Basics of superconductivity and applications

(Part 1)

M. Marchevsky,

Lawrence Berkeley National Laboratory

Credits and thanks

The material of this course is largely based upon:

- S. Prestemon and S. Gourlay "Basics of superconductivity" (Unit 3 of USPAS 2018)
- A. Gurevich, "General aspects of superconductivity" (2007 SRF Workshop, Beijing, China, Oct. 11, 2007)

- V. V. Scmidt, The Physics of Superconductors (Springer, 1997)
- M. Tinkham, "Introduction to Superconductivity" (1980)

- An excellent video course from Univ. of Cambridge:

  Lectures on Superconductivity - Introduction (cam.ac.uk)

  https://www.ascg.msm.cam.ac.uk/lectures/introduction.html

Outline

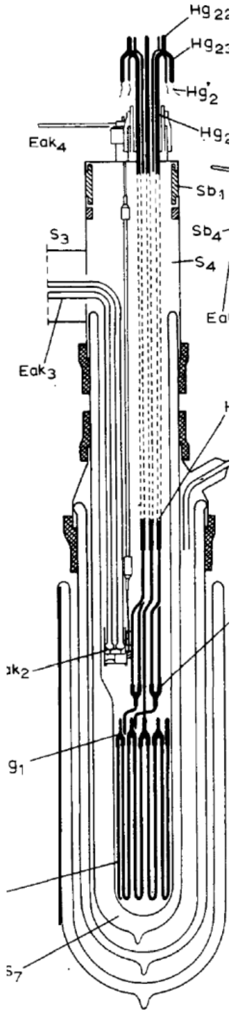
Part 1

- Discovery of superconductivity
- Basic phenomenology of superconductors and characteristic lengths
- Microscopic theory of superconductivity

Part 2

- Type I and Type II superconductors
- Phenomenology of type-II superconductors: flux lines, pinning, flux flow, critical state
- Practical scaling for technological superconductors
- Summary on applications

Helium liquefaction opened up a new era in condensed matter physics



Helium liquefier built in Leiden in 1908 produced ~0.28 liters/hour

(summary *by S. Presetmon)*

- Faraday (~ 1820's) demonstrates an ability to liquify gases by first cooling with a bath of ether and dry ice, followed by pressurization. He was unable to liquify oxygen, hydrogen, nitrogen, carbon monoxide, methane, and nitric oxide
- The noble gases, helium, argon, neon, krypton and xenon had not yet been discovered (many of these are critical cryogenic fluids today)
- In 1848 Lord Kelvin determined the existence of absolute zero: 0 K = -273 C (= - 459 F)
- In 1877 Louis Caillettet (France) and Raoul-Pierre Pictet (Switzerland) succeed in liquifying **air**
- In 1883 Von Wroblewski (Cracow) succeeds in liquifying **oxygen**
- In 1898 James Dewar succeeded in liquifying **hydrogen** (~20 K!); he then went on to freeze hydrogen (14 K).
- Helium remained elusive; it was first discovered in the spectrum of the sun. In 1908 H. Kamerlingh Onnes succeeded in liquifying Helium (4.2 K)

Resistivity of metals

Resistivity in a conductor stems from the scattering of electrons off thermally activated ions

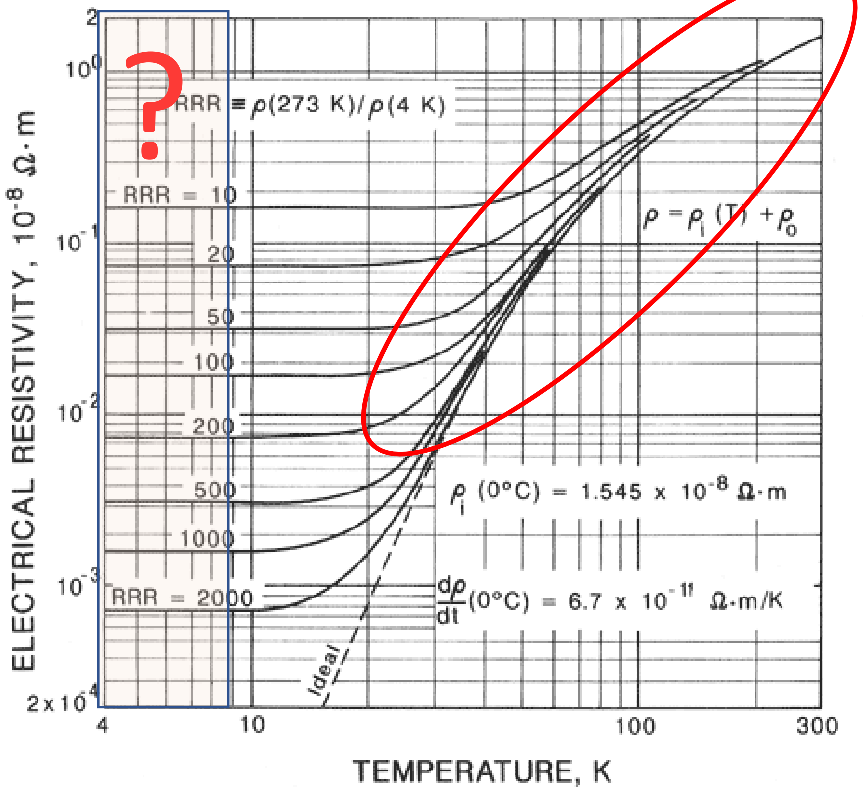
Resistance, therefore, **goes down as the temperature decreases** (from the high-temperature regime in which $\rho \propto T$ to a low-temperature regime in which $\rho \propto T^5$**)**

The decrease in resistance in normal metals reaches a minimum due to the presence of irregularities and impurities in the lattice, hence the concept of RRR (Residual Resistivity Ratio). RRR is a rough measure of electron scatterers (dislocations and impurities) in a metal.

$$\rho(T) = A\left(\frac{T}{\Theta_R}\right)^n \int_0^{\Theta_R/T} \frac{t^n}{(e^t-1)(1-e^{-t})}dt$$

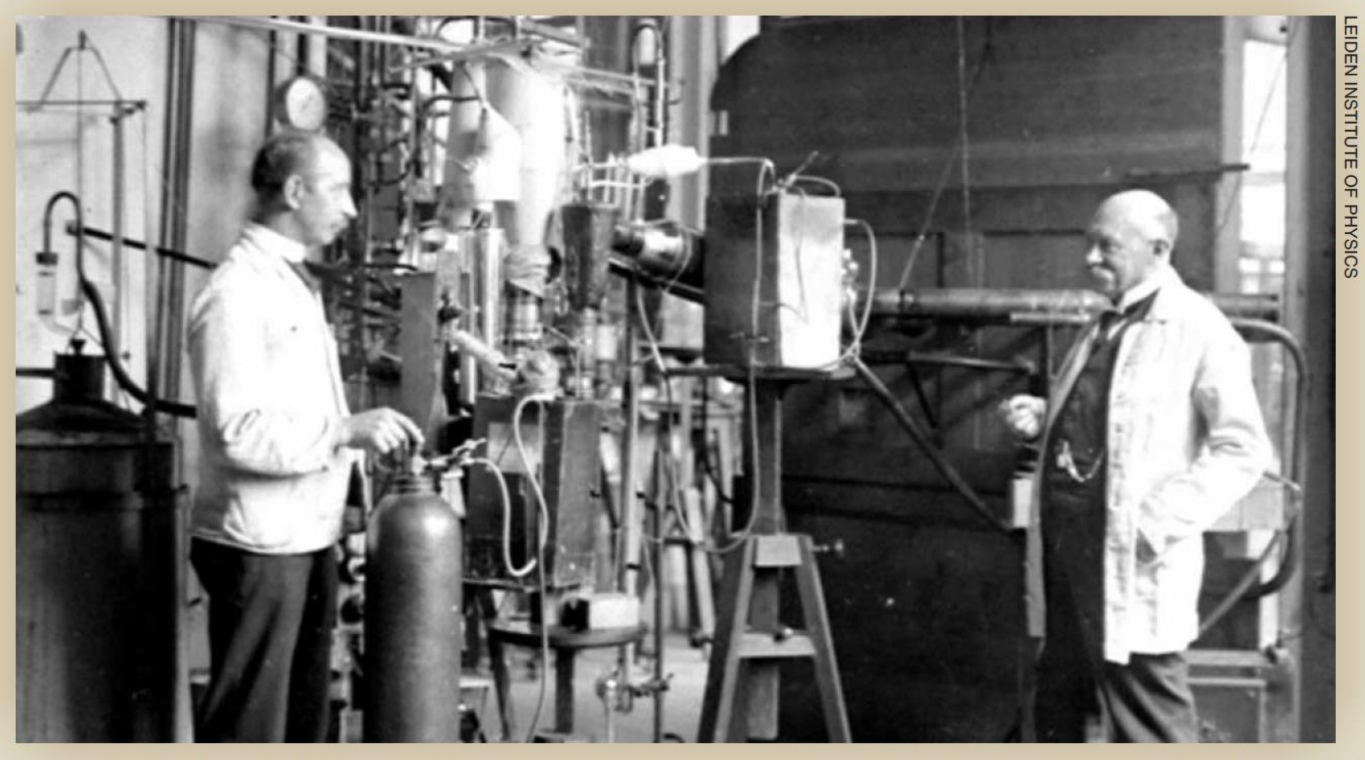
(Bloch – Gruneisen, 1930)

Copper resistivity



In 1911 several theories (by Debye, Einstein, Matthieson, etc.) co-existed describing the resistivity behavior of metal close to absolute zero. The successful liquefaction of Helium allowed verifying those theories for the first time.

Discovery of superconductivity

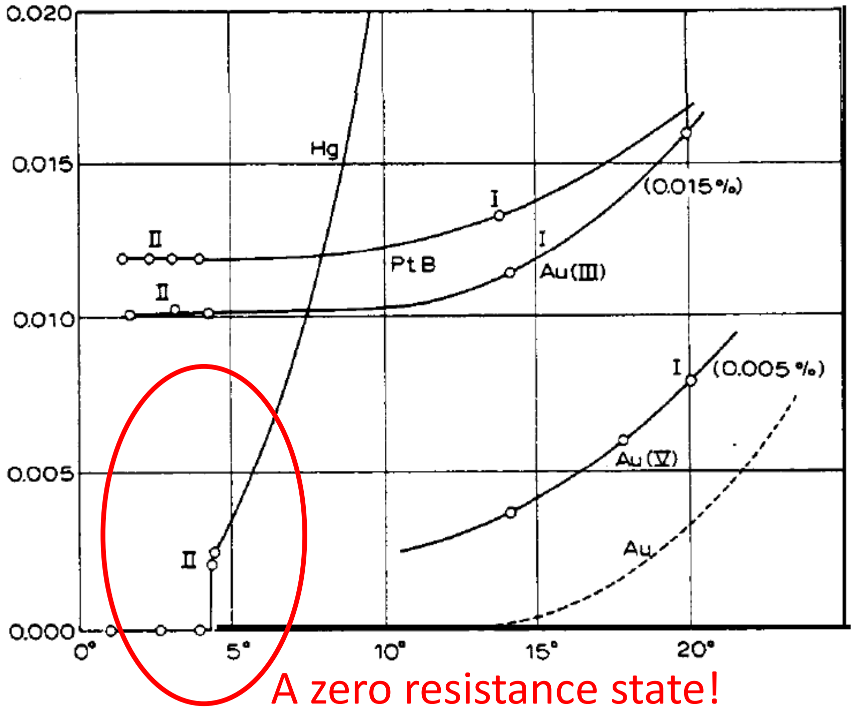


LEIDEN INSTITUTE OF PHYSICS

Figure 1. Heike Kamerlingh Onnes (right) and Gerrit Flim, his chief technician, at the helium liquefier in Kamerlingh Onnes's Leiden laboratory, circa 1911.

Figure 2. A terse entry for 8 April 1911 in Heike Kamerlingh Onnes's notebook 56 records the first observation of superconductivity. The highlighted Dutch sentence *Kwik nagenoeg nul* means "Mercury['s resistance] practically zero [at 3 K]." The very next sentence, *Herhaald met goud,* means "repeated with gold." (Courtesy of the Boerhaave Museum.)

The resistivity of a superconductor is truly ZERO, accurate to 10^{-26} Ω m. (Purest copper at 4.2 K is ~10^{-11} Ω m)



H. K. Onnes, Commun. Phys. Lab. 12, 120, (1911)

https://doi.org/10.1016/j.physc.2012.02.046

https://physicstoday.scitation.org/doi/10.1063/1.3490499

Dirk van Delft, Peter Kes (2010)

Superconductor in a magnetic field

' …100,000 Gauss could then be obtained by a coil of say 30 centimeters in diameter and the cooling with helium would require a plant which could be realized in Leiden with a relatively modest support…'
Third International Congress of Refrigeration, Chicago Sept 1913

Lead wire wound coil

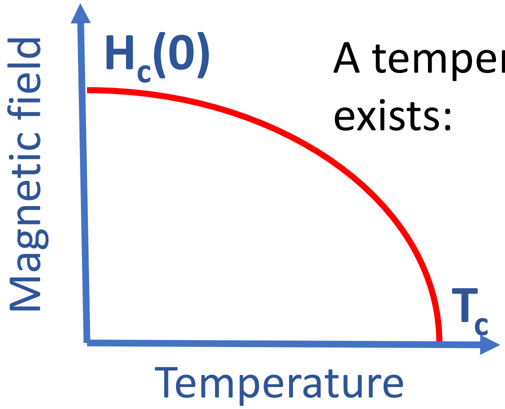
Leiden, 1912

H. Kamerlingh Onnes, *"The sudden disappearance of the ordinary resistance of tin, and the supraconductive state of lead"*, Commun. 133d (1913)

But…

"Using sections of wire soldered together to form a total length of 1.75 meters, a coil consisting of some 300 windings, each with a cross-section of 1/70 mm^2, and insulated from one another with silk, was wound around a glass core. Whereas in a straight tin wire the threshold current was 8 A, in the case of the coil, it was just 1 A. Unfortunately, the disastrous effect of a magnetic field on superconductivity was rapidly revealed. Superconductivity disappeared when field reached 60 mT."

H. Kamerlingh Onnes, *KNAW Proceedings* 16 II, (1914), 987. *Comm.* 139f.



A temperature-dependent "critical field" $\mathbf{H_c(T)}$ exists:

$$H_c(T) = H_c(0)[1 - (T/T_c)^2]$$

- empirical dependence

Magnetic field destroys superconductivity!

Maxwell's equations

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon} \quad \text{Gauss' law}$$

The flux of the electric field out of an arbitrary closed surface is proportional to the electric charge enclosed by the surface

$$\nabla \cdot \vec{B} = 0$$

Magnetic field lines "curl" around currents, magnetic monopoles do not exist

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{Faraday's law}$$

dB/dt induces voltage (and so in conductors that generates electric current)

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{Ampere's law (corrected by Maxwell)}$$

Moving charges generate magnetic field

$$\mu_0 = 4\pi \times 10^{-7} \quad N/A^2 \quad \text{Permeability of free space}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \quad C^2/(Nm^2) \quad \text{Permittivity of free space}$$

Differential operators

a. The ***gradient*** of a scalar-valued function $f(x, y, z)$ is the vector field

$$\text{grad } f = \nabla f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{\mathbf{k}}$$

Note that the input, f, for the gradient is a scalar-valued function, while the output, ∇f, is a vector-valued function.

b. The ***divergence*** of a vector field $\mathbf{F}(x, y, z)$ is the scalar-valued function

$$\text{div } \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Note that the input, $\mathbf{F}$, for the divergence is a vector-valued function, while the output, $\nabla \cdot \mathbf{F}$, is a scalar-valued function.

c. The ***curl*** of a vector field $\mathbf{F}(x, y, z)$ is the vector field

$$\text{curl } \mathbf{F} = \nabla \times \mathbf{F} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\hat{\imath} - \left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right)\hat{\jmath} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\hat{\mathbf{k}}$$

Note that the input, $\mathbf{F}$, for the curl is a vector-valued function, and the output, $\nabla \times \mathbf{F}$, is a again a vector-valued function.

d. The Laplacian[2] of a scalar-valued function $f(x, y, z)$ is the scalar-valued function

$$\Delta f = \nabla^2 f = \nabla \cdot \nabla f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

The Laplacian of a vector field $\mathbf{F}(x, y, z)$ is the vector field

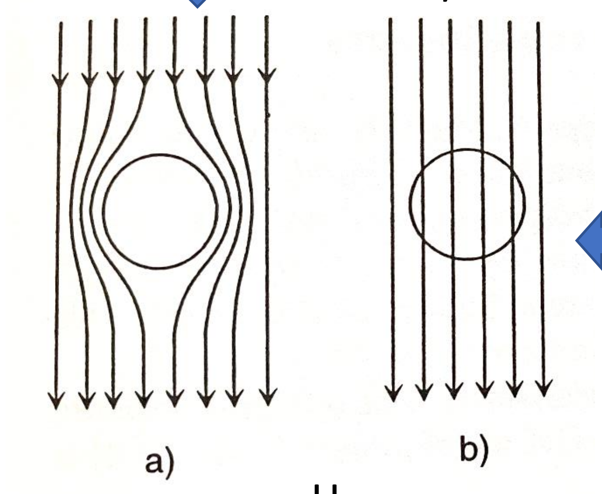
$$\Delta \mathbf{F} = \nabla^2 \mathbf{F} = \nabla \cdot \nabla \mathbf{F} = \frac{\partial^2 \mathbf{F}}{\partial x^2} + \frac{\partial^2 \mathbf{F}}{\partial y^2} + \frac{\partial^2 \mathbf{F}}{\partial z^2}$$

Note that the Laplacian maps either a scalar-valued function to a scalar-valued function, or a vector-valued function to a vector-valued function.

Superconductor vs ideal conductor: Meissner effect and intermediate state

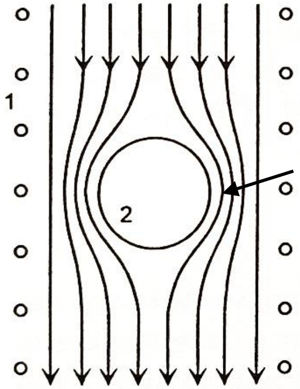
dB/dt !

dB/dt =0



If field is applied to the ordinary conductor, it would penetrate it slowly, with a relaxation time τ ~ L/R (->∞ for an ideal conductor, so the field will never penetrate)

But if "ideal conductivity" is "turned on" while the magnetic field is already present, field lines would just stay in the conductor.



Superconductor expels magnetic flux!

Fig. 1.5. Superconducting sphere in the homogeneous field of a solenoid; *1* – the winding of the solenoid, *2* – the superconducting sphere

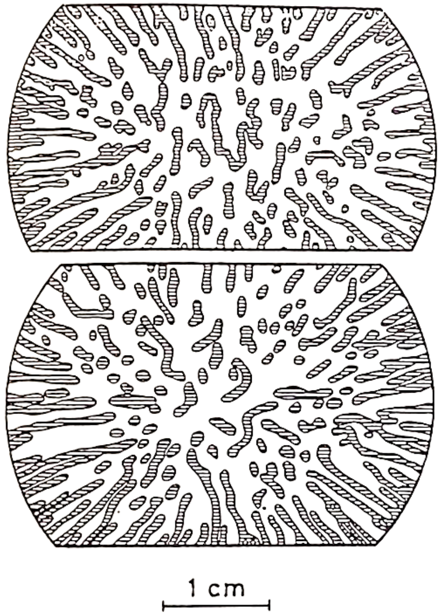
W. Meissner and R. Ochsenfeld (1933)

"Intermediate state"

Table 1.2. The demagnetizing factor n for various geometries

Sample geometry	n
Cylinder in parallel field	0
Cylinder in transverse field	1/2
Sphere	1/3
Thin plate in perpendicular field	1

$$H_m = \frac{H_0}{1-n}$$



Superconducting regions: H=0

Normal regions: H=H_c

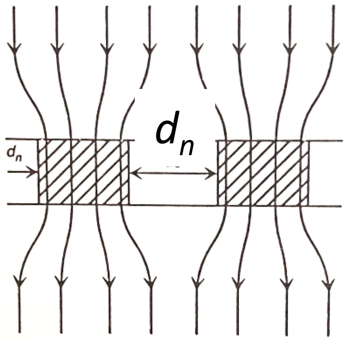


Fig. 1.7. Superconducting and normal regions in a tin sphere [15]. Shaded region: are superconducting

Normal region size d_n would adjust to provide a correct value of the field.

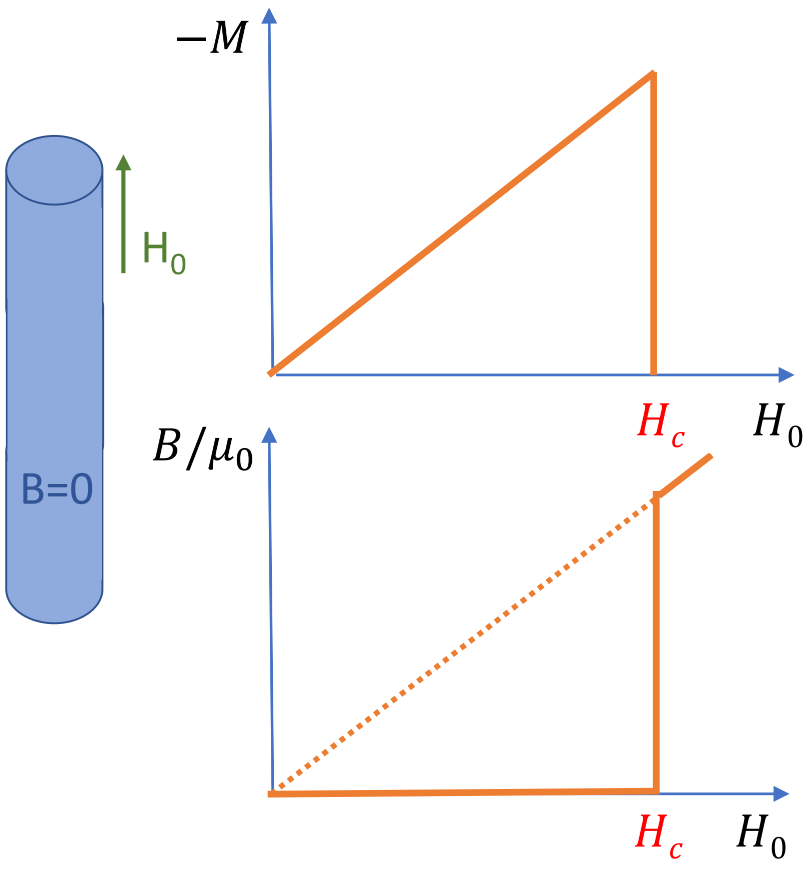
Meshkovskii, Shalnikov (1947)

Magnetization of a superconductor

The magnetic induction $\boldsymbol{B}$ and the magnetic field $\boldsymbol{H}_0$ in the material are related with each other as: $\boldsymbol{B} = \mu_0(\boldsymbol{H}_0 + \boldsymbol{M})$, where $\boldsymbol{M}$ is the magnetic moment per unit volume (magnetization).

$\boldsymbol{M} = \chi \boldsymbol{H}, B = \mu \boldsymbol{H} = \mu_0(1+\chi)\,\boldsymbol{H}$, χ - magnetic susceptibility

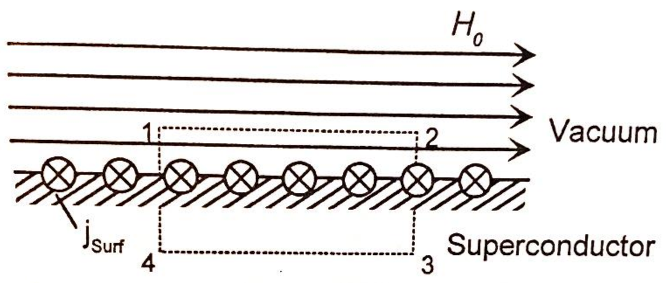
*SI unit of **B** is Tesla*
*SI unit of **H** is A/m*
One often used T, mT to define H, as it is a more practical unit.
In than case "true" $H = B/\mu_0$



- *Magnetic field outside of a superconductor is always tangential to its surface*

$\nabla \cdot \boldsymbol{B} = 0$ ⇒ Component of **B** normal to the surface must be equal on both sides of that surface. As inside the superconductor **B** = 0, so in B_n=0

- *Superconductor in an external field always carries an electric current near its surface*



$\nabla \times \boldsymbol{B} = \mu_0 \mathbf{j}$

$$\oint_{1\to2\to3\to4} \boldsymbol{B} d\boldsymbol{l} = \mu_0 I$$

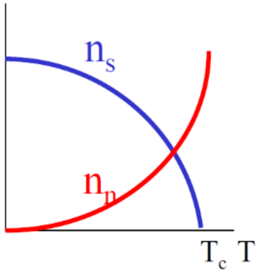
$$\mu_0 H_0 l_{12} = \mu_0 j_{surf} l_{12}$$

⇓

$$\boldsymbol{j}_{surf} = [\boldsymbol{n} \times \boldsymbol{H}_0]$$

- *Superconductor below H_c is an ideal diamagnetic* $(\chi = -1)$

London equations



Two-fluid model: assuming coexisting “fluids” of normal and superconducting electrons with densities $n_s(T) + n_n(T) = n$
Electric field $\boldsymbol{E}$ accelerates only the SC component, the normal component is short-circuited


Fritz London


Heinz London

Assuming ballistic flow for the superconducting electrons, one can write second Newton law for the SC component as:

$$m\frac{dv_s}{dt} = eE$$

As current $\boldsymbol{J}_s = n_s(T)e\boldsymbol{v}_s$, by substituting we obtain: $\frac{d\boldsymbol{J}_s}{dt} = \left(\frac{e^2 n_s(T)}{m}\right)\boldsymbol{E}$ (First London equation)

Now, using two Maxwell equations:

$\nabla\times\boldsymbol{E} = -\mu_0\frac{\partial H}{\partial t}$ and $\nabla\times\boldsymbol{H} = \boldsymbol{J}_s$ and the known identity: $a\cdot\nabla\times b = b\cdot\nabla\times a - \nabla\cdot[a\times b]$

$\Downarrow$

$\lambda^2\nabla\boldsymbol{H} - \boldsymbol{H} = 0$ (Second London equation)

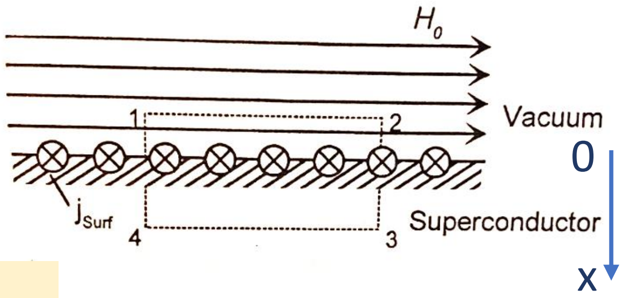
where $\lambda = \left(\frac{m}{e^2 n_s(T)\mu_0}\right)^{1/2}$ has a dimensionality of length and is called London penetration depth

$$\lambda(T) = \frac{\lambda(0)}{(1-(T/T_c))^4}$$

The empirical formula for the temperature dependence of λ

Magnetic penetration depth

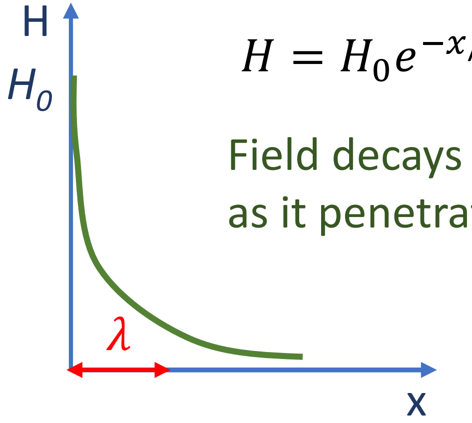
Re-writing the second London equation for the familiar problem of semi-space occupied by a superconductor with field applied parallel to the surface, we get:



$H(0) = H_0$

$H(\infty) = 0$ (Meissner effect)

$$\frac{d^2H}{dx^2} - \lambda^{-2}H = 0$$



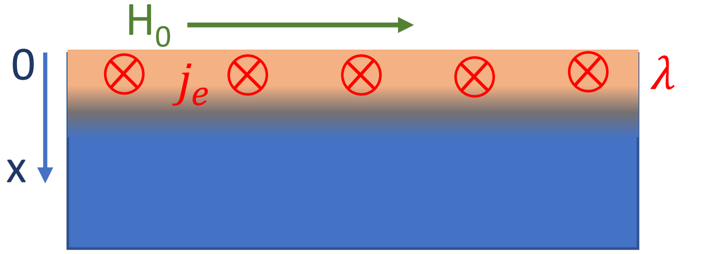
$$H = H_0 e^{-x/\lambda}$$

Field decays exponentially with distance as it penetrates the superconductor

λ is a characteristic depth of field penetration

Table 2.1. London penetration depths for some superconductors [2]

Element	Al	Cd	Hg	In	Nb	Pb	Sn	Tl	$YBa_2Cu_3O_7$
$\lambda(0)$ /Å	500	1300	380–450 (anisotropy)	640	470	390	510	920	1700



$j_e = dH/dx$ and then $j_e = \frac{H_0}{\lambda} e^{-x/\lambda}$

Supercurrent completely *screens* the applied field from the interior of the superconductor

Limiting value for the surface current density: $j_d = \frac{H_c(T)}{\lambda(T)} \cong j_o \left(1 - \frac{T^2}{T_c^2}\right)^{3/2}$

Thermodynamics of the superconducting transition

Free energy of the superconductor changes at the superconducting transition

$$F_n - F_s = \mu_0 \frac{H_c^2}{2}$$

"Condensation energy"

Free energy is lower in the superconducting state!

$F_n - F_s = \mu_0 \frac{H_c^2}{2} + \mu_0 \int_0^H \boldsymbol{M} d\boldsymbol{H}$ - Generalized equation when external magnetic filed is varied. Useful for calculating **M**!

Entropy: $S = -\frac{\partial F}{\partial T} \quad \Rightarrow \quad S_n - S_s = -H_c \frac{dH_c}{dT}$

We can then determine the amount of heat absorbed when a unit volume transitions from the superconducting to the normal state:

$$Q = T(S_n - S_s) = -TH_c \frac{dH_c}{dT}$$

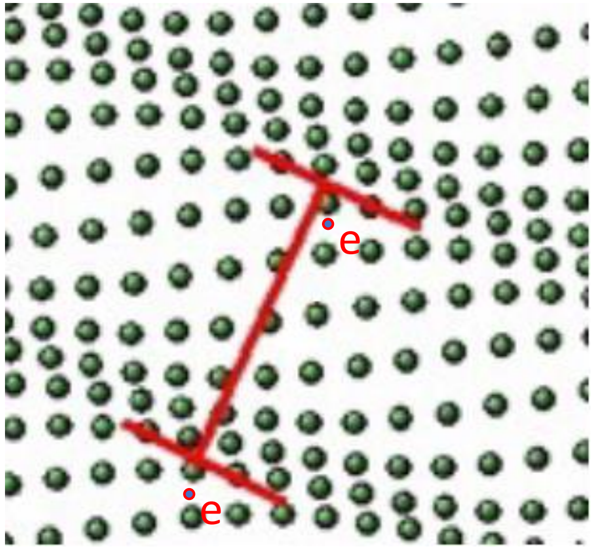
As $\frac{dH_c}{dT} < 0$ the heat balance is positive (meaning the superconductor is cooling when it transitions **in magnetic field** to the normal state). This is a hallmark of a first-order phase transition

Specific heat: $c = T\frac{dS}{dT} \quad \Rightarrow \quad \Delta c = c_s - c_n = T\left(H_c \frac{d^2 H_c}{dT^2} + \left(\frac{dH_c}{dT}\right)^2\right)$

Note that at critical temperature T_c we have $H_c = 0$, and so $S_n - S_s = 0$ and $\Delta c_{T_c} = T_c \left(\frac{dH_c}{dT}\right)^2$

Superconducting transition **at $\boldsymbol{T_c}$** is a second-order phase transition

Microscopic origins of superconductivity

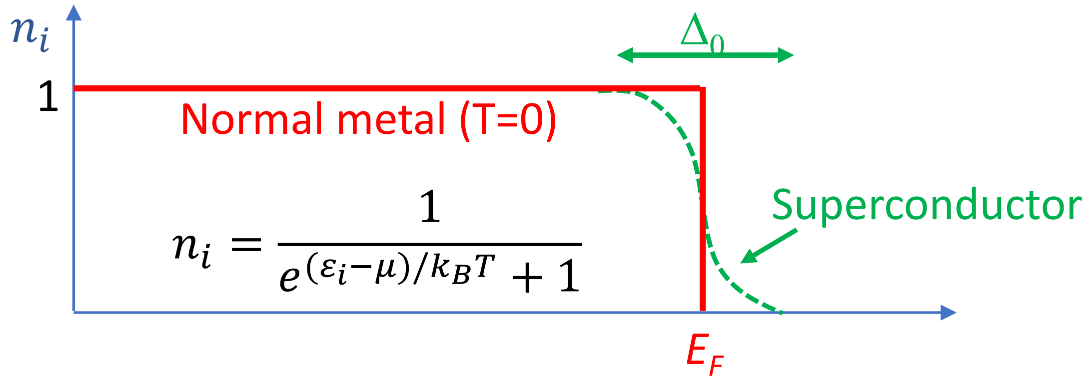


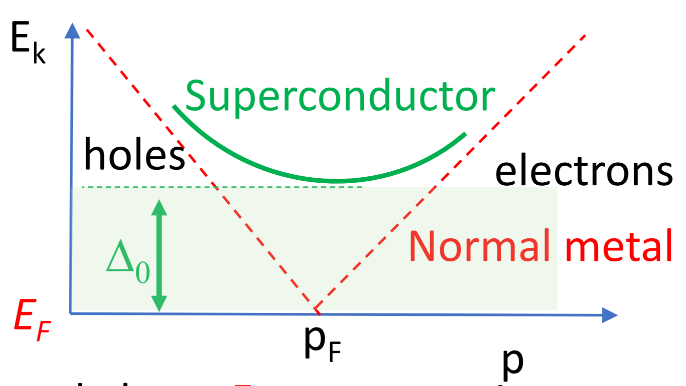
- The atomic lattice vibrates all the time; lattice vibrations are called "phonons"
- An electron moving in the vibrating lattice of ions interacts with it via Coulomb forces (electron-phonon interaction)
- A "trace" of the lattice distortion remains behind a moving electron.
- Another electron can take advantage of this distortion to move through the lattice easier (aka "flocking"). Through interaction with phonons, electrons can influence each other over long distances of many lattice constants.

In a superconductor, when the temperature descends below the critical temperature, electrons find it energetically preferable to form "Cooper pairs" (Cooper, 1953) of electrons with anti-parallel momenta and spins. Due to the long-range nature of the interaction, many Cooper pairs may coexist in the same volume of the superconductor. Since free energy is lower in the superconducting state, the interaction responsible for the pair forming is **attractive**.

Superconducting energy gap and coherence length

Electrons are ½ spin particles and hence are fermions. The average number of fermions in a single-particle state *i* is given by the **Fermi–Dirac distribution:**





***Δ_0* - superconducting energy gap**

(Typically only ~1 meV or *10^{-22} J* (compared to several eV of Fermi energy)

In normal metal, at T=0 all states up to E_F are occupied, and above E_F are empty. In a superconductor, even at T = 0, there is a characteristic broadening of the distribution near E_F due to the electron-phonon interaction.

As a result, the net energy of the electron ensemble decreases by: $\Delta E = F_n - F_s \simeq \frac{N_{E_F}}{2} \Delta_0^2 = \frac{\mu_0 H_c^2}{2}$

At the same time, *only electrons within ~$k_B T_c$ of the Fermi energy* can be expected to play a role in a phenomenon that sets in at T_c. These electrons have a momentum range $\Delta p \approx \frac{kT_c}{v_F}$. By analogy to the famous Heisenberg's uncertainty principle $\boldsymbol{\Delta p \Delta x \sim h}$, the characteristic dimensions of the superconducting wavefunction can be estimated as $\Delta \mathrm{x} \sim \frac{h}{\Delta p}$, where *h* is the Plank constant. This defines another characteristic length for a superconductor, called **superconducting coherence length:**

$\xi_0 = \alpha \frac{h v_F}{k T_c}$ (α is a constant ~1). (Pippard, 1953)

Ginzburg-Landau's (GL) theory of superconductivity

Complex superconducting order parameter: $\psi = \left(\frac{n_s}{2}\right)^2 e^{i\theta}$

amplitude *phase*

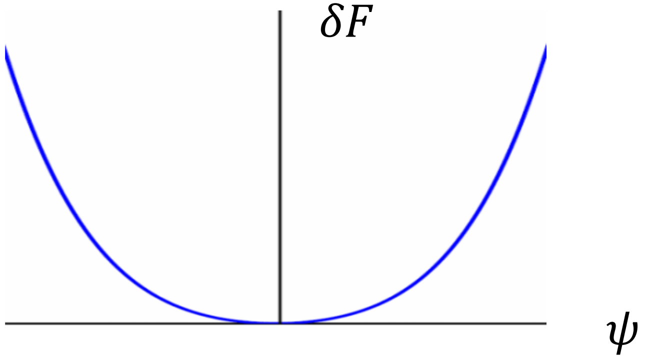
Generalization of the London equations to <u>nonlinear</u> problems

Near T_c the order parameter ψ is expected to be small, so free energy can be written as Taylor series

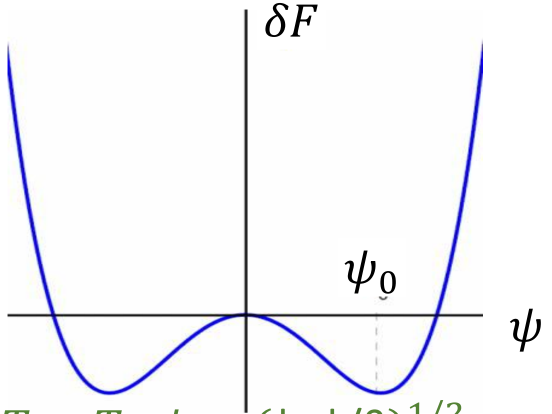
$$F = F_n + \int dV \left[\alpha(T)|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{\hbar^2}{2m^*}\left|\left(\nabla + \frac{2\pi i \vec{A}}{\phi_0}\right)\Psi\right|^2 + \frac{\mu_0 H^2}{2} \right]$$

nonlinear inhomogeneity magnetic

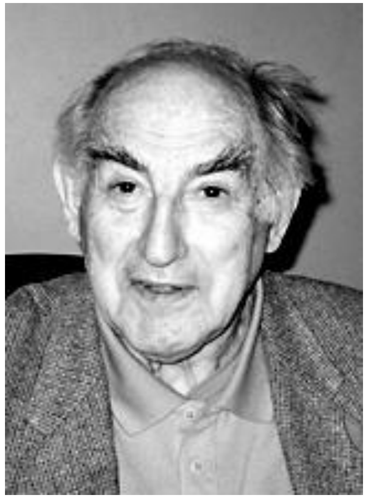
$\alpha(T) = \alpha_0(T - T_c)/T_c$ - changes sign at T_c



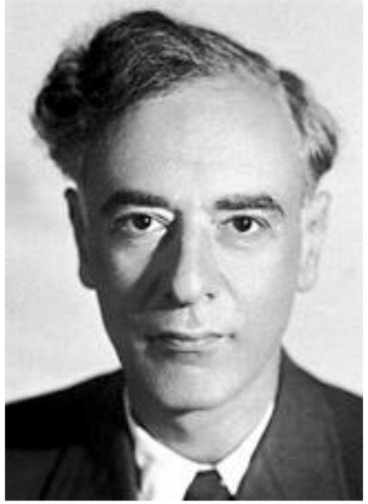
$T > T_c, \psi = 0$
(normal state)



$T < T_c, \psi = (|\alpha|/\beta)^{1/2}$
(superconducting state)



V. L. Ginzburg



L. D. Landau

(1950, Nobel prize 2003)

GL theory is one of the most widely used theories

Phase coherence and flux quantization

All superconducting electrons are paired in a coherent quantum state described by the macroscopic complex wave function:

$$\psi = \left(\frac{n_s}{2}\right)^2 e^{i\theta}$$

The **same phase** θ for all superconducting electrons!

Phase gradient $\nabla\theta$ results in a superconducting current:

$$J = -\left(\frac{e\hbar n_s}{m}\right)\nabla\theta$$

"analog" of Ohm's law for superconductors! → SQUIDs, Qubits

Vector potential

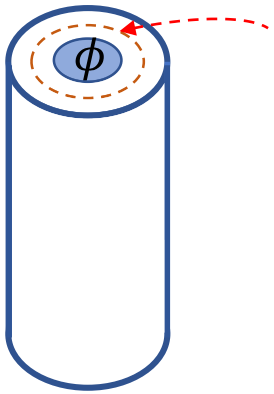
$$\boldsymbol{A} = \nabla\times B = \mu_0 \boldsymbol{J}$$ (Ampere's law)

$$\boldsymbol{J_s} = \frac{1}{\mu_0\lambda^2}\left(\frac{\phi_0}{2\pi}\nabla\theta + \vec{\boldsymbol{A}}\right)$$

$$\lambda = \left(\frac{m}{e^2 n_s(T)\mu_0}\right)^{1/2}$$ London penetration depth

$$\phi_0 = \frac{\pi\hbar}{|e|}$$ - magnetic flux quantum

- ➢ *What is the magnetic flux trapped in a hollow superconducting cylinder?*



If one integrates $\boldsymbol{J_S}$ around the contour taken in the bulk of the superconductor where J_S = 0, then:

$$\frac{1}{\mu_0\lambda^2}\oint\left(\frac{\phi_0}{2\pi}\nabla\theta + \vec{\boldsymbol{A}}\right)d\vec{\boldsymbol{l}} = 0$$

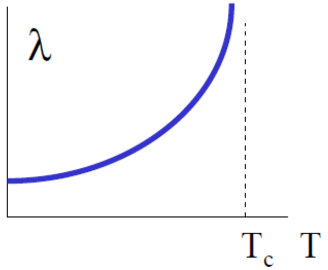
Using Gauss theorem: $\Phi = \int \nabla A dS = \oint A dl$ and the fact that ψ must be single-valued, one comes up with a periodic solution: $\oint \nabla\theta dl = 2\pi n$, where $n = 0, \pm 1, \pm 2, \ldots$

$$\Phi = \pm n\phi_0,\ \phi_0 = \pi\hbar/|e| = 2.07 \times 10^{-15}\ \text{Wb}$$

Important results of the GL theory

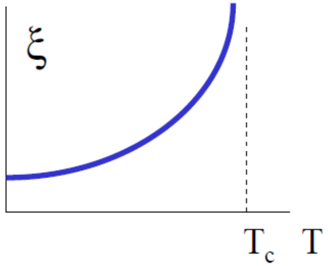
Magnetic London penetration depth near T_c:

$$\lambda(T) = \left(\frac{m\beta}{2e^2\mu_0 a_0}\right)^{1/2}\sqrt{\frac{T_c}{T_c - T}}$$



Coherence length near T_c – a scale of spatial variation of the superconducting electron density $n_s(r)$ or superconducting gap Δ(r):

$$\xi(T) = \left(\frac{\hbar^2}{4ma_0}\right)^{1/2}\sqrt{\frac{T_c}{T_c - T}}$$



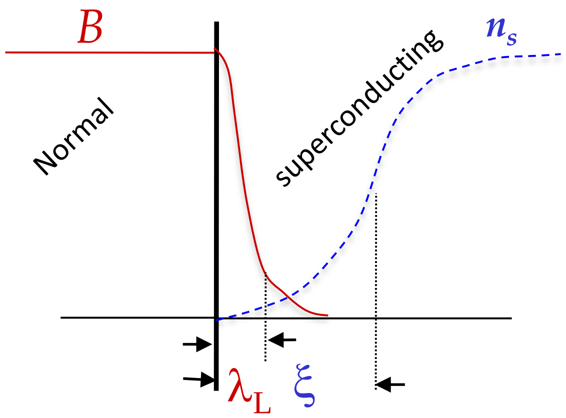
Critical field $B_c(T)$ neat T_c, in terms of $\xi(T)$ and $\lambda(T)$:

$$B_c(T) = \frac{h}{4\sqrt{2}\pi e\xi(T)\lambda(T)}$$

The Ginzburg-Landau parameter:

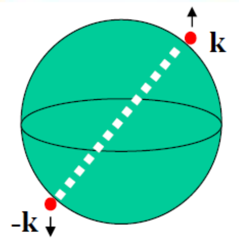
$$\boldsymbol{\kappa = \lambda/\xi}$$ -

an essential temperature -independent characteristic of superconducting materials



BCS theory

In 1957 Bardeen, Cooper, and Schrieffer publish microscopic theory (BCS) of Cooper-pair formation that continues to be held as the standard for low-temperature superconductors. Nobel prize 1972.



Cooper pair on the Fermi surface

Attraction between electrons with antiparallel momenta k and spins due to exchange of lattice vibration quanta (phonons)

A rigorous microscopic theory explaining the formation of Cooper pairs and superconducting energy gap

Bose condensation of overlapping Cooper pairs in a coherent superconducting state. (Cooper pairs are bosons!)


John Bardeen


Leon Cooper


John Robert Schrieffer

Critical temperature T_c is connected with the electron-phonon coupling constant and the energy gap:

$T_c = 1.13\, T_D\, e^{-1/\gamma}$, where $\gamma \approx$ 0.1-1 is a dimensionless coupling constant between phonons and electrons

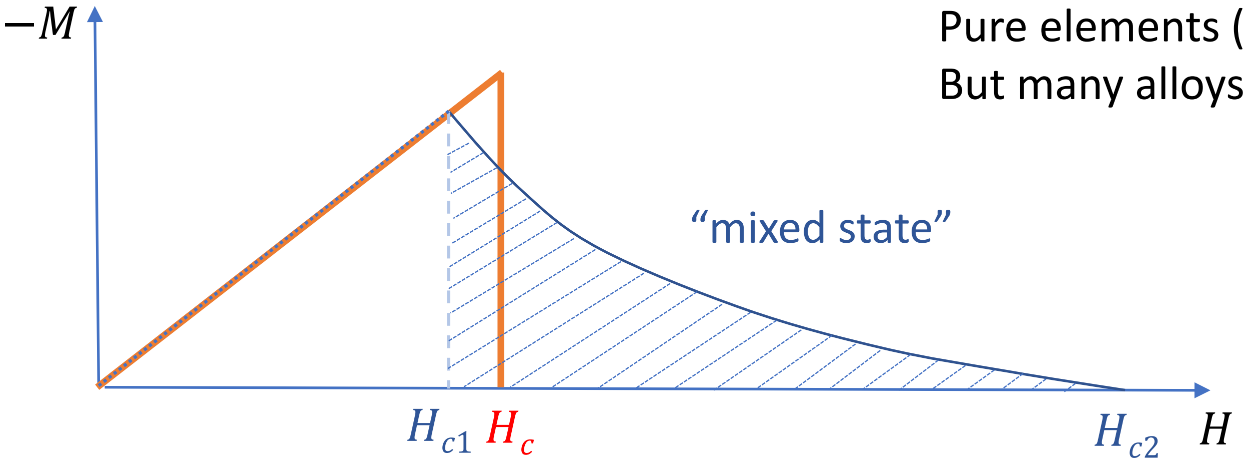
$2\Delta = 3.52\, k_B T_c$, $T_c \ll T_D$ (where T_D is the Debye temperature ~300 K)

Basics of superconductivity and applications

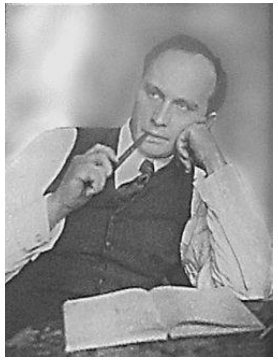
(Part 2)

M. Marchevsky,
Lawrence Berkeley National Laboratory

Type-I and Type-II superconductors



Pure elements (Hg, Sn, Pb, In, etc. are usually "type I"
But many alloys were exhibiting "type II" behavior"…



Lev Shubnikov

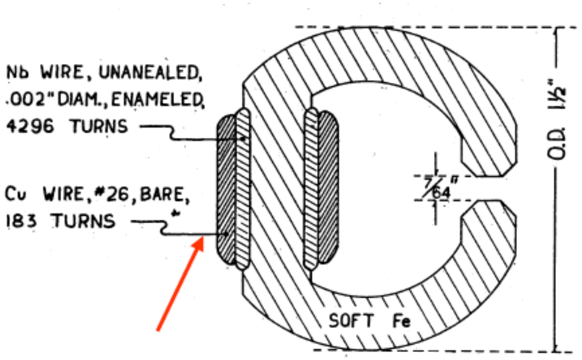
Type-I: Field penetrates the superconductor at H_c destroying bulk superconductivity at once

Type-II: Field penetrates the superconductor at H_{c1} but superconductivity is fully destroyed at a much higher field H_{c2}

What happens in a superconductor between H_{c1} and H_{c2}?

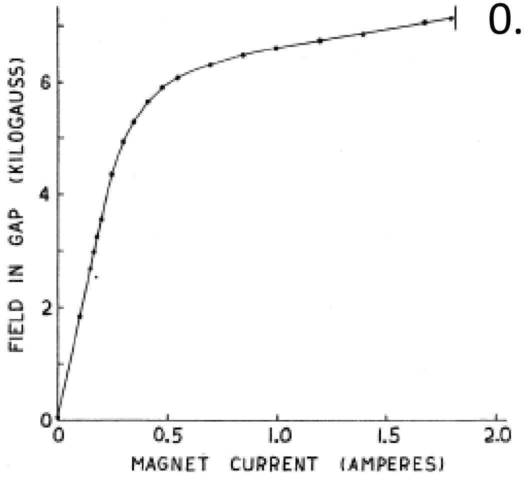
Superconductor	T_c, (K)	ξ_0 (nm)	λ_0 (nm)	GL parameter $\kappa = \lambda/\xi$	$\mu_0 H_c$ (or B_{c2}), (T)
Pb	7.19	83	37	0.445	0.08
In	3.14	70	40	0.57	0.03
Sn	3.72	230	14	0.06	0.03
Nb	9.26	38	39	1.026	0.82
NbTi	10	4	240	60	15
Nb_3Sn	18.3	3	65	21.7	30
MgB_2	39	6	140	23.3	74
YBCO	92	1.5	150	100	~100
$Bi_2Sr_2Cu_2O_{8+\delta}$	85	1.5	200	117.6	~120

First type-II superconductor magnet



for quench protection

George Yntema, Univ. of Illinois, 1954



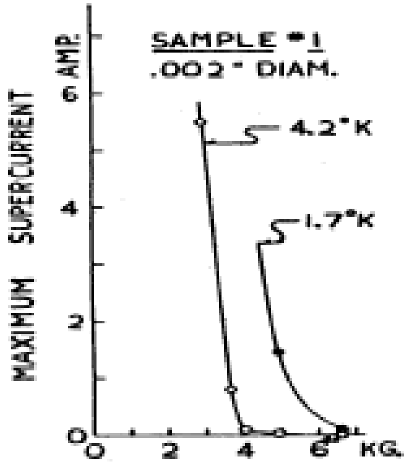
0.7 T field

- The first successful type-II superconductor magnet was wound with Nb wire

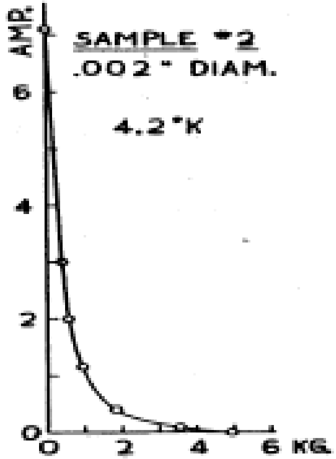
It was also noted that "cold worked" Nb wire yielded better results than the annealed one.

Material defects in the conductor seemed to help improve the magnet performance!

hard



annealed

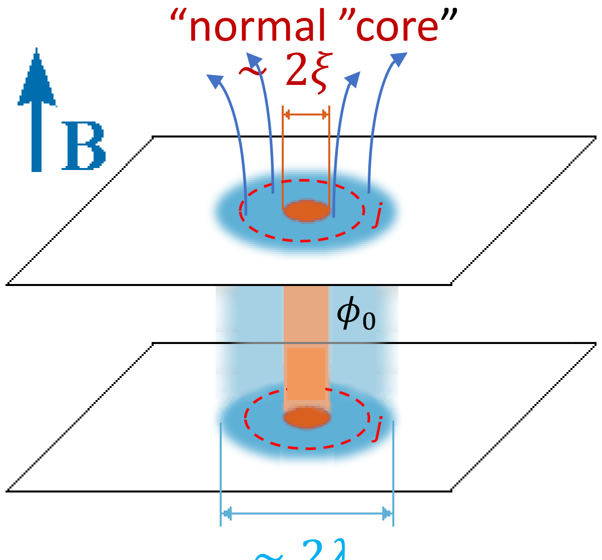


Flux lines (vortices) and flux-line lattice

In 1956 A. Abrikosov applied GL theory to the case of "extreme" superconductors where $\lambda \gg \xi$ $(k \gg 1)$. He found that in this case it should more energetically favorable for the magnetic field to penetrate the superconductor at H_{c1} in a form of flux lines or "vortices" carrying a single flux quantum ϕ_0 rather than forming macroscopic domains of normal phase like in type-I materials.

Nobel prize, 2003

Alexei Abrikosov



According to the solution:

$\kappa < 1/\sqrt{2}$ - type I superconductor

$\kappa > 1/\sqrt{2}$ - type II superconductor

$$H_{c1} = \frac{\phi_0}{4\pi\mu_0\lambda^2}\left(ln\frac{\lambda}{\xi} + 0.5\right)$$

$$H_{c2} = \frac{\phi_0}{2\pi\mu_0\xi^2}$$ - when normal cores overlap, superconductivity disappears

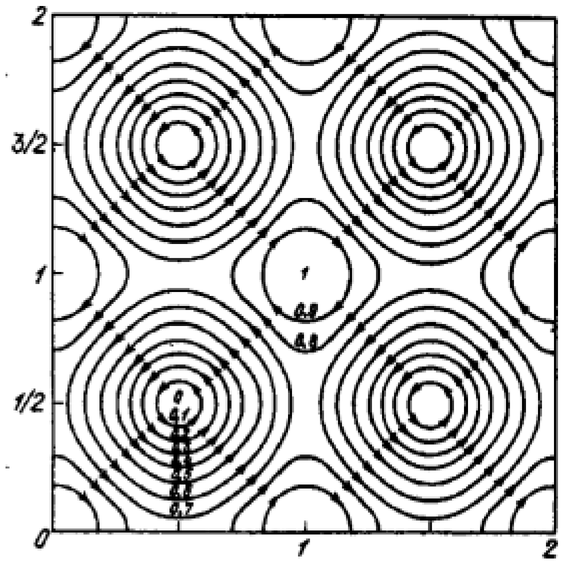


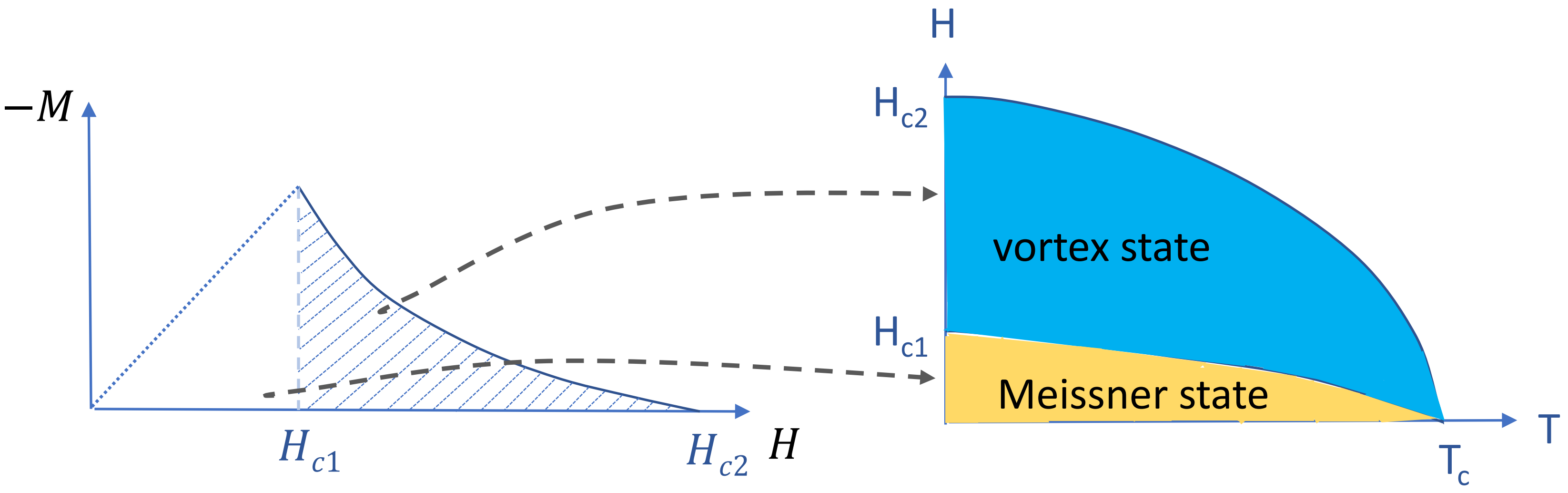
FIG. 2. The lines of current coinciding with the lines of constant $|\Psi|$ for a square lattice.

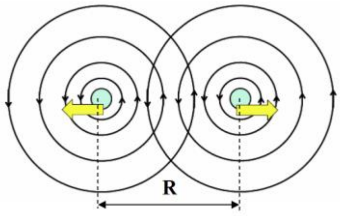
To further minimize energy, vortices will form a **hexagonal** lattice

Vortex density $n(B) = \phi_0/B$ defines the magnetic induction B in the material

Spacing between vortices: $a_0 = (\phi_0/B)^{1/2}$

Vortex state in type-II superconductors





Vortices are magnetic dipoles, they repel each other (locally), while macroscopic currents flowing in the superconductor "pull" them in, thus balancing the net force.

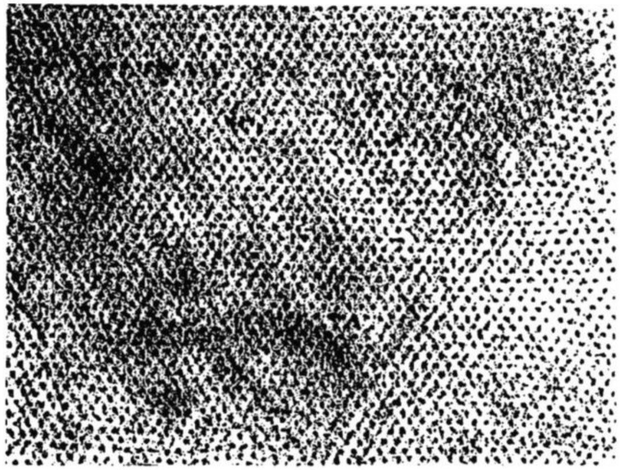
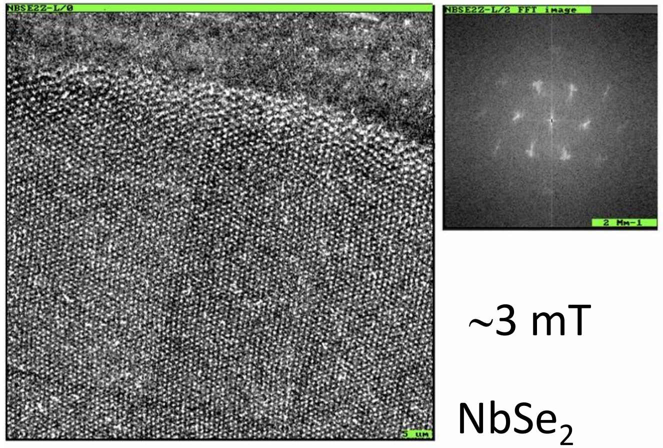


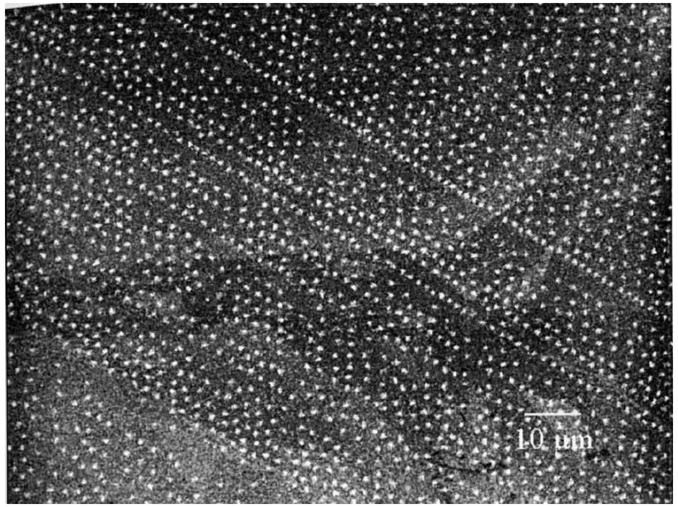
FIG. 5. First decoration picture of vortices by Essmann and Traeuble (1967).

Vortex lattice in different materials

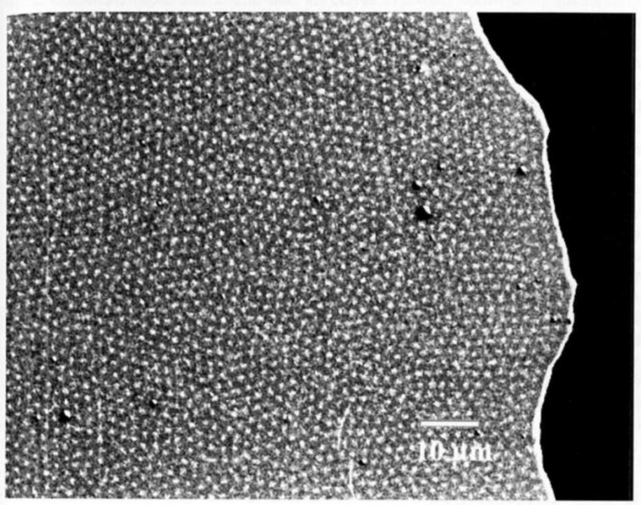


~3 mT

$NbSe_2$



Bi-2212 ~ 0.5 mT

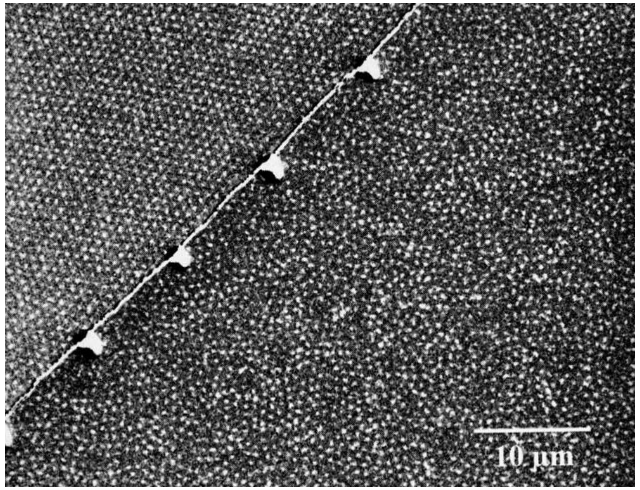


MoGe ~ 0.5 mT

~4.5 mT

$NbSe_2$

$NbSe_2$



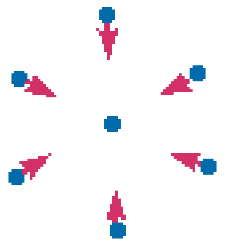
Nb

~5 mT

Magnetic imaging methods like decoration only work at low fields ~H_{c1}, where interior-vortex separation is greater than λ, so they do not "overlap".

MM, PhD thesis

Vortex matter: elastic properties

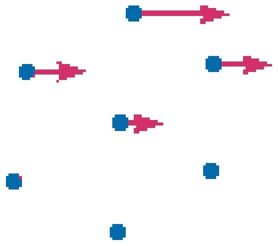


Compression modulus:

$$c_{11}(k) = \frac{B^2}{4\pi}\frac{1}{1+\lambda^2 k^2} \sim 1/\lambda^2$$

- Like the real matter, "vortex matter" has elastic properties and elastic energies associated with them!

(B ~ B_{c1})

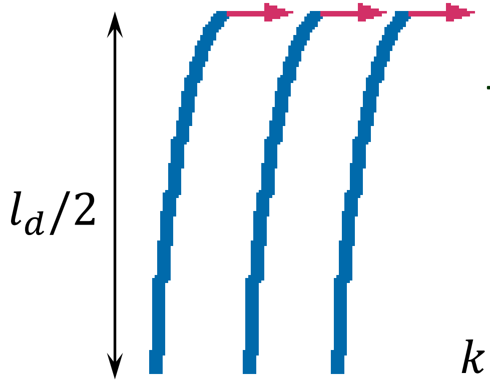


Shear modulus:

$$c_{66} = \frac{\phi_0 B}{(8\pi\lambda)^2} \sim 1/\lambda^2$$

$$\lambda(T) = \frac{\lambda_0}{(1-T/T_c)^{1/2}}$$

Vortex lattice "stiffness" vanishes at T_c is approached



Tilt modulus:

$$c_{44}(k) = \frac{B^2}{4\pi}\frac{1}{1+\lambda^2 k^2} \sim 1/\lambda^2$$

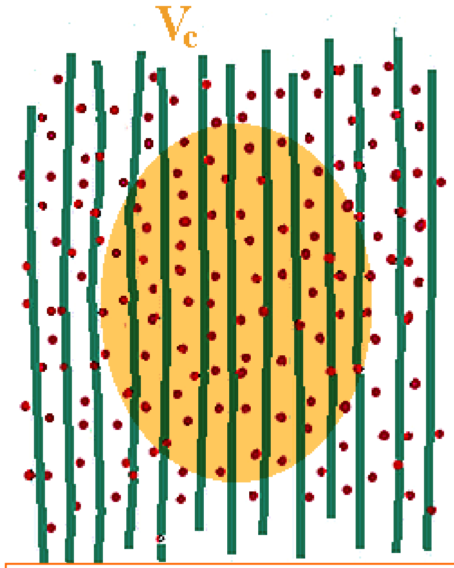
k is the wavevector of the deformation, $k = 2\pi/l_d$

What can distort elastically-coupled vortex lattices in a material?

Vortex pinning

weak pinning

collective pinning



$$F_p = \left[\frac{1}{2} n_p f_p^2 \Big/ V_c \right]^{1/2}$$

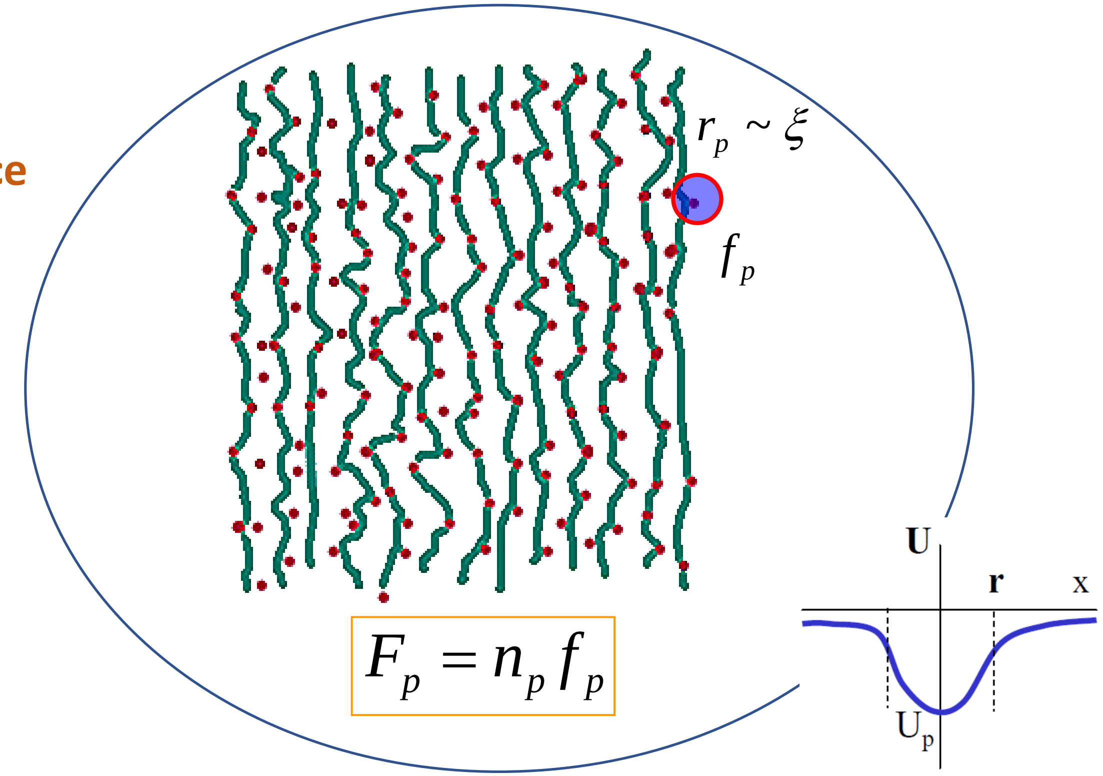
$$V_c \sim c_{66}^2 c_{44}$$

Larkin and Ovchinnikov,1979.

Pining action of various material defects "competes" against flux line lattice elasticity

strong pinning

individual (single vortex) pinning)



$$F_p = n_p f_p$$

It is favorable for a flux line to "sit" on a defect, as normal core would be going through the volume where superconductivity is already suppressed locally by the defect. Energy gain!

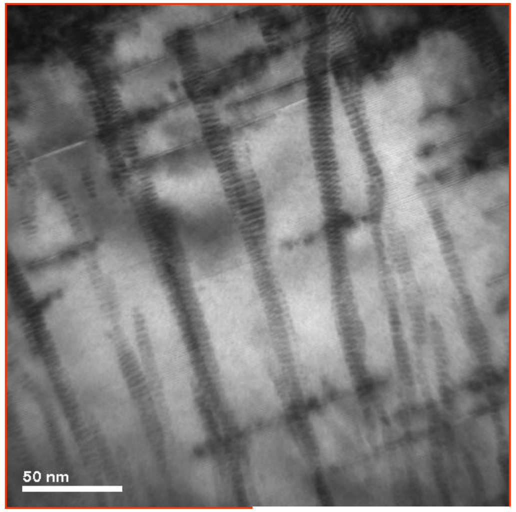
Flux pinning

Flux lines can be pinned by a wide variety of material defects

- Inclusions
  - Under certain conditions, small inclusions of appropriate materials can serve as pinning site locations; this suggests tailoring the material artificially through manufacturing
- Lattice dislocations / grain boundaries
  - These are known to be primary pinning sites. Superconductor materials for wires are severely work hardened so as to maximize the number and distribution of grain boundaries.
- Precipitation of other material phases
  - In NbTi, mild heat treatment can lead to the precipitation of an a-phase Ti-rich alloy that provides excellent pinning strength.
  - In high-temperature superconductor Y-Ba-Cu-O nanorods can be formed to pin vortices **along the length** (very strong!)

Fig. 1: Microstructure of a NbTi filament (Courtesy of P.J. Lee, University of Wisconsin at Madison).

Nanorods in Zr-doped YBCO tape conductor



TEM by A. Goyal, ORNL

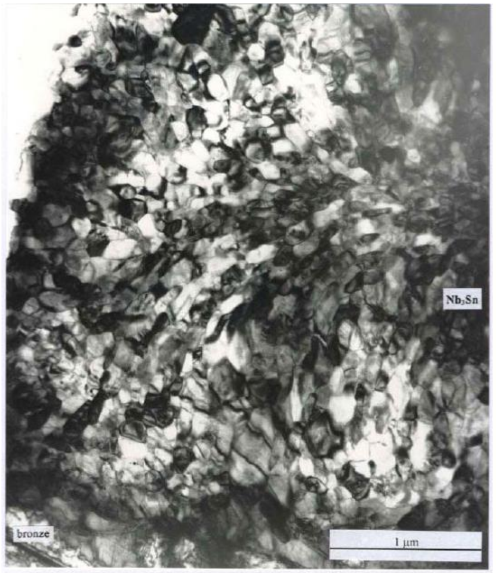
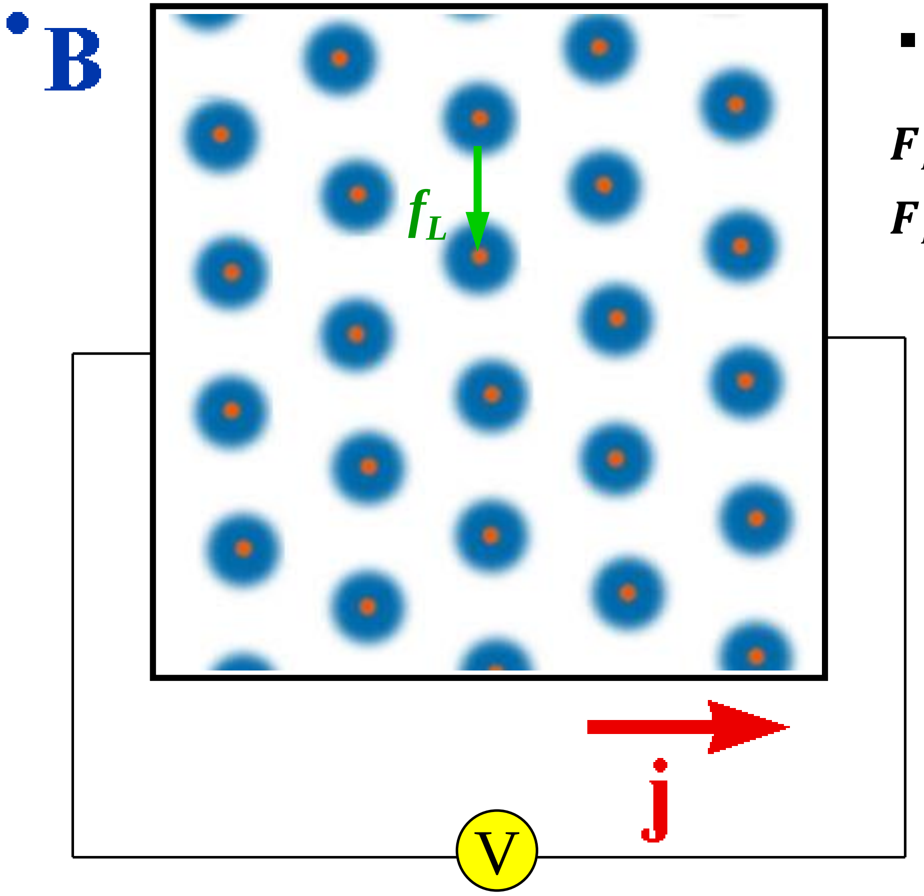


Fig. 6: Microstructure of a Nb_3Sn filament (Courtesy of C. Verwaerde, Alstom/MSA).

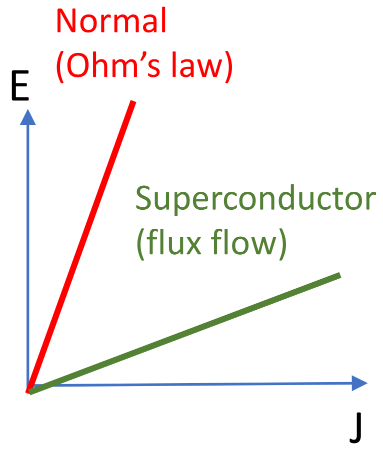
Flux flow in absence of pinning



- Viscous flow of vortices due to Lorentz force

$\boldsymbol{F}_L = \phi_o[J \times \boldsymbol{n}]$ - Lorentz force acting on a single vortex

$\boldsymbol{F}_L = \eta\boldsymbol{v}$ where η is a viscosity coefficient



Together with the Faraday's law: $\boldsymbol{E} = v \times \boldsymbol{B}$

this yields the flux-flow relation:

$\boldsymbol{E} = \rho_f \boldsymbol{J}$ $\rho_f = \rho_f B/B_{c2}$ *volume fraction of normal vortex cores!*

Vortex flow viscosity appears due to dissipation in the vortex core and can be expressed in terms of the normal state resistivity:

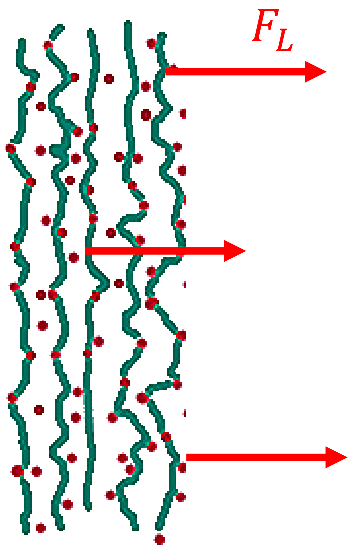
$$\eta = \phi_o B_{c2}/\rho_n$$

Example: $E = 1$ μV/cm and B= 1 T => vortex velocity is

$$v = \frac{E}{B} = 0.1 \text{ mm/s}$$

Depinning. Critical current.

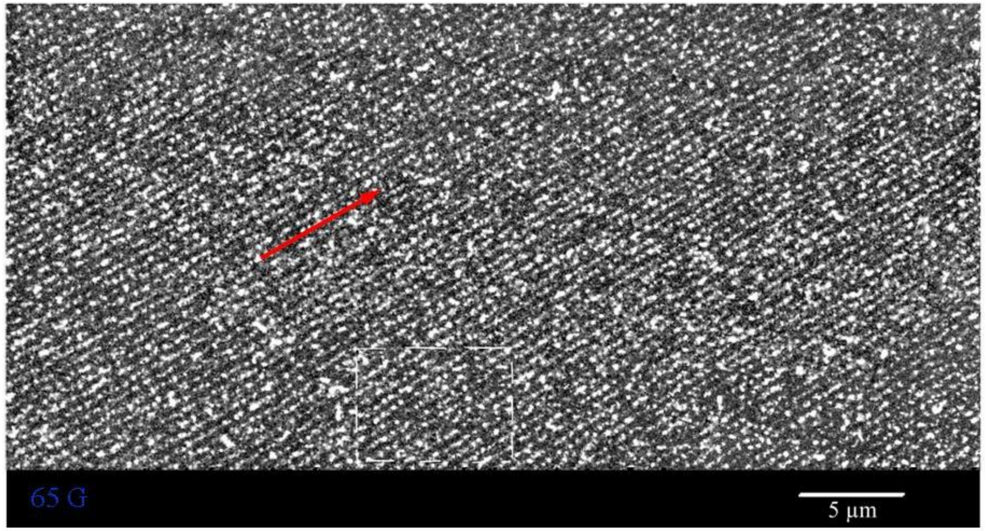
Let us now bring pinning back!



$$BJ_c = F_p(T, B)$$

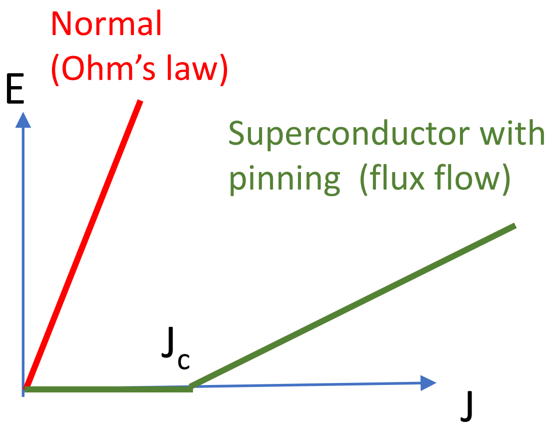
Balance of the volume Lorentz and pinning forces defines the critical current density J_c

- Defects pin vortices and restore *almost* zero resistivity for currents J smaller than the critical current density J_c
- For currents J > J_c flux flow is restored
- J_c is strongly sample dependent

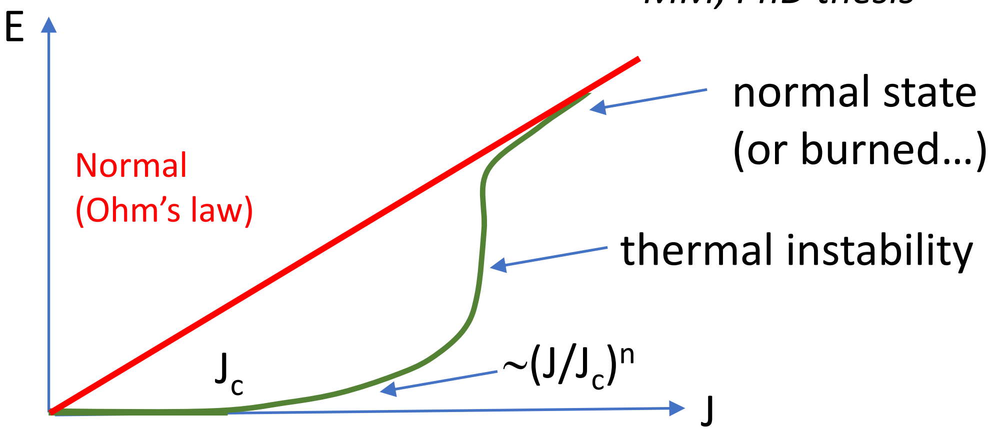


Decoration image of the flowing vortex lattice in $NbSe_2$
MM, PhD thesis

Ideal case ➔

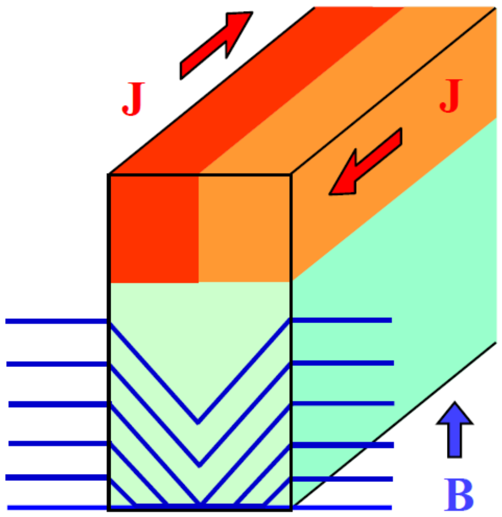


In reality ➔



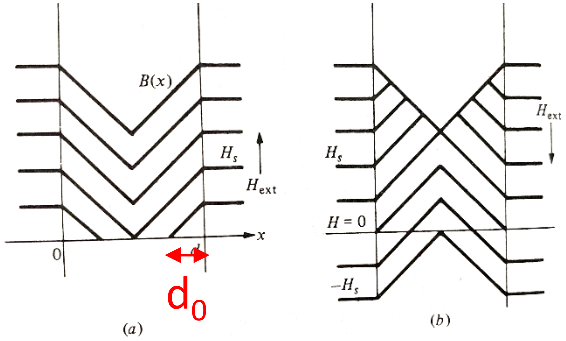
➢ J_c variation range: 10^2 A/cm^2 – (MoGe, $NbSe_2$) -> 10^5 A/cm^2 (NbTi, Nb_3Sn)

Bean critical state and magnetization



Assume a slab of type-II superconductor where field is applied parallel to its surfaces.

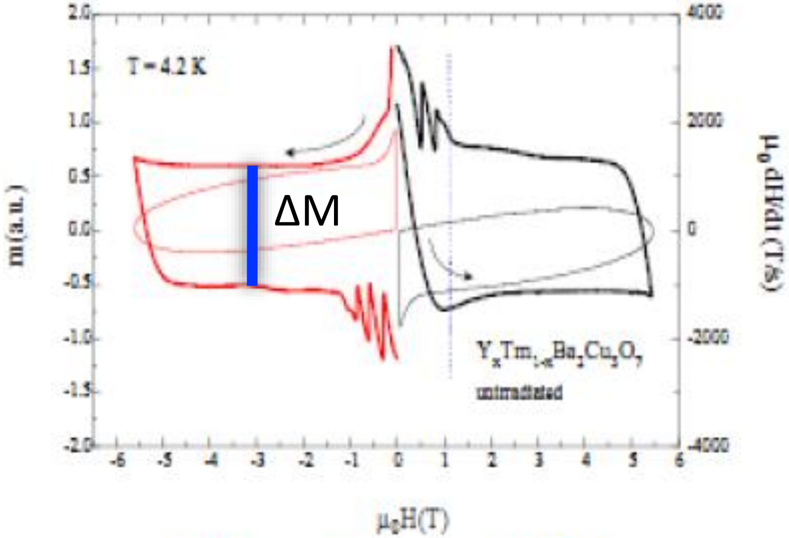
Screening currents will then flow along the surfaces. If current density reaches J_c, flux lines will be "pulled" into the slab. The process will stop when current density equals to J_c **everywhere where flux lines are present**, resulting in the Bean critical state (C.P. Bean, 1962)



$\mu_0 J_y$ = - *dB/dx, and assuming* $J_y = J_c$ one can find the depth of initial flux front penetration into the slab (or cylinder) as:

$$d_0 = \frac{H_{ext}}{J_c} = \frac{B_{ext}}{\mu_0 J_c}$$

Magnetization measurements can provide insight into flux pinning and flux motion, key concepts governing the performance of superconducting materials.



J. Vanacken, et. al, 1999.

$$\Delta M \cdot B \propto F_p(T, B)$$

Often used to evaluate $J_c(B,T)$!

Magnetization in type-II superconductors is mainly defined by pinning and formation of critical state (rather than by the Meissner effect as in in type-I materials). It is because penetration fields in high J_c *materials are typically >>* H_{c1}

Flux jumping

- When pinning is strong, a significant amount of flux is trapped in superconductor (= magnetization)
- When current exceeds critical, instead of a gradual de-pinning of vortices, an "avalanche-like" instability may occur that is called **"flux jump"**

The mechanism of "flux jumping":

A small "bundle" of flux initially moves -> temperature rises->critical current density (pinning strength) is reduced -> more flux moves -> temperature rises further ->> a flux "avalance" forms, seen as a spike in voltage across the conductor…

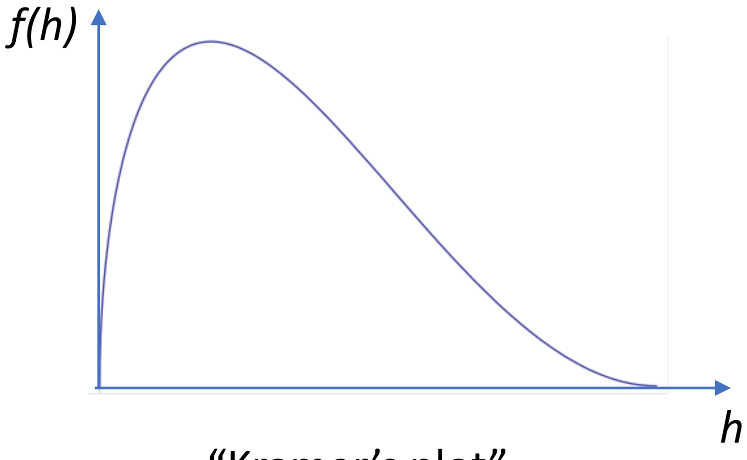
"Cure" for flux jumping: weaken the link in the feedback loop. This is primarily done by reducing diameter of a superconducting wire. Use many fine filaments instead of a large diameter wire. For NbTi the stable diameter is ~ 50 μm.

Modeling pinning

- Precise first-principles physical descriptions of overall pinning strength (and hence critical current) of real superconductors is difficult due to various mechanisms of intrinsic in pinning
- Nevertheless, models based on sound physics minimize free parameters needed to fit measured data and provide reliable estimates for classes of materials
- One of the most cited correlations is that of Kramer:

$$F_p = F_{\max} f(h) \propto \frac{H_{c2}^{\nu}}{\kappa^{\gamma}} f(h)$$

$$f(h) = h^{1/2}(1-h)^2; \quad h = H / H_{c2}$$



"Kramer's plot"

The fitting coefficients ν and γ depend on the type of pinning.
Temperature dependence is through:

$$H_c(T) \simeq H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

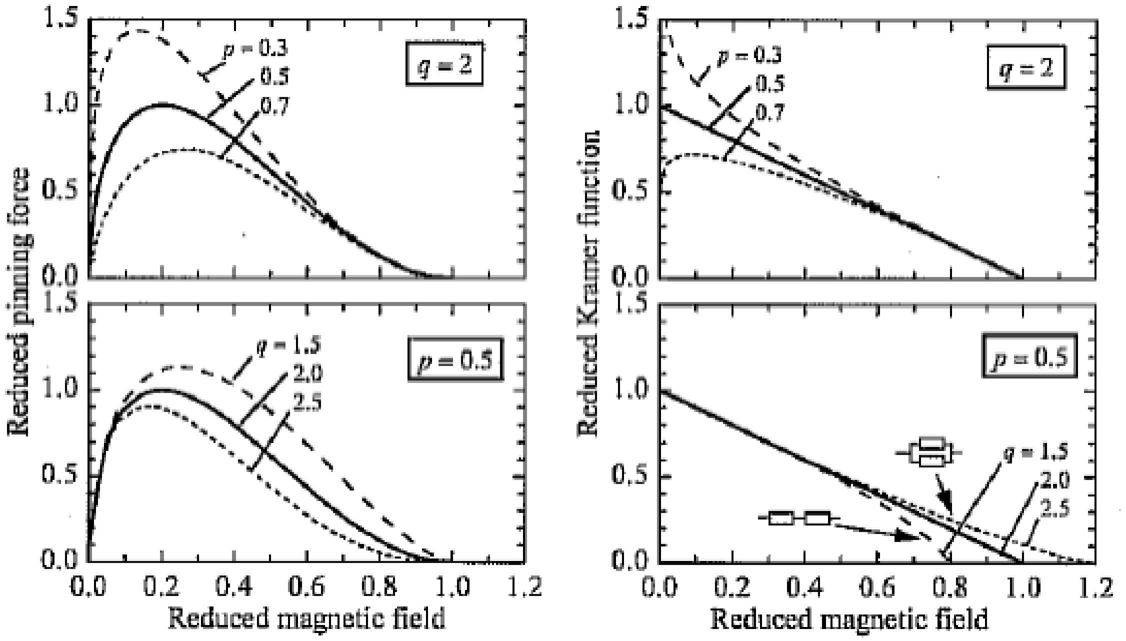
Scaling of critical current: field dependence

- The Kramer formulation provides excellent fits in the region 0.2<h<0.6 for Nb_3Sn; it is appropriate for regimes where the number of flux lines exceeds the number of pinning sites
- Outside this region, a variety of effects (e.g. inhomogeneity averaging) can alter the pinning strength behavior, so the pinning strength is often fitted with the generalization

$$f_p(h) \propto h^p (1-h)^q ; \quad h = H / H_{c2}$$

- It is preferable to stay with the Kramer formulation, yielding:

$$J_c^{1/2} B^{1/4} \simeq \frac{1.1\times10^5}{\kappa} \mu_0 \left(H_{c2} - H\right)$$



Strain dependence of J_c in Nb_3Sn: physics-based model

- A physics-based model of strain dependence has been developed using the frequency-dependent electron-phonon coupling interactions (Eliashberg; Godeke , Markiewitz)

Phonon density of states

$$\lambda_{ep}(\varepsilon) = 2\int \frac{\alpha^2(\omega)F(\omega)}{\omega} d\omega$$

- From the interaction parameter the strain dependence of T_c can be derived
- Experimentally, the strain dependence of H_{c2} behaves as

$$\frac{H_{c2}(4.2,\varepsilon)}{H_{c2m}(4.2)} \cong \frac{T_c(\varepsilon)}{T_{cm}}$$

- The theory predicts strain dependence of J_c for all LTS materials, but the amplitude of the strain effects varies (e.g. very small for NbTi)
- The resulting model describes quite well the asymmetry in the strain dependence of B_{c2}, and the experimentally observed strong dependence on the deviatoric strain

Scaling of critical current, Nb_3Sn,empirical strain dependence

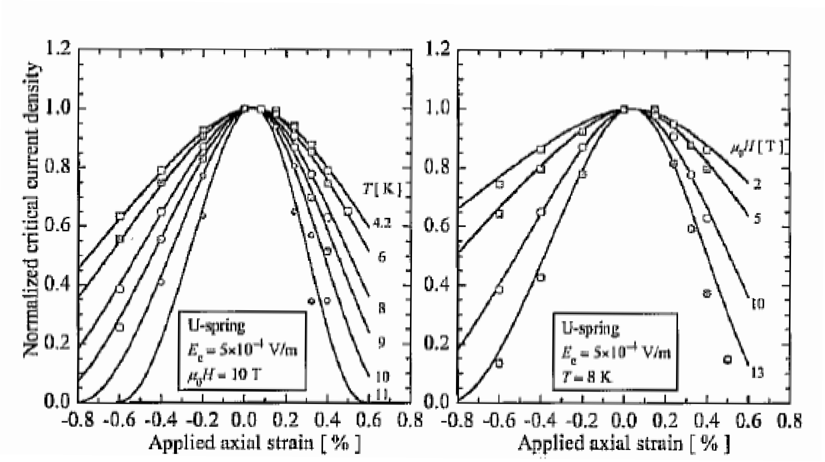
- The critical current of Nb_3Sn is strain dependent, particularly at high field
- The strain dependence is typically modeled in terms of the normalized critical temperature:

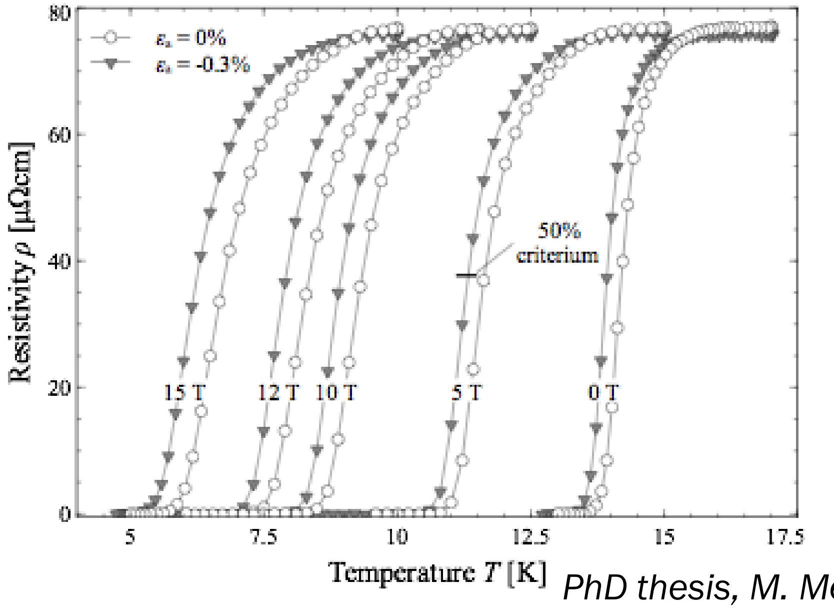
$$\frac{H_{c2}(4.2,\varepsilon)}{H_{c2m}(0)} \simeq \left[\frac{T_c(\varepsilon)}{T_{cm}}\right]^3 = s(\varepsilon)$$

- The term T_{cm} and H_{c2m} refer to the peaks of the strain-dependent curves
- A strain model proposed by Ekin:

$$s(\varepsilon) = 1 - a\left|\varepsilon_{axial}\right|^{1.7}$$

$$a = \begin{cases} 900 & \varepsilon_{axial} < 0 \\ 1250 & \varepsilon_{axial} > 0 \end{cases}$$





PhD thesis, M. Mentink

Critical surface: example fit for NbTi

Critical surface: critical current density plotted as 3D plot against B and T

- NbTi parameterization
  - Temperature dependence of B_{C2} is provided by Lubell's formulae:

$$B_{C2}(T) = B_{C20}\left[1 - \left(\frac{T}{T_{C0}}\right)^{1.7}\right]$$

where B_{C20} is the upper critical flux density at zero temperature (~ 14.5 T)

- Temperature and field dependence of *Jc* can be modeled, for example, by Bottura's formula

$$\frac{J_C(B,T)}{J_{C,ref}} = \frac{C_{NbTi}}{B}\left[\frac{B}{B_{C2}(T)}\right]^{\alpha_{NbTi}}\left[1 - \frac{B}{B_{C2}(T)}\right]^{\beta_{NbTi}}\left[1 - \left(\frac{T}{T_{C0}}\right)^{1.7}\right]^{\gamma_{NbTi}}$$

where $J_{C,Ref}$ is critical current density at 4.2 K and 5 T (e.g. ~3000 A/mm^2) and C_{NbTi} (~30 T), α_{NbTi} (~0.6), β_{NbTi} (~1.0), and γ_{NbTi} (~2.3) are fitting parameters.

J_c universal scaling for NbTi & Nb3Sn

$$J_c(H,T,\varepsilon) \cong \frac{C_1}{\mu_0 H} s(\varepsilon)\left(1-t^{n_1}\right)\left(1-t^{n_2}\right)h^p(1-h)^q,$$

with

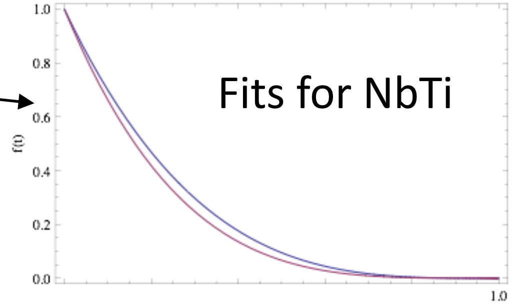
$$t \equiv T/T_c^*(\varepsilon), \quad h \equiv H/H_{c2}^*(T,\varepsilon),$$

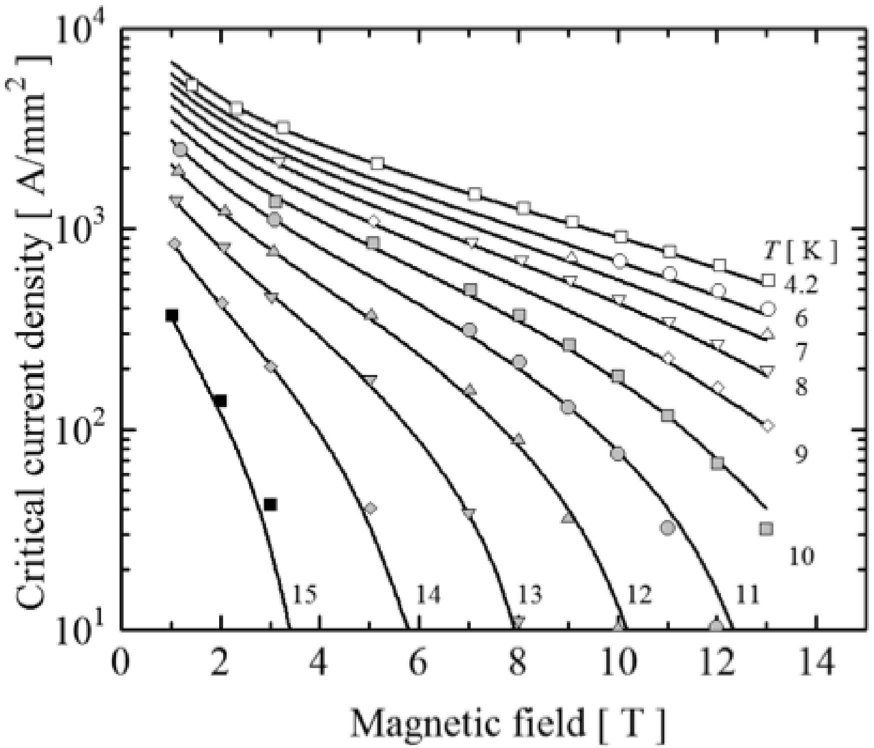
$$H_{c2}^*(T,\varepsilon) \cong H_{c2m}^*(0)s(\varepsilon)\left(1-t^{n_1}\right),$$

$$T_c^*(\varepsilon) = T_{cm}^* s(\varepsilon)^{\frac{1}{3}}$$

Godeke et al., SUST 19 (2006)

Fits for NbTi





Nb_3Sn

Godeke, SuST **19**

- $n_1 \cong 1.52$
- $n_2 = 2$
- $p = 0.5$
- $q = 2$
- $s(\varepsilon)$ = strain dependence

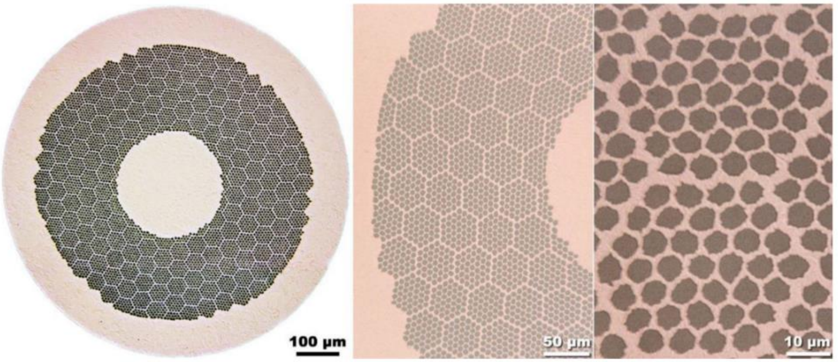
NbTi

Bottura, TAS **19**

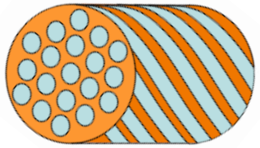
- $n_1 = n_2 \cong 1.7$
- $p \cong 0.73$
- $q \cong 0.9$
- $s(\varepsilon) \cong 1$

Practical conductor architecture

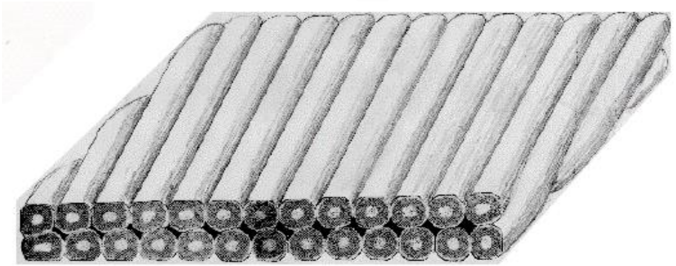
Stabilization: solid superconductor wire is replaced with an array of thin superconducting filaments embedded in copper matrix. Should a filament lose superconductivity, current will be redirected into the surrounding copper stabilizer



Twisting filaments along the strand length allows to reduce their stray magnetization



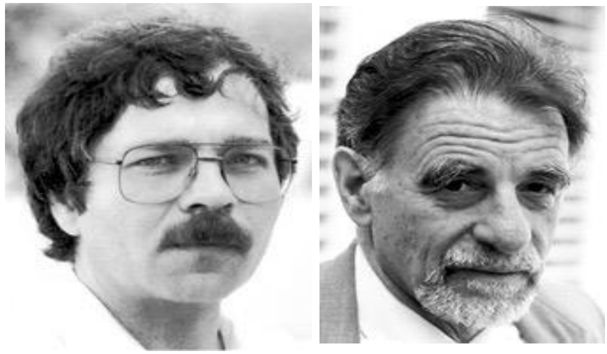
Transposition: superconducting strands forming a cable allow for a uniform current sharing, re-distribution in case of quenching, as well as additional magnetization reduction



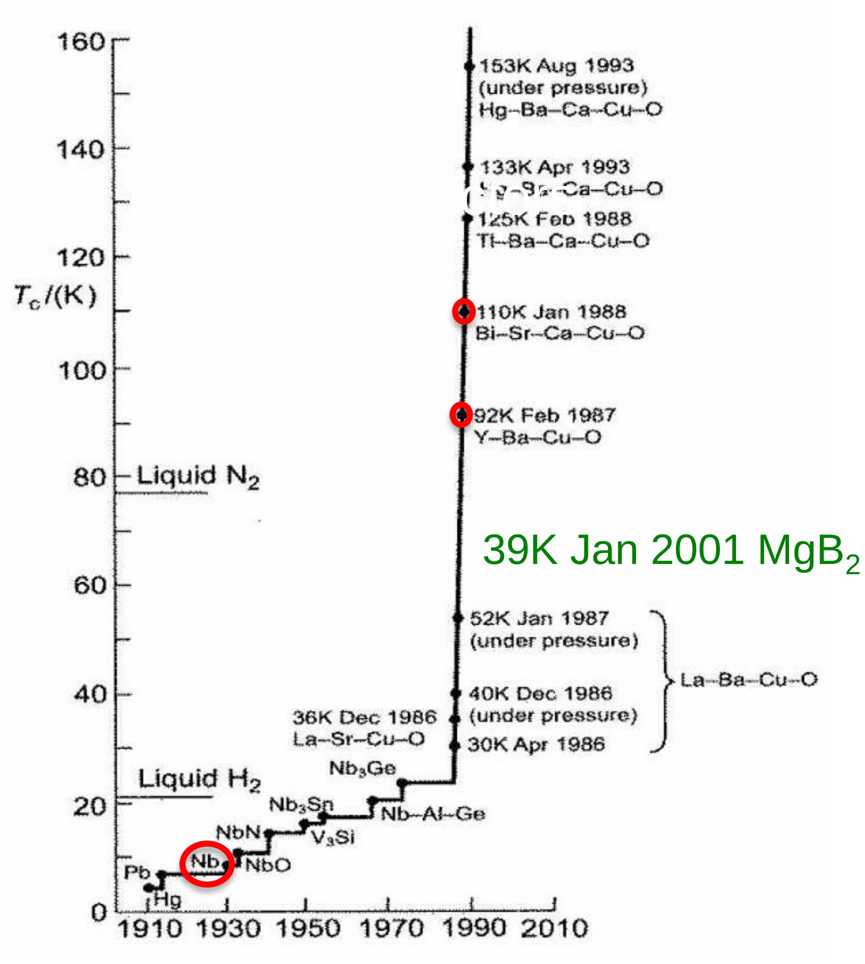
"**Nb-Ti - from beginnings to perfection**", *Peter J. Lee NHMFL, Florida State University* and *Bruce Strauss U.S. Department of Energy, in "100 Years of Superconductivity*" ", CRC Press 2011

High temperature superconductors

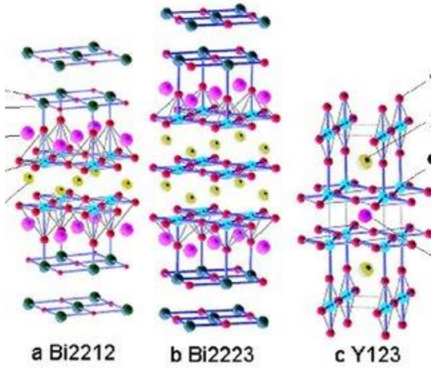
1986: Bednorz and Muller discover superconductivity at high temperatures in layered materials comprising copper oxide planes



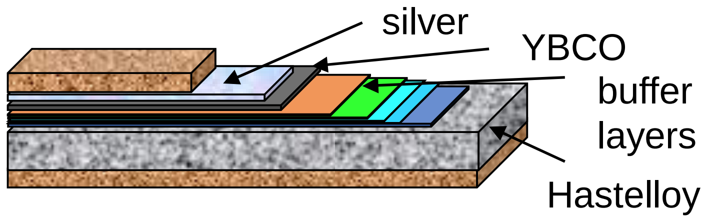
George Bednorz and Alexander Muller Nobel prize for Physics (1987)



- **Extreme type-II materials, k>>1**
- **T_c above liquid nitrogen**
- **B_{c2} is > 100 T**
- **Layered -> anisotropy!**
- **Brittle (ceramics)**



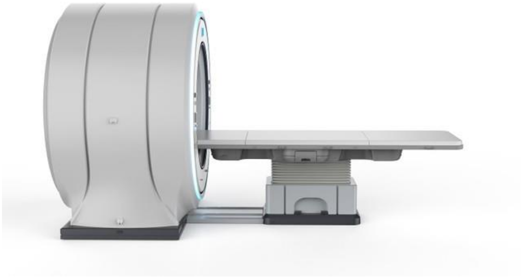
- **Critical current density improved dramatically since the discovery**
- **Our only path towards 20+ T superconducting magnets**

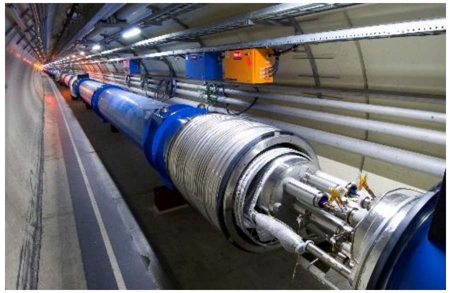


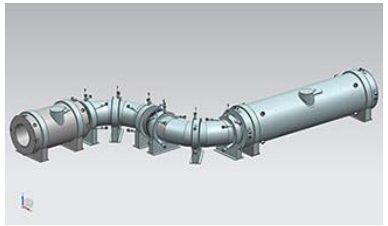
Geometry of the modern YBCO tape conductor

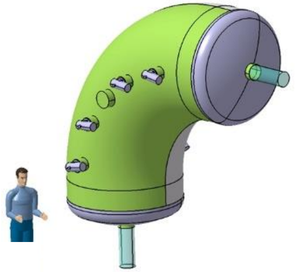
Superconducting applications (magnets)

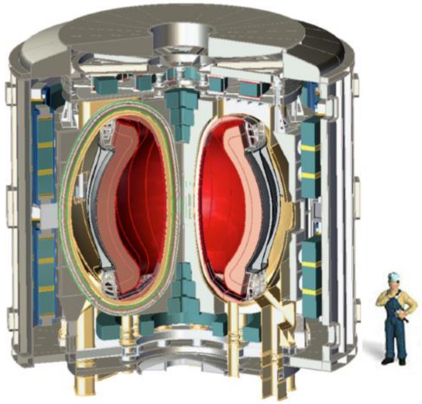
- MRI
- Particle accelerators (dipoles, quadrupoles, correctors)
- Other HEP experiments (detectors, particle guiding, etc…)
- Medical radiation treatment (gantries)
- Nuclear Fusion







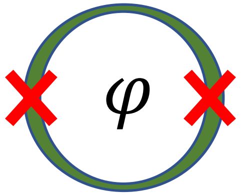


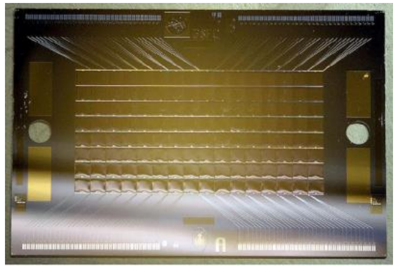


The fusion power produced in a tokamak is proportional to the strength of the magnetic field to the fourth power!

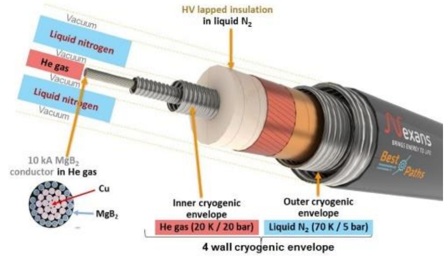
Superconducting applications (non-magnets)

- Quantum computing





- Bolometers, IR and THz detectors (astronomy, dark matter, security applications, etc…)

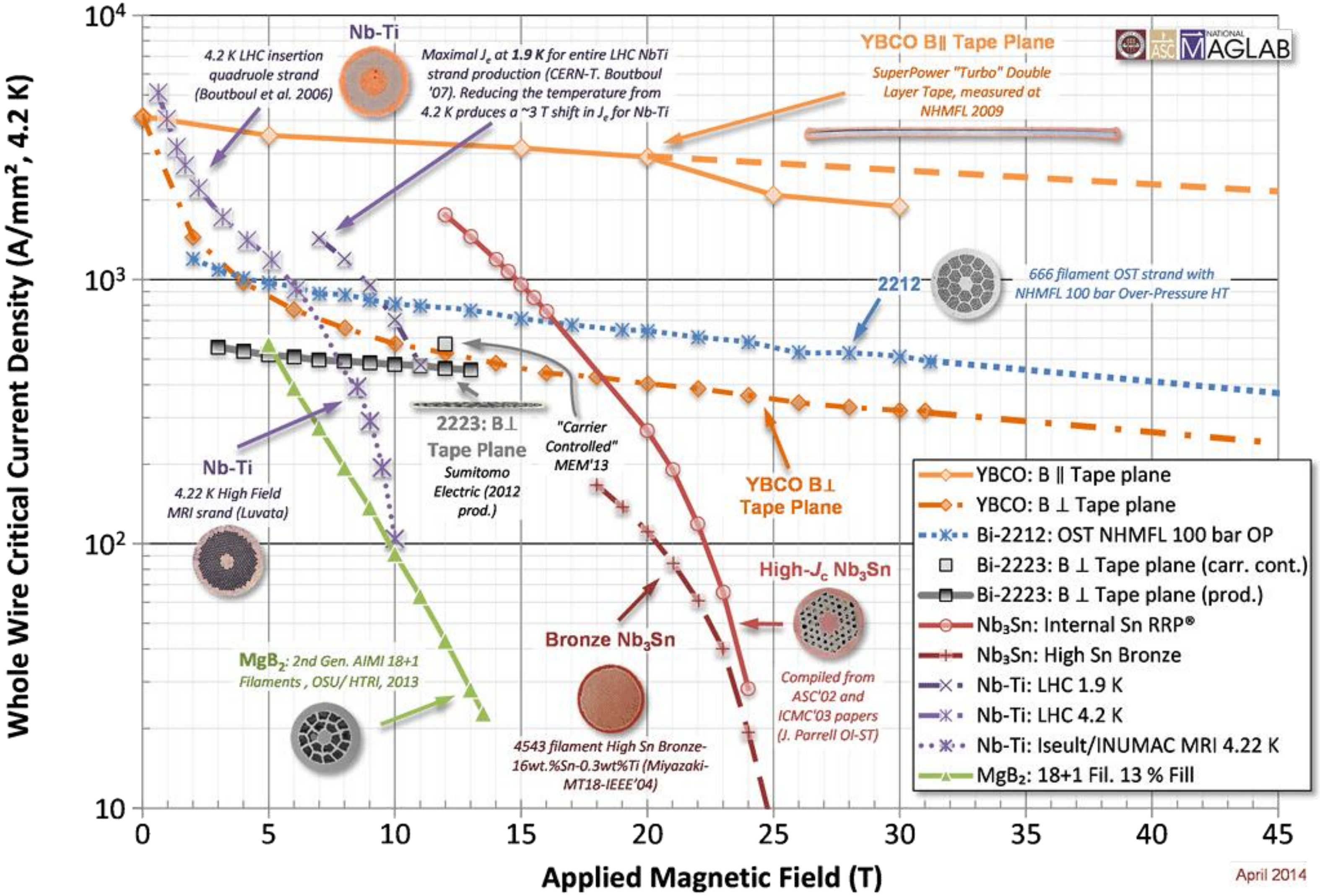


- Power applications (superconducting grid: cables, SMES, SFCLs)

- Transportation (MAGLEV, electric airplanes)

More are coming!

Superconductors for high-field applications: summary



P. Lee, NHMFL

Thank you!