Unit 2
Requirements for magnets in accelerators

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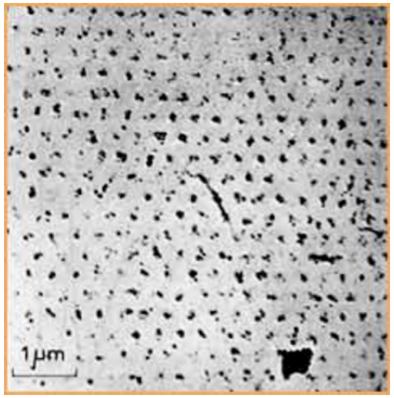
All the units will use International System (meter, kilo, second, ampere) unless specified

FOREWORD

- The science (or the art …) of superconducting magnets is a exciting, fancy and dirty mixture of physics, engineering, and chemistry
  - Chemistry and material science: the quest for superconducting materials with better performances
  - Quantum physics: the key mechanisms of superconductivity and superfluidity
  - Classical electrodynamics: magnet design
  - Mechanical engineering: support structures
  - Electrical engineering: powering of the magnets and their protection
  - Cryogenics: keep them cool …
- The cost optimization is a key element
  - Keep them cheap …

FOREWORD

- An example of the variety of the issues to be taken into account
  - The field of the LHC dipoles (8.3 T) is related to the critical field of Niobium-Titanium (Nb-Ti), which is determined by the microscopic quantum properties of the material



Quantized fluxoids penetrating a superconductor used in accelerator magnets

A 15m truck unloading a 27 tons LHC dipole

  - The length of the LHC dipoles (15 m) has been determined by the maximal dimensions of (regular) trucks allowed on European roads
- This makes the subject complex, challenging and complete for any physicist or engineer who has the chance of working in this field

AIM OF THE LECTURES

- Superconducting magnets for particle accelerators are a vaste domain
- These lectures will be especially focused on magnets for colliders, with a special eye on the CERN high energy infrastructures
  - The Large Hadron Collider, 27 km tunnel with 8.3 T magnets based on Nb-Ti (CERN-2004-0039, the blue book)
  - The HL-LHC, upgrade in luminosity of the LHC based on Nb3Sn quadrupole – ongoing project with installation foreseen in 2025 (CERN-2017-007)
  - We will give some examples for the studies related to FCC (Future Circular Colliders)
    - A ~100 TeV collider based on a ~16 T Nb_3Sn dipole in a ~100 km tunnel (CERN-ACC-2018-058)

CONTENTS

- Principles of synchrotron
- How to keep particles on a circular orbit
  - Relation between energy, dipolar field, machine length
- The beam size and principles of focusing
  - Aperture requirements for arc dipoles and arc quadrupoles
  - Gradient requirements for arc quadrupoles
- The beam size in the interaction regions
- Some examples
- Conclusions

- Reference: https://indico.cern.ch/category/12408/ Units 1 and 2 (see also the attached notes Chapter1 and Chapter2)

PRINCIPLES OF A SYNCHROTRON

- Electro-magnetic field accelerates particles

$$\vec{F} = e\vec{E}$$

- Magnetic field steers the particles in a ~circular orbit

$$\vec{F} = e\vec{v} \times \vec{B} \qquad p = eB\rho$$

  - Driving particles in the same accelerating structure several times
  - Particle accelerated $\rightarrow$ energy increased $\rightarrow$ magnetic field increased ("synchro") to keep the particles on the same orbit of curvature ρ

$$p = eB\rho$$

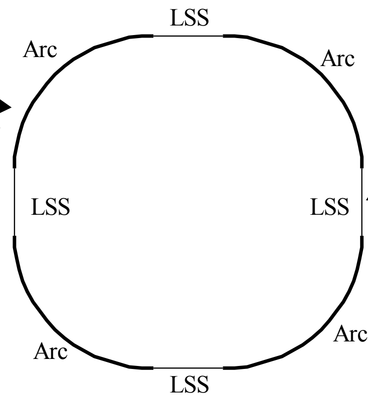
Constant

- Limits to the increase in energy
  - The maximum field of the dipoles (proton machines)
    - This is why high field magnets are important to get high energies!
    - Paradox: B does not accelerate but limits the energy
  - The synchrotron radiation due to bending trajectories (electron machines) – ignored in these lectures

PRINCIPLES OF A SYNCHROTRON

$$\vec{F} = e\vec{v} \times \vec{B}$$

$$p = eB\rho$$

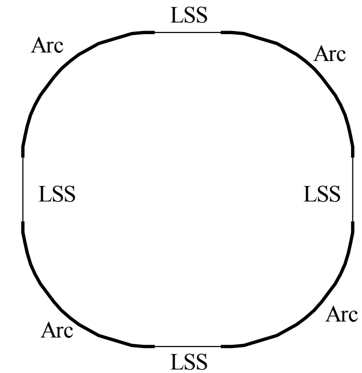


$$\vec{F} = e\vec{E}$$

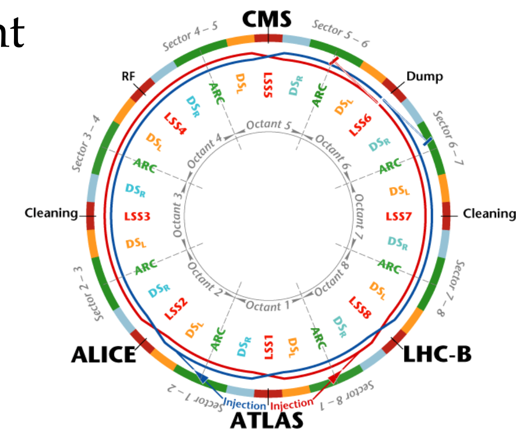
- Colliders: two beams with opposite momentum collide
  - This doubles the energy !
  - One pipe if particles collide their antiparticles (LEP, Tevatron)
  - Otherwise, two pipes (ISR, RHIC, HERA, LHC)

PRINCIPLES OF A SYNCHROTRON

- The arcs: region where the beam is bent
  - Dipoles for bending
  - Quadrupoles for focusing
  - Correctors
- Long straight sections (LSS)
  - Interaction regions (IR) where the experiments are housed
    - Quadrupoles for strong focusing in interaction point
    - Dipoles for beam crossing in two-ring machines
  - Regions for other services
    - Beam injection (dipole kickers)
    - Accelerating structure (RF cavities)
    - Beam dump (dipole kickers)
    - Beam cleaning (collimators)
- LHC: 27.6 km of tunnel, 20.5 km in the arcs, 7.2 km in the insertions



A schematic view of a synchrotron



The lay-out of the LHC

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HOW TO KEEP PARTICLES ON A CIRCLE

- Kinematics of circular motion

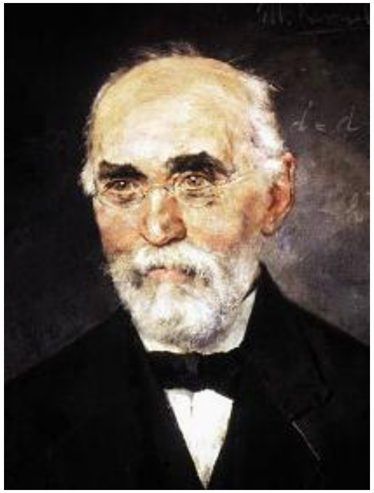
$$\left|\frac{d\vec{v}}{dt}\right| = \frac{v^2}{\rho}$$

- Relativistic dynamics

$$\vec{p} \equiv m\gamma_r \vec{v} \qquad \gamma_r \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \vec{F} = \frac{d\vec{p}}{dt} = m\frac{d(\gamma_r \vec{v})}{dt}$$

- Lorentz (?) force

$$\vec{F} = e\vec{v} \times \vec{B}$$



Hendrik Antoon Lorentz, Dutch

(18 July 1853 – 4 February 1928), painted by Menso Kamerlingh Onnes, brother of Heinke

Putting all together

- Hyp. 1 - longitudinal acceleration<<transverse acceleration

$$m\gamma_r\left|\frac{d\vec{v}}{dt}\right| >> m\frac{d\gamma_r}{dt}\left|\vec{v}\right| \qquad \left|\vec{F}\right| = \left|\frac{d\vec{p}}{dt}\right| = m\left|\frac{d(\gamma_r\vec{v})}{dt}\right| \approx m\gamma_r\left|\frac{d\vec{v}}{dt}\right| = m\gamma_r\frac{v^2}{\rho}$$

$$\left|\vec{F}\right| = \left|e\vec{v}\times\vec{B}\right| = evB = m\gamma_r\frac{v^2}{\rho} \qquad eB = m\gamma_r\frac{v}{\rho} = \frac{p}{\rho}$$

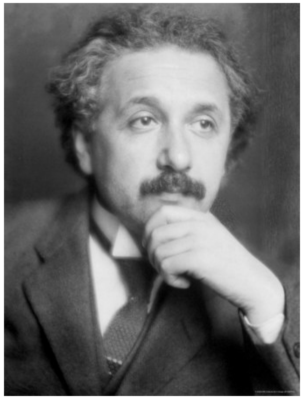
$$p = eB\rho$$

A DIGRESSION ON "RELATIVISTIC MASS"

- Note: in several books
  - The mass is called the rest mass $m \to m_0$
  - A new relativistic mass is defined $m \equiv m_0 \gamma_r$
  - So that the momentum is still defined as $\vec{p} \equiv m\vec{v}$
  - This is why you see in several texts that accelerating corresponds to increase the mass (?)
    - This is a rather controversial intepretation of the formalism
    - Einstein was not in favour of it – I will follow his advice

"It is not good to introduce the concept of the mass of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the 'rest mass' *m*. Instead of introducing *M* it is better to mention the expression for the momentum and energy of a body in motion."

— *Albert Einstein in letter to Lincoln Barnett, 19 June 1948 (quote from L. B. Okun (1989), p. 42*[1]



Albert Einstein, German

(14 March 1879 – 18 April 1955)

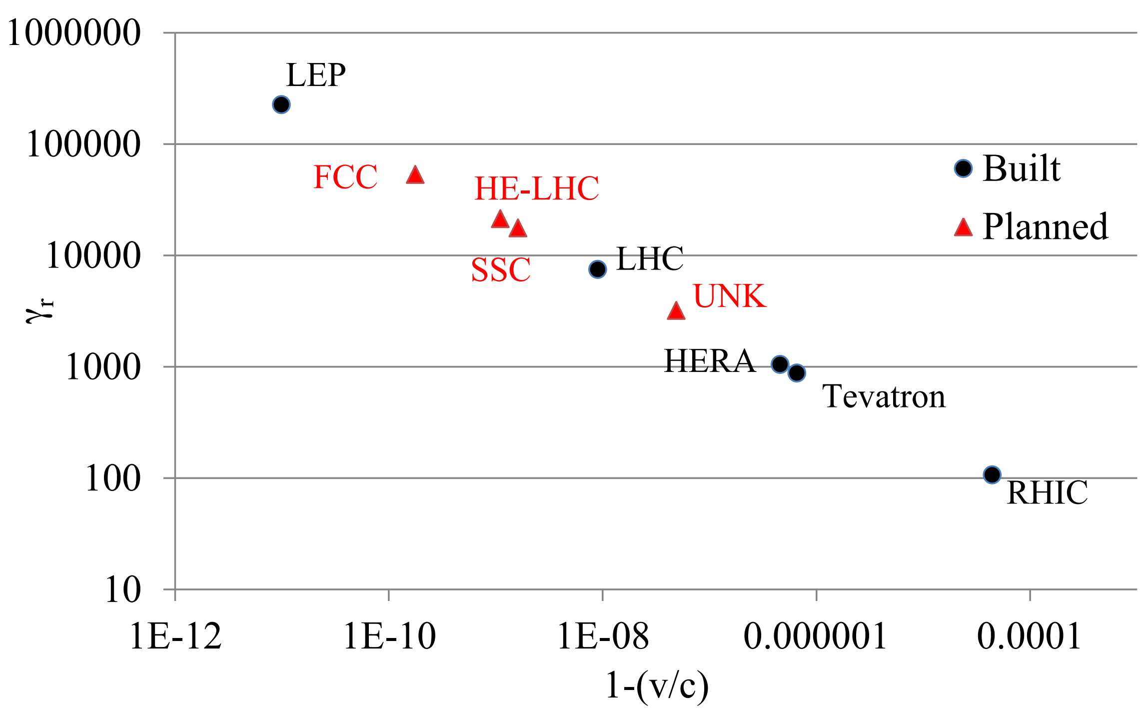
- Other authors against this concept

"The concept of "relativistic mass" is subject to misunderstanding. That's why we don't use it. First, it applies the name mass - belonging to the magnitude of a 4-vector - to a very different concept, the time component of a 4-vector. Second, it makes increase of energy of an object with velocity or momentum appear to be connected with some change in internal structure of the object. In reality, the increase of energy with velocity originates not in the object but in the geometric properties of spacetime itself."[18]

E. F. Taylor, J. A. Wheeler (1992), *Spacetime Physics, second edition*, New York: W.H. Freeman and Company, ISBN 0-7167-2327-1

RELATION BETWEEN GAMMA, VELOCITY AND ENERGY

- What are the ranges of velocity obtained in particle accelerators?
  - From 10^{-4} to 10^{-11} from the speed of light !



$$\vec{p} \equiv m\gamma_r \vec{v}$$

$$\gamma_r \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E = m\gamma_r c^2$$

$$\gamma_r = \frac{E}{mc^2}$$

- Please note e^- in LEP (100 GeV) go faster than p^+ in LHC (7 TeV)
- This happens since 7000/100 < 2000, ie the ratio between p and e mass

- Preservation of 4-momentum

$$E = m\gamma_r c^2$$

$$E^2 - p^2c^2 = m^2c^4 \qquad E = \sqrt{m^2c^4 + p^2c^2}$$

Rest energy $E_0 = mc^2$

- Hyp. 2 Ultra-relativistic regime $pc >> mc^2$ $E \sim pc$ $p = eB\rho$

$$E = ceB\rho$$

- Using practical units for particle with charge as electron, one has

$$E[GeV] = 0.3 \times B[T] \times \rho[m]$$

magnetic field in Tesla …

- Remember 1 eV=1.602×10^{-19} J
- Remember 1 e= 1.602×10^{-19} C

TESLA DIGRESSION

Nikolai Tesla (10 July 1856 - 7 January 1943)

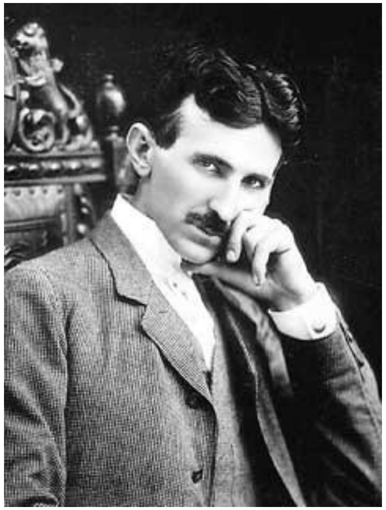
- Born at midnight during an electrical storm in Smiljan near Gospić (now Croatia)
- Son of an orthodox priest
- A national hero in Serbia

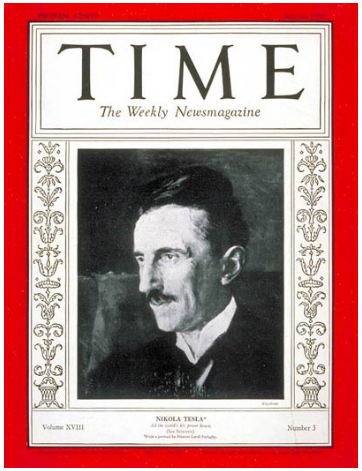
Career

- Polytechnic in Gratz (Austria) and Prague
- Emigrated in the States in 1884
- Electrical engineer
- Inventor of the alternating current induction motor (1887)
- Author of 250 patents

Miscellaneous

- Strongly against marriage
- Considered sex as a waste of vital energy





Tesla, man of the year

ALL YOU HAVE TO KNOW ABOUT TAYLOR SERIES

- For small ε, one has

$$(1+\varepsilon)^{\alpha} \approx 1+\alpha\varepsilon$$

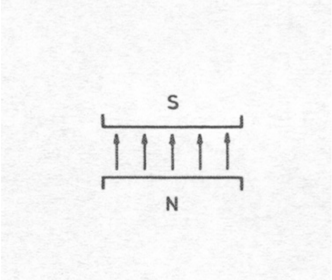
- Example
  - α=1 is trivial $(1+\varepsilon) = 1+\varepsilon$
  - α=2 is a bit less trivial $(1+\varepsilon)^2 = 1+2\varepsilon+\varepsilon^2$
  - For ε=0.01 (1+0.01)^2=1.0201 1+2×0.01+ .. = 1.02+ …
  - What is neglected? Order of ε^2, so my result is precise up to 0.0001

- Application to relativity
  - We find Newton: energy is mv^2/2

$$\gamma_r \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \left(1-\frac{v^2}{c^2}\right)^{-1/2} = 1+\frac{v^2}{2c^2}$$

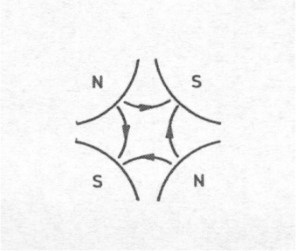
$$E = m\gamma_r c^2 = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = mc^2\left(1+\frac{v^2}{2c^2}+\dots\right) = mc^2+\frac{m}{2}v^2+\dots$$

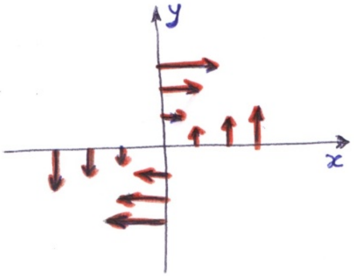
- The magnet that we need should provide a constant (over the space) magnetic field, to be increased with time to follow the particle acceleration
  - This is done by dipoles



$$\begin{cases} B_y = B_1 \\ B_x = 0 \end{cases}$$

- As the particle can deviate from the orbit, one needs a linear force to bring it back (like a spring)
  - We will show in the next section that this is given by quadrupoles





$$\begin{cases} B_y = Gx \\ B_x = Gy \end{cases}$$

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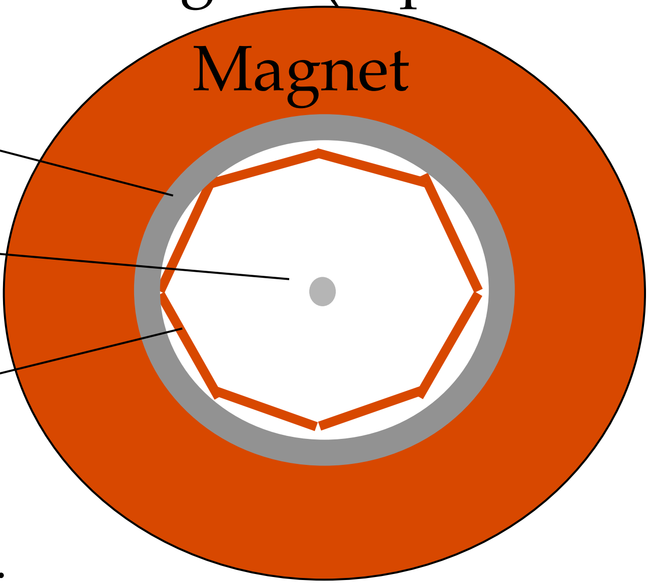
- Requirements for arc dipole and quadrupoles
  - Aperture requirements for dipole/quadrupole
  - Gradient requirements for quadrupoles
  - Some theoretical insight, and some examples
- Requirements for interaction region quadrupoles
- Conclusions

APERTURE REQUIREMENT IN THE ARC

- The aperture (diameter) needed in an arc magnet (dipole or quadrupole) is given by



$$\phi = \phi_0 + 2n\sigma$$

  - A constant accounting ϕ_0 for the cold bore, beam screen, shielding, tolerances …
  - A part proportional to the beam size σ (the spread of a Gaussian distribution of the transverse beam) times n (the number of sigma we take, usually more than 15)
- Example: LHC beam size (injection) is σ=1.2 mm (see next slide), we have 56 mm aperture diameter, ~15 mm for ϕ_0 and 40 mm for the beam i.e. >15 σ
  - Note that initially magnet aperture in LHC was 50 mm and was later enlarged to 56 mm for fears on field quality

SIZE OF THE BEAM: MAIN DEPENDENCES

- Size of the beam is given by
- Three relevant dependences:

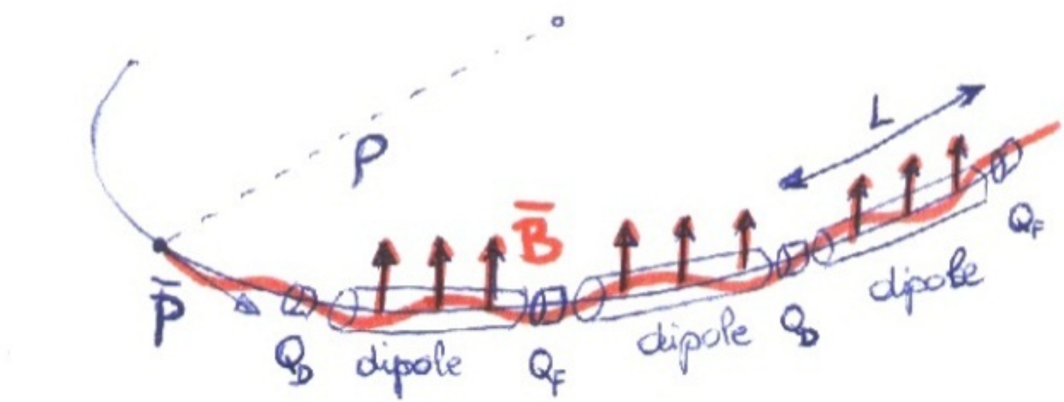
$$\sigma = \sqrt{\frac{\varepsilon\beta(s)}{\gamma_r}}$$

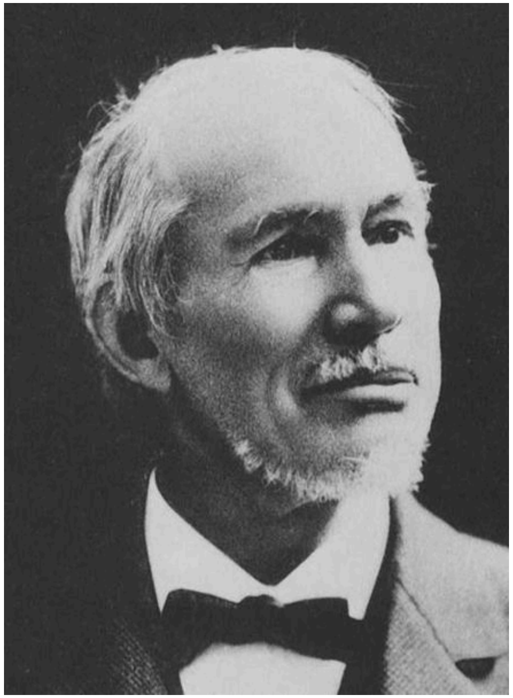
  - ε: emittance [m] invariant along the ring, invariant from ring to ring
    - This is a property of the injectors!
  - γ_r Beam energy: the larger the energy, the smaller the beam
  - β is the beta function [m], its square root giving the envelope of the beam along the ring (the sequence of magnets define the beta functions – what is also called beam optics)

  - LHC: ε_n =3.75 10^{-6} m rad
    - Beta is 170 m (see next slides)
    - High field E=7 TeV, γ=7460, σ=0.29 mm
    - Injection E=450 GeV, γ=480, σ=1.2 mm

SIZE OF THE BEAM: TRAJECTORY SHAPE

- A typical structure is the FODO cell: alternating quadrupoles spaced by length L of similar gradient
- This structure can ensure stability for certain combinations of quadrupole strength and quadrupole distance
- Ruled by Hill equation $$\frac{d^2x}{ds^2} + K(s)x = 0$$
  - s is the coordinate along the accelerator



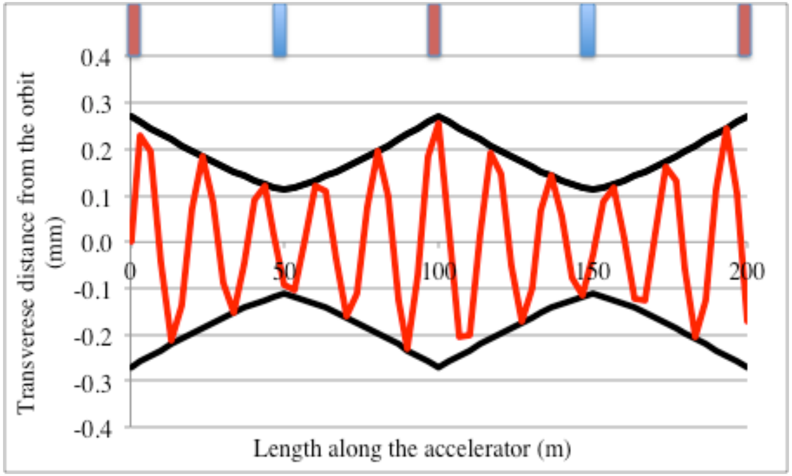


G. W. Hill, American
3 March 1838 – 16 April 1914

SIZE OF THE BEAM: TRAJECTORY SHAPE

- Hill equation solution is a pendulum where both amplitude and phase depend on *s*

$$x(s) = \sqrt{\frac{\varepsilon\beta(s)}{\gamma_r}}\cos(\psi(s))$$

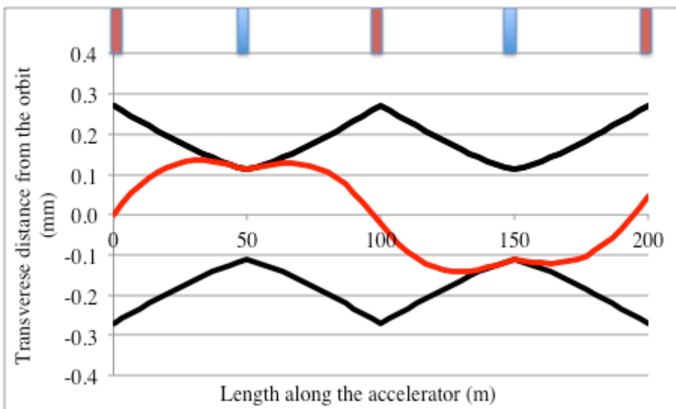


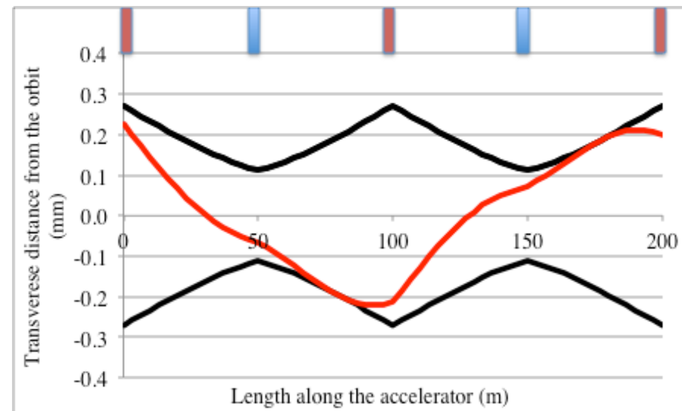
  - There are good reason to use the square root (see appendix)

- We define the phase advance Ψ_L as the angle done along one cell
- In the figure, we count 5 oscillations between red and red quadrupoles, so phase advance is 5*360=1800 degrees

SIZE OF THE BEAM: TRAJECTORY SHAPE

- In reality, the phase advance is much smaller, i.e. in one cell one does not complete one oscillation – here we show a 180 degrees phase advance (two cases shown with different starting points)





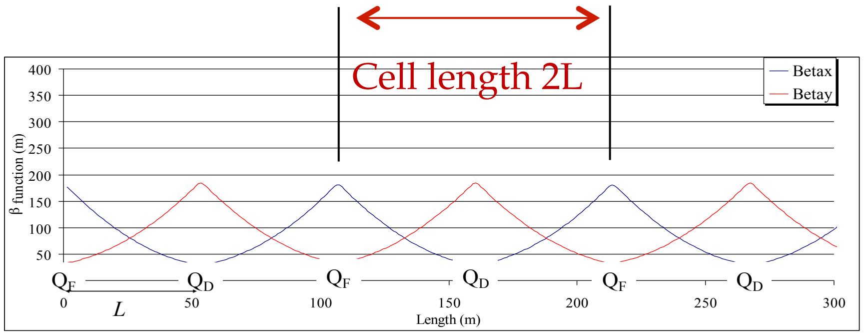
- In the LHC, the phase advance is 90 degrees so one complete oscillation is done every four cells (you go through 8 quadrupoles)

SIZE OF THE BEAM: MAIN DEPENDENCES

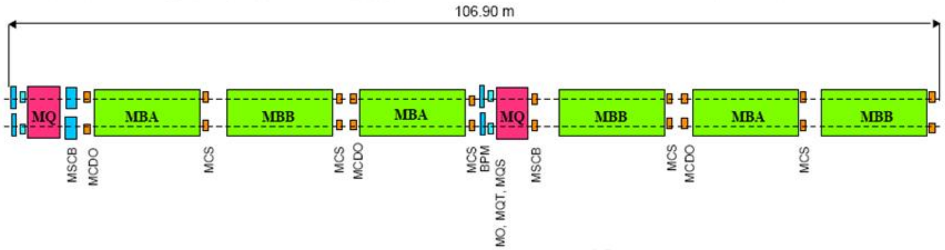
- The dependence of beta function in FODO cell
  - Beta function (the envelope of beam trajectories) is proportional to the distance between the quadrupoles L – the maximum value is achieved in the focusing quadrupole
  - ψ_L is the phase advance
  - For a 90 degrees phase advance cell, one has $\psi_L=\pi/2$

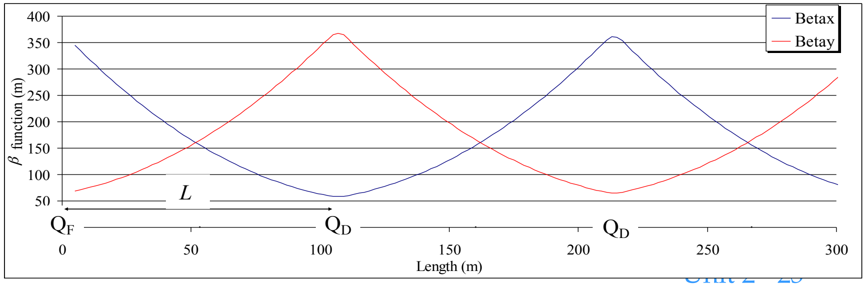
$$\beta_f = \frac{2+\sqrt{2-2\cos\psi_L}}{\sin\psi_L} L$$

$$\beta_f = \left(2+\sqrt{2}\right) L$$



  - LHC: L=53.5, β~180 m





GRADIENT REQUIREMENT FOR ARC QUADRUPOLES

- The requirements on the integrated gradient of the cell quadrupoles is

$$Gl_q \approx \frac{B\rho}{L}\sqrt{2-2\cos\psi_L}$$

- Two relevant dependences:
  - Proportional to the energy of the machine (the larger the energy, the larger the integrated gradient
  - Inverse proportional to the spacing of the quadrupoles L
- For a 90 degrees phase advance cell, one has $\psi_L=\pi/2$

$$Gl_q = \sqrt{2}\,\frac{B\rho}{L}$$

- LHC: $$G\ell_q \approx \frac{\sqrt{2}B\rho}{L} = \frac{1.4\times 8.3\times 2800}{53.5} = 620\ \mathrm{T}$$
- This is realized via ~3.15 m long quadrupoles giving ~200 T/m gradient

OPTIMIZING THE CELL LENGTH

- The cell length is the main parameter that can be optimized
  - ☺ The larger L, the less quads you need
  - ☺ The larger L, the lower integrated gradient
  - ☹ The larger L, the larger the beam
    - Usually, for longer machines (higher energy) you use longer cells lengths

- Case of 60° phase advance: linear dependence on *L*, different constants
  - It looks interesting: same beam size, 30% less focusing required

$$Gl_q = \frac{B\rho}{L}\sqrt{2-2\cos\psi_L} = \frac{B\rho}{L}$$

$$\beta_d = 2\sqrt{3}L \sim 3.4L$$

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APERTURE REQUIREMENT IN INTERACTION REGION

- In the experiments the beta functions are made as small as possible to get smaller beams and highest probability of collisions

$$x(s) = \sqrt{\frac{\varepsilon\beta(s)}{\gamma_r}}\cos(\psi(s))$$

  - In the LHC interaction point, beta functions go down to 0.3 m, from the 30/170 m of arcs (three orders of magnitude)
  - So beam size is reduced from 1.2 mm (injection, arc) to 0.3 mm (collision, arc) to 13 μm (collisions, interaction point)
- Since the experiments are empty of accelerator magnets, beta functions drift according to this unfortunate equation

$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$

  - So the smaller the beam in the experiment the larger the beam in the first magnets outside the experiment (IR magnets)
  - The first IR magnets are a set of quadrupoles to bend the beta functions back to the values of the arc (triplet)

APERTURE REQUIREMENT IN INTERACTION REGION

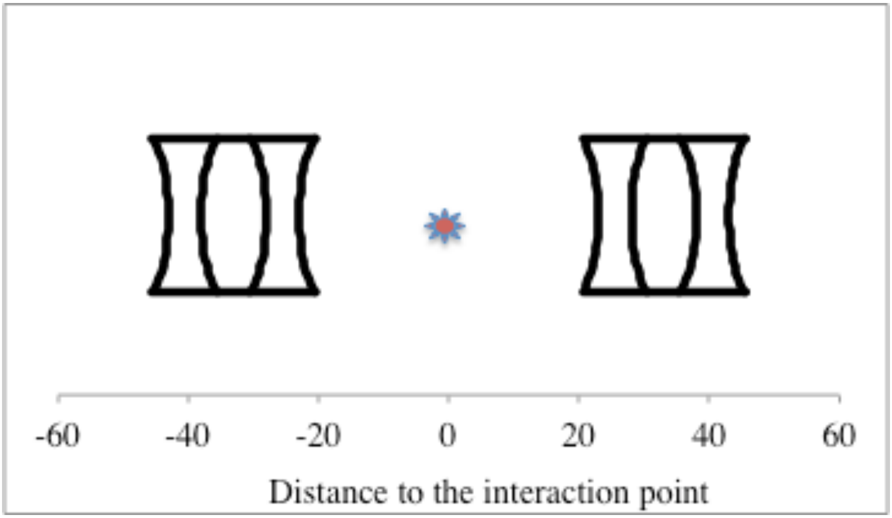
- For the LHC first magnets are at 23 m from the collision point, b is 0.3 m, and therefore the beta function is $23^2/0.3$=1500 m
- In reality the requirement on the interaction region magnets is larger since it takes some space to bent the beta functions back to the values in the arc – in LHC the beta function in the IR magnets reaches 8 km at β^*=30 cm

- High luminosity ? Large apertures (in HL-LHC is 150 mm)
- Dependence of aperture of IR magnets
  - Proportional to the inverse of the square root of the beta function in the interaction region
  - Proportional to the longitudinal size of the experiment *l**
  - And as usual, inverse proportional to the square root of the energy
  - Proportional to the square root of the emittance
  - Plus an offset ϕ_0 due to cold bore, tolerances, shielding …

$$\beta_{\max,IR} \propto \frac{l^{*2}}{\beta^*} \qquad \phi_{IR} = \phi_0 + k\sqrt{\frac{\varepsilon l^{*2}}{\beta^* \gamma_r}}$$

GRADIENT REQUIREMENT IN INTERACTION REGION

- The gradient requirement scales as
  - Proportional to the beam energy
  - Inverse proportional to the longitudinal size of the experiment



$$G_{IR} l_{qIR} \propto \frac{B\rho}{l^*}$$

  - At first order is as an optical system with focusing length equal to the distance of the quadrupoles to the interaction point
  - Note that in the gradient requirement there is no dependency on β*

CONTENTS

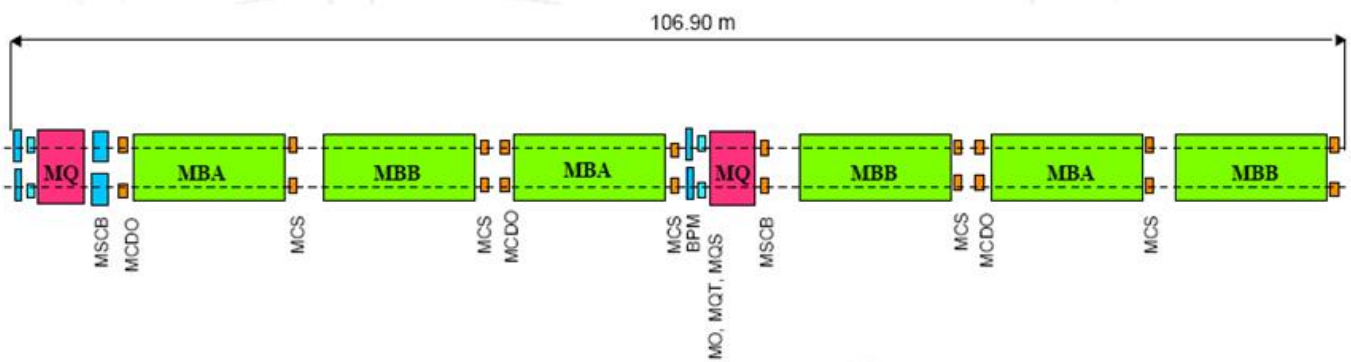
FIRST EXAMPLE: LHC

- The Large Hadron Collider, E=7000 GeV
  - Nb-Ti magnets, dipole field B=8.3 T, 14.3 m long, 1232 dipole
    - ➔Total (magnetic) length of dipoles 1232×14.3=17.6 km

$$L_d = 2\pi\rho = 2\pi \frac{E}{0.3B}$$

  - Cell semilength L=53.5 m, three dipoles per semicell
    - ➔Beta function in the focusing quad β_f=180 m
    - ➔Filling factor 14.3×3/53.5=80%

$$\beta_f = (2+\sqrt{2})L \sim 3.4L$$



- 27.6 km of tunnel, 20.5 km in the arcs, 7.2 km in the insertions

  - Emittance ε=3.75×10^{-6}m rad
  - Injection energy 450 GeV, γ=480
    - ➔Beam size σ=0.0012 m (at injection)
    - ➔Coil aperture of 0.056 m (coil-to-coil)

$$\sigma = \sqrt{\frac{\varepsilon_n \beta_f}{\gamma}}$$

FIRST EXAMPLE: LHC

- LHC arc quadrupoles
  - 56 mm aperture, Nb-Ti, gradient of 200 T/m
    - Corresponds to 200×0.028=5.6 T field on the bore
  - LHC cell phase advance of 90 degrees

$$f = \frac{L}{\sqrt{2}} \qquad \frac{G\ell_q}{B\rho} = \frac{1}{f} \qquad G\ell_q = \sqrt{2}\,\frac{B\rho}{L} = 1.41\,\frac{8.33\times 2808}{53.5} = 620$$

  - Integrated gradient of 620 T required, with 3.15-m-long quadrupoles
    - Therefore, over a cell whose length is $2L$ = 107 m we have
    - Six 14.3-m-long dipoles → total 86 m (80%)
    - Two 3.15-m-long quadrupoles → total 6.3 m (6%)
    - Plus interconnections and correctors
  - Note that all this equations are relying on several approximations, and are intended to give the main picture, not the details

SECOND EXAMPLE: HE-LHC

- A higher energy collider in the LHC tunnel (HE-LHC)
  - Assume a 16 T dipole available
  - Which energy ? First easy answer, assume same filling factor as LHC, we just scale the energy with the field
    - E=7.0/8.33*16 = 13.45 TeV
  - If we assume the same injection energy, and the same cell semilength, we obtain the same aperture of 56 mm
  - What about the quadrupoles ? We need

$$G\ell_q = \sqrt{2}\frac{B\rho}{L}$$

  - Now *B* nearly doubles, so integrated gradient becomes 1270 T (it was 660 T for the LHC)
  - Since Nb_3Sn cannot do more than 300 T/m in 56 mm aperture (we will see this much later), we will have 4.2 m long quadrupoles
    - We lose 2.5% in the filling factor (need 2.5 meters more for two quadrupole in 100 m cell) → the energy shall be not 13.45, but 13.11 TeV

SECOND EXAMPLE: HE-LHC

- A higher energy collider in the LHC tunnel (HE-LHC)
  - What about if we want to save on the cost of the quadrupoles and make them in Nb-Ti ?
  - 1270 T of integrated gradient is done with 5.7 m long quadrupoles giving 220 T/m
  - Now with respect to the LHC we lose 2.6 m per quadrupole, 5.2 m per cell, i.e. 5% in filling factor → energy will be 12.77 TeV instead of 13.44 TeV
    - … unless I can increase the dipole field by 5%, i.e. going from 16 T to 16.8 T – but this looks extremely difficult with Nb_3Sn …

SECOND EXAMPLE: HE-LHC

- A higher energy collider in the LHC tunnel (HE-LHC)
  - I could also work on the cell length – let us have a cell with L=105, then I just need 620 T of integrated gradient

$$G\ell_q = \sqrt{2}\frac{B\rho}{L}$$

  - So I can use cheaper Nb-Ti with 200 T/m – I could even reuse LHC quadrupoles
  - But the peak beta function will double to 320 m

$$\beta_f = (2+\sqrt{2})L \sim 3.4L$$

  - And I could probably need more aperture – not the double, since it depends with a square root – but to be verified

$$\sigma = \sqrt{\frac{\varepsilon_n \beta_f}{\gamma}}$$

  - … and the optimization goes on…

THIRD EXAMPLE: FCC

- A 100 km, 100 TeV collider with 16 T magnets (FCC)
  - E=50,000 GeV
  - Nb-Ti magnets, dipole field B=16 T
    - Curvature radius 10.4 km
    - Total length of dipoles L_d=65.4 km – looks reasonable for 100 km tunnel

$$L_d = 2\pi\rho = 2\pi \frac{E}{0.3B}$$

  - Cell semilength L=100 m
    - Better to increase the LHC value otherwise quadrupole needs will become huge
    - Beta function in the focusing quad β_f=340 m

$$\beta_f = (2+\sqrt{2})L \sim 3.4L$$

  - Emittance ε=3.75×10^{-6}m rad
  - Injection energy 1 TeV GeV
    - Beta has doubled, but I also double gamma so the aperture is the same as in the LHC

$$\sigma = \sqrt{\frac{\varepsilon_n \beta_f}{\gamma}}$$

THIRD EXAMPLE: FCC

- FCC arc quadrupoles

$$G\ell_q = \sqrt{2}\,\frac{B\rho}{L}$$

  - We need 2350 T of integrated gradient
  - Even with Nb_3Sn quadrupoles, we have 300 T/m and 7.8 m long
  - Since we increased the cell length, we have only one quadrupole per 100 m, and therefore quadrupole uses 7.8% of the cell length

- Summarizing
  - 65.4 km of 4573 dipoles (14.3 m long)
  - 200 m long cell, 1 quadrupole every 6 dipoles: total 762 quadrupoles, occupying 6.0 km
  - Interconnection and correctors: 1 m per magnet → 6 km more
  - Total length of the arcs: 77 km, the rest for insertions (topic of next unit)

FOURTH EXAMPLE: HL-LHC

- LHC has triplet quadrupoles with 70 mm aperture (in Nb-Ti, with ~8 T peak field
  - Integrated gradient of the triplet: 200 T/m over 24 m total (4800 T)

$$\phi_{IR} = \phi_0 + k\sqrt{\frac{\varepsilon l^{*2}}{\beta^* \gamma_r}} \qquad G_{IR} l_{qIR} \propto \frac{B\rho}{l^*}$$

- HL-LHC has 150 mm aperture quadrupoles (in Nb_3Sn, with ~11.5 T peak field)
  - Doubling the aperture → four times larger beta function in the triplet, four times smaller beta*
  - Integrated gradient: 132 T/m over 32 m total (4100 T) – energy and experiment size stays the same, but the baricentre of triplet shifted away from the experiment

CONCLUSIONS

- Equation for a synchrotron

$$p = eB\rho$$

  - And for a ultrarelativistic proton/electron one has

$$E[GeV] = 0.3 \times B[T] \times \rho[m]$$

approximation

- More energy?
  - Either brute force (longer collider)
  - Or technological development (higher magnetic field)

- Speed world record is not of LHC, but of LEP

$$E = m\gamma_r c^2$$

- Do not use relativistic mass

$$\gamma_r \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

  - Fast particles do not get fatter
  - If you travel by plane you do not weight more

CONCLUSIONS

- Equation for the transverse particle motion
  - Like an pendulum, but frequency and amplitude vary along the accelerator
  - Alternating sign quadrupoles guarantee stability of oscillations
- Aperture requirement is given by
  - More energy requires less aperture
  - More beta requires more aperture

$$x(s) = \sqrt{\frac{\varepsilon\beta(s)}{\gamma_r}}\cos(\psi(s))$$

- Essential parameter: spacing between quadrupoles *L*
  - L=53.5 m in the LHC (cell semilength)
  - This gives the amplitude of oscillations and the requirements for magnet aperture
  - The function beta is proportional to quadrupole spacing
  - For a 90 degrees cell

$$\beta_f = (2+\sqrt{2})L \sim 3.4L$$

approximation

CONCLUSIONS

- The integrated gradient of the quadrupole is inverse proportional to the quadrupole spacing L

$$G\ell_q = \sqrt{2}\,\frac{B\rho}{L}$$

approximation

- The lattice design implies a balance between
  - Large spacing L: less quadrupoles in number and in strength, more space for dipoles but larger aperture magnets
  - Small spacing L: more quadrupoles in number and in strength, less space for dipoles, but smaller aperture

REFERENCES

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  - P. Schmuser, et al, Ch. 9.
  - F. Asner, Ch. 8.
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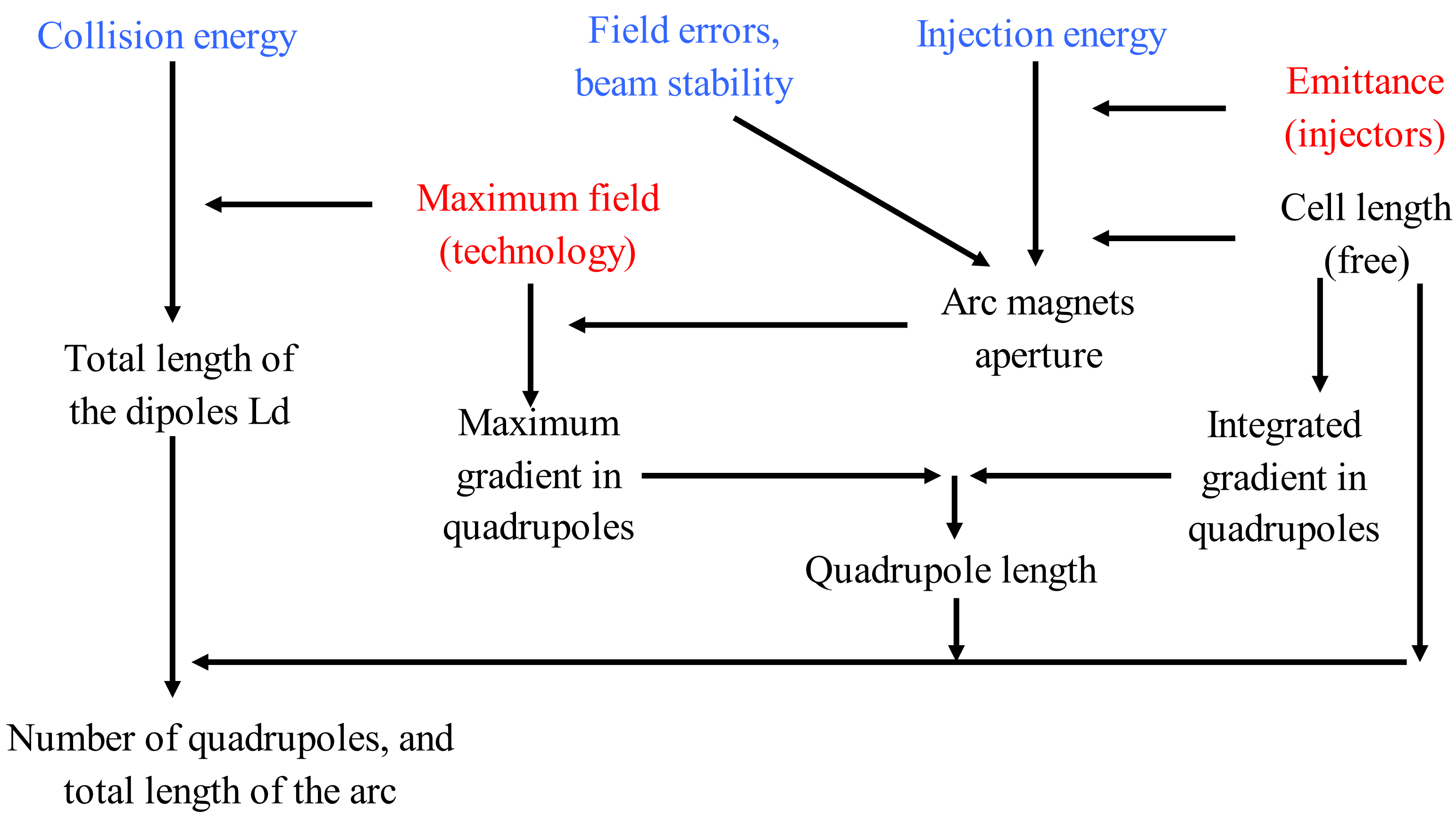
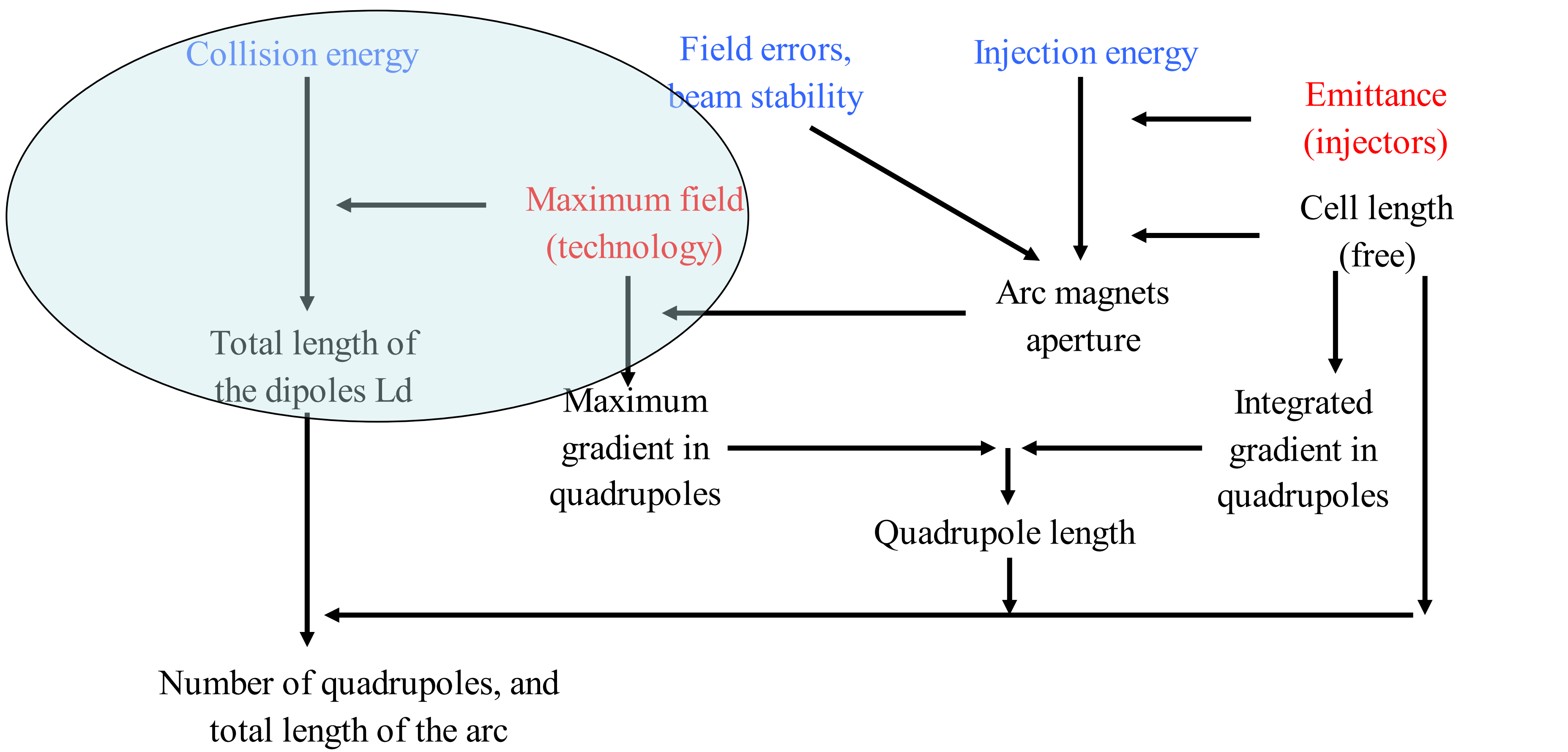
- www.wikipedia.org for most of the pictures

FLOWCHART FOR MAGNET PARAMETERS



FLOWCHART FOR MAGNET PARAMETERS



FLOWCHART FOR MAGNET PARAMETERS

- Input 1. Collision energy E_c
  - Gives a relation between the dipole magnetic field B and the total length of the dipoles L_d

$$E[GeV] = 0.3 \times B[T] \times \rho[m]$$

- Technology constraint 1. Dipole magnetic field B
  - Does not depend on magnet aperture
  - B_t <2 T for iron magnets
  - B_t <13 T for Nb-Ti superconducting magnets (10 T in practice)
  - B_t <25 T for Nb_3Sn superconducting magnets (16 T in practice)

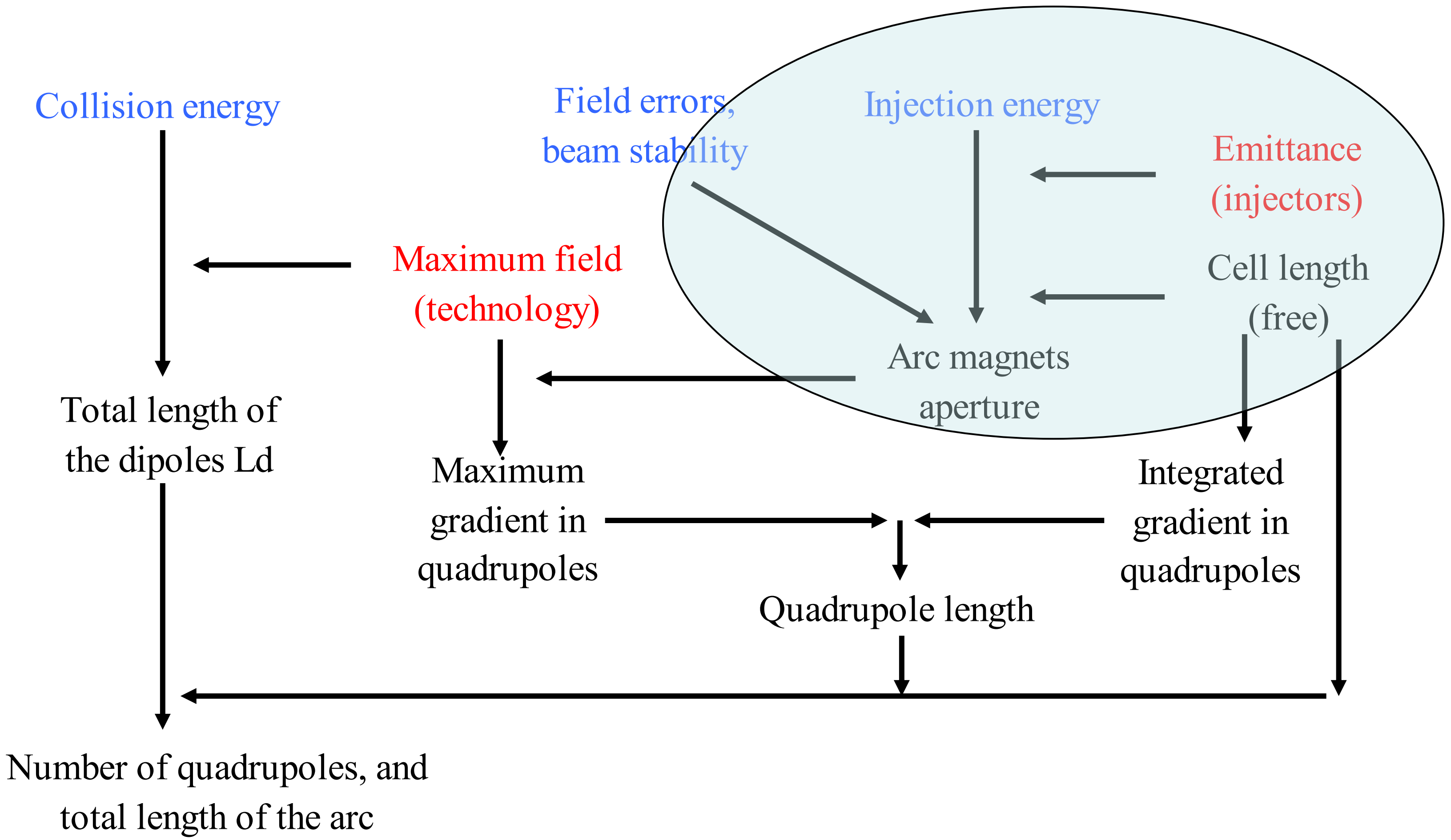
$$B < B_t$$

- **Output 1. Length of the dipole part**

$$L_d = 2\pi\rho = 2\pi \frac{E}{0.3B}$$

Length in m, B in T, energy in GeV

FLOWCHART FOR MAGNET PARAMETERS

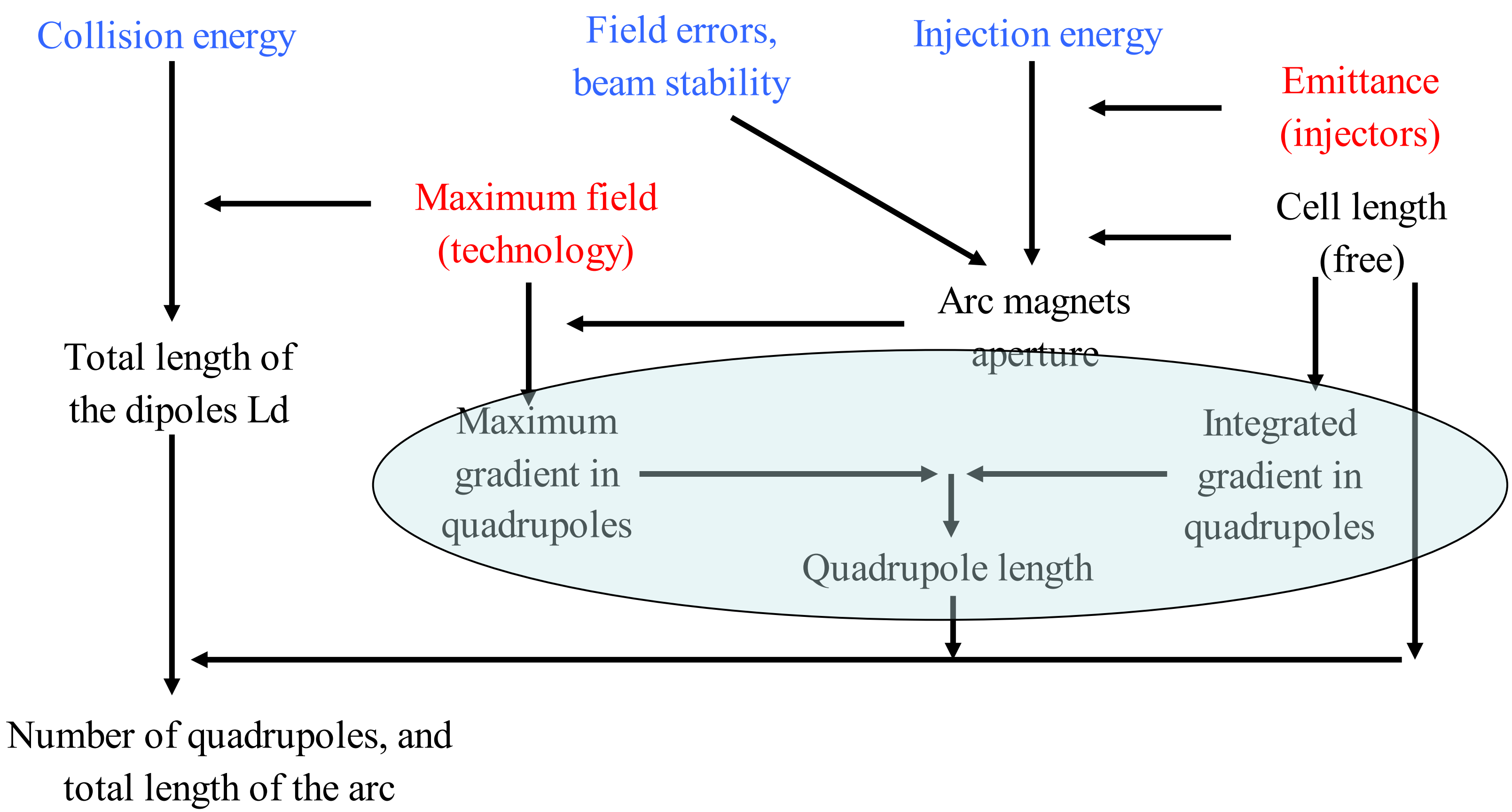


FLOWCHART FOR MAGNET PARAMETERS

- Input 2. Injection energy E_i
  - Determines the relativistic factor, that affect the beam size
- Constraint 2. Normalized beam emittance ε
  - Determined by the beam properties of the injectors
- Semi-cell length L
  - This is a free parameter that can be used to optimize
  - Determines the beta functions $\beta_f = (2+\sqrt{2})L \sim 3.4L$
- **Output 2. Aperture of the arc magnets** (also determined by field errors and beam stability)
  - Size of the beam at injection $\sigma = \sqrt{\dfrac{\varepsilon\beta_f}{\gamma_{r,i}}}$
  - Magnet aperture (diameter)

$$\phi_a = a + b\sigma = a + b\sqrt{\frac{\varepsilon\beta_f}{\gamma_{r,i}}} = a + b\sqrt{\frac{\varepsilon\left(2+\sqrt{2}\right)L}{\gamma_{r,i}}}$$

FLOWCHART FOR MAGNET PARAMETERS



FLOWCHART FOR MAGNET PARAMETERS

- Technology constraint 1. Quadrupole magnetic field vs aperture

$$\frac{G\phi_a}{2} < B_t$$

- **Output 3. Gradient of the quadrupoles**

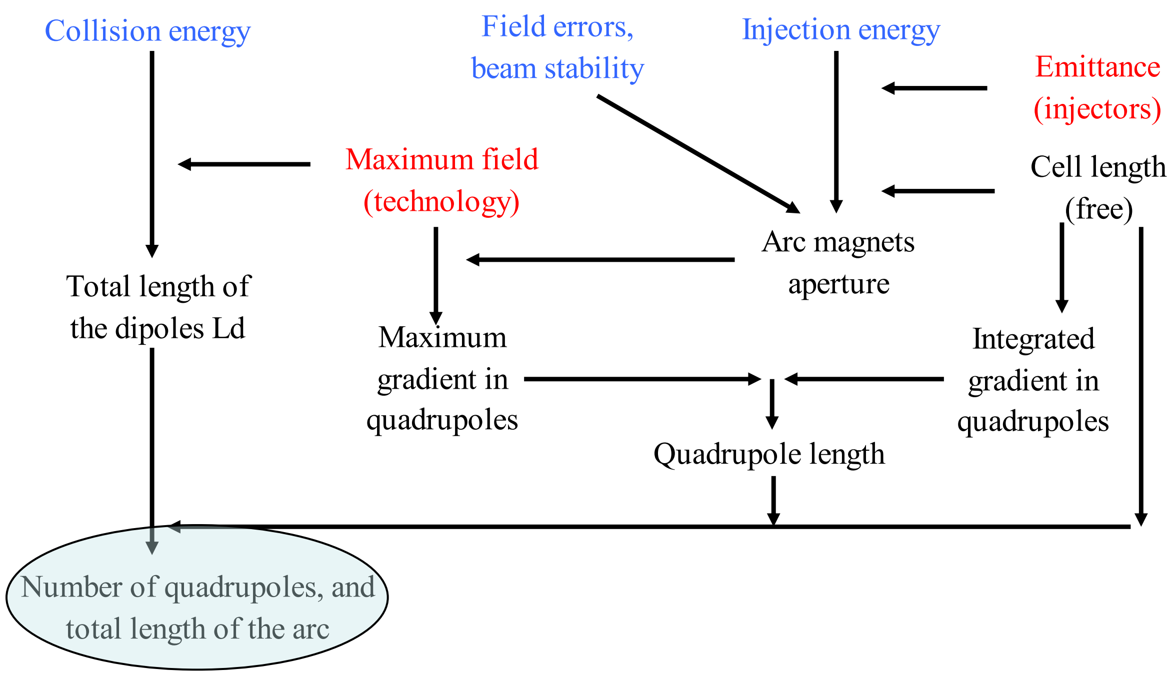
$$G = G(B_t, \phi_a) < \frac{2B_t}{\phi_a}$$

- Semi-cell length *L*
  - Also determines the focusing, i.e. the integrated gradient

$$G\ell_q = \frac{\sqrt{2}B\rho}{L} = \frac{\sqrt{2}BL_d}{2\pi L} = \frac{\sqrt{2}E}{0.3L}$$

- **Output 4. Length of the quadrupoles**

$$\ell_q = \frac{\sqrt{2}B\rho}{GL} = \frac{\sqrt{2}BL_d}{2\pi GL} = \frac{\sqrt{2}E}{0.3GL}$$

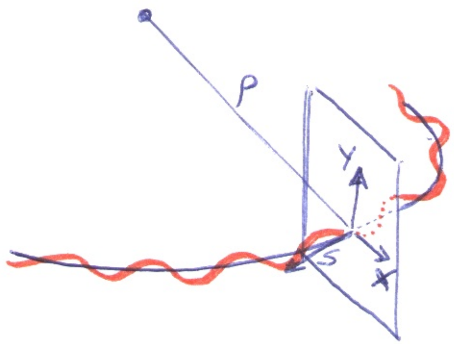


- **Output 5. Number of semi-cells and arc length**
  - Number of semi-cells = number of quadrupoles = n_q
  - Length of the arc L_a

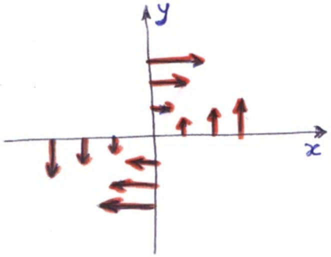
$$n_q = \frac{L_d}{L - \ell_q}$$

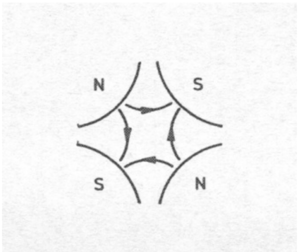
$$L_a = n_q L$$

- The force necessary to stabilize linear motion is provided by the quadrupoles
  - Quadrupoles provide a field which is proportional to the transverse deviation from the orbit, acting like a spring



$$\begin{cases} B_y = Gx \\ B_x = Gy \end{cases}$$





  - If we define the gradient as $$K_1(s) \equiv \frac{G(s)}{B\rho}$$
  - the motion equation will read as $$\begin{cases} \dfrac{d^2x}{ds^2} + K_1(s)x = 0 \\ \dfrac{d^2y}{ds^2} - K_1(s)y = 0 \end{cases}$$

SOME THEORETICAL INSIGHT

- In this way the motion equation in the transverse space is similar to a harmonic oscillator

  where the force depends on time …
  $$\frac{d^2x}{ds^2} + K(s)x = 0$$

  - Solution: a oscillator whose amplitude and frequency are modulated

  $$x(s) = A(s)\cos(\psi(s))$$

  - We write the amplitude as a *s*-dependent part (beta function) plus an invariant along the ring

  $$x(s) = \sqrt{a\beta(s)}\cos(\psi(s))$$

  - The invariant is inverse proportional to the energy – so one can define a new invariant, also independent of the energy

  $$x(s) = \sqrt{\frac{\varepsilon\beta(s)}{\gamma_r}}\cos(\psi(s))$$

SOME THEORETICAL INSIGHT

- It may appear strange to use a square root

$$x(s) = \sqrt{\frac{\varepsilon\beta(s)}{\gamma_r}}\cos(\psi(s))$$

- The big advantage of using a square root is that there is a relation between the beta function and the phase advance
  - Large beta fuction, small phase advance
  - Small beta function, large phase advance

$$\psi'(s) = \frac{1}{\beta(s)}$$

- The tune is the phase advance over the ring

$$Q_x = \psi_x(l_T)$$

$$Q_y = \psi_y(l_T)$$