ch} \propto N_{part}$$

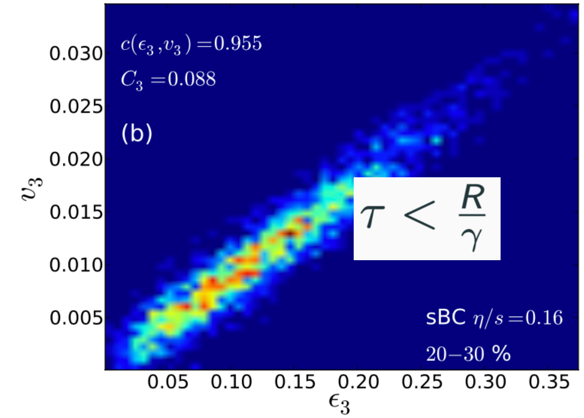
$$\frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp}$$

$$V_n \propto \mathcal{E}_n$$









nice correlation at very high energy
breaks down at low energy