AC losses in superconductors

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Introduction

- Superconductors subjected to varying magnetic fields see multiple heat sources that can impact conductor performance and stability
- All of the energy loss terms can be understood as emanating from the voltage induced in the conductor:

The hysteretic nature of magnetization in type-II superconductors, i.e. flux flow combined with flux pinning, results in a net energy loss when subjected to a field cycle

But, in addition:

- ➢ The combination of individual superconducting filaments and a separating normal-metal matrix results in a coupling Joule loss
- ➢ The normal-metal stabilizer sees traditional eddy currents

Hysteresis losses – basic model

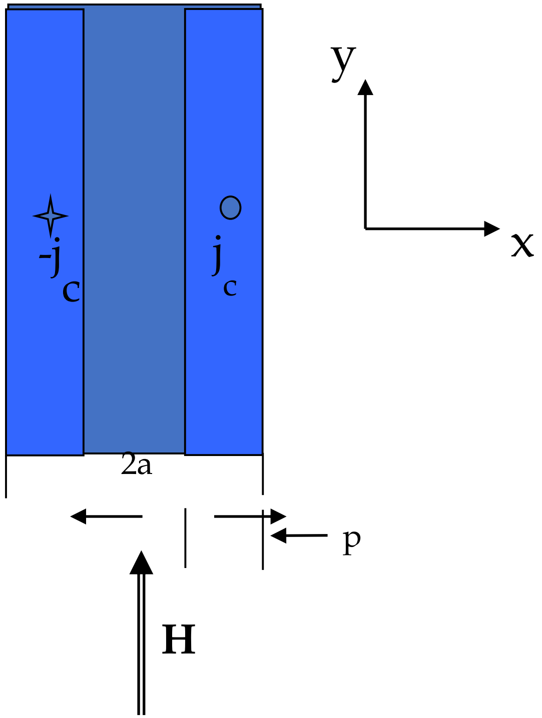
Hysteresis loss is:

$$Q = \int HdM = \int MdH$$

Note that magnetization generated by a current loop *I* enclosing an area *A* is defined as

$$m = \mu_0 AI$$

Assume $j=j_c$ in the region of flux penetration in the superconductor (Bean Model), then

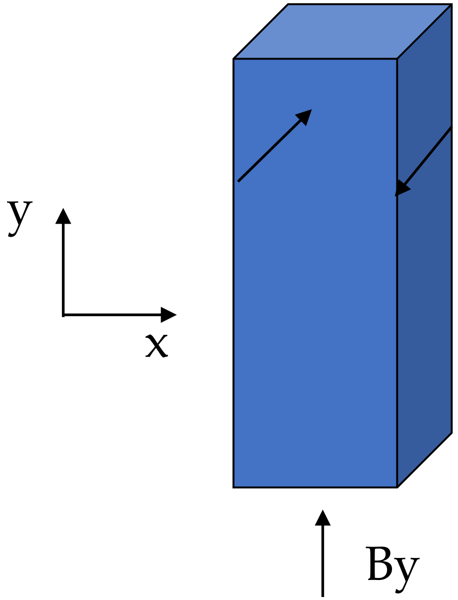


$$m = \mu_0 \int_{a-p}^{a} j_c x dx$$

$$= \frac{\mu_0 j_c}{2}\left[p^2 - 2ap\right]$$

- Below H_{c1} the superconductor is in the Meissner state and the magnetization from dH/dt corresponds to pure energy storage, i.e. there is no energy lost in heat;
- Above H_{c1} flux pinning generates hysteretic B(H) behavior; the area enclosed by the B(H) curve through a dB/dt cycle represents thermal loss

Calculating hysteresis losses



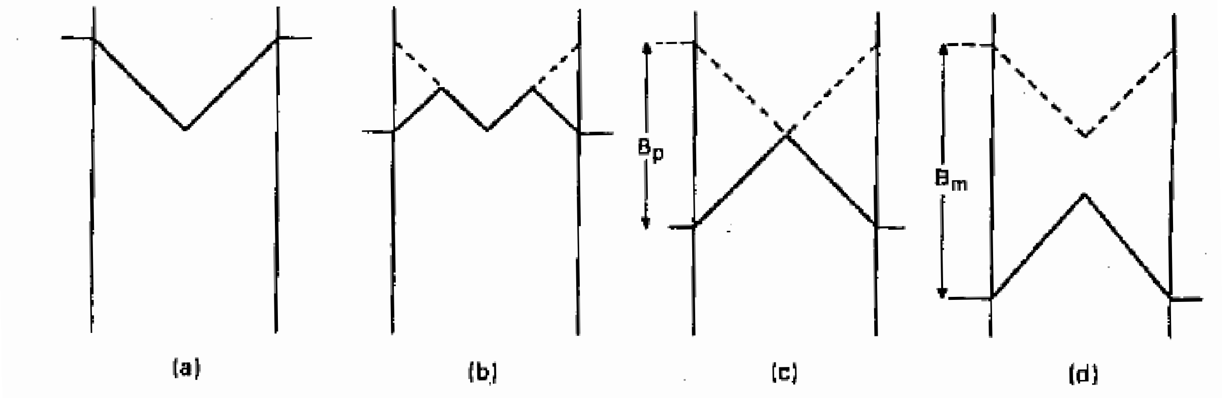
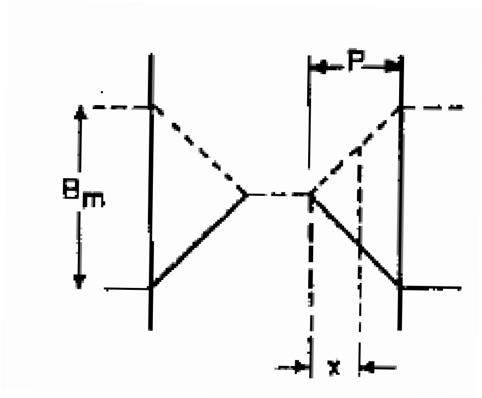


Fig. 8.4. (a) Field pattern within a superconducting slab subjected to large field change; (b) as the field is reduced; (c) when the field change penetrates to centre of slab; (d) when the field reaches a minimum value before rising again.



Some basic definitions:

B_p = Penetration field

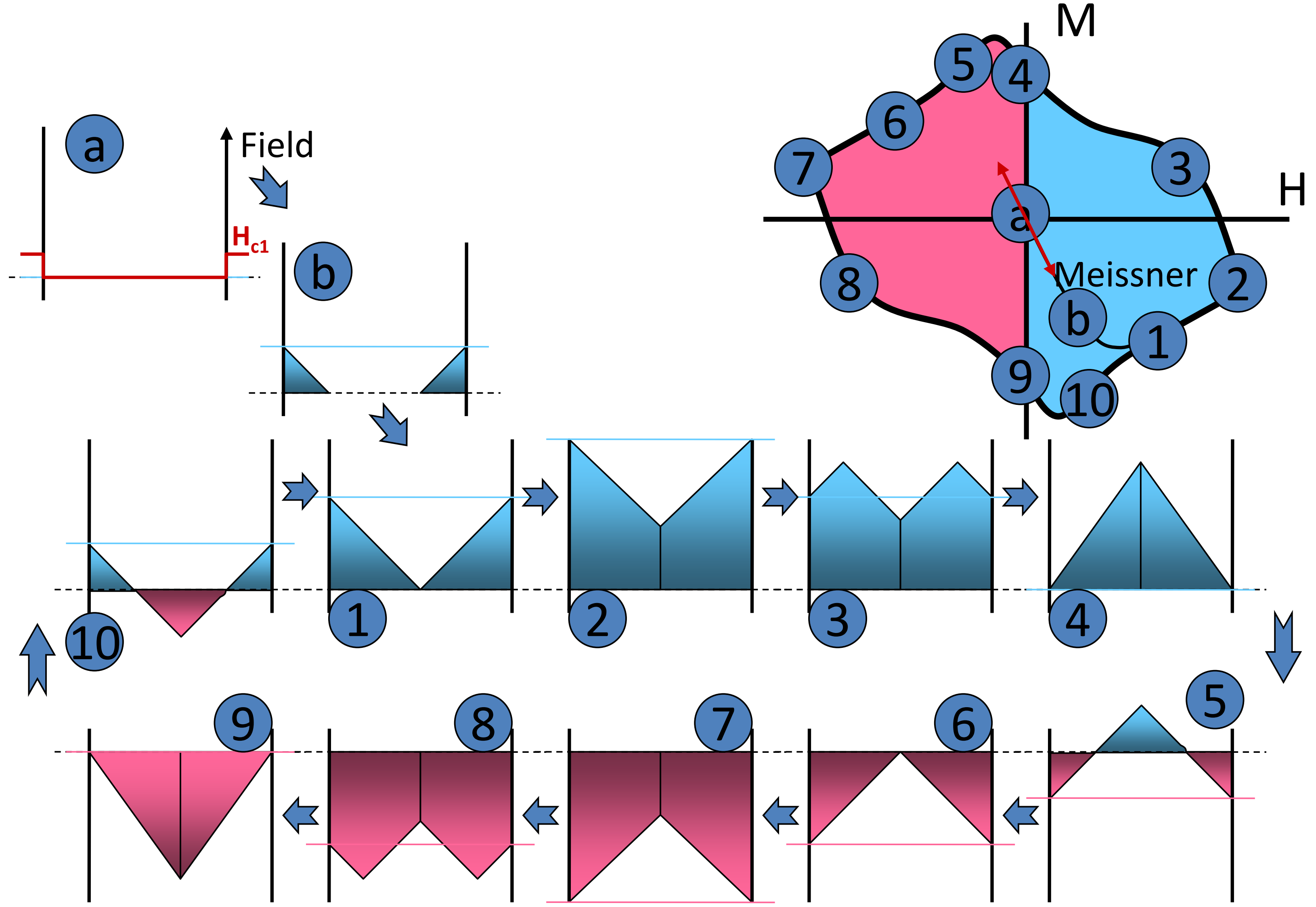
B_m = Field modulation

$B_m = 2\mu_0 J_c p$ for $p < a$, p is the field penetration distance

The power generated by the penetrating field is

$$P = EJ_c = J_c \frac{d\Phi}{dt}$$

Magnetization cycle



Slide taken from Lance Cooley, USPAS

Calculating hysteresis losses

- The total heat generated for a half-cycle is then

$$P = EJ_c = J_c \frac{d\Phi}{dt}$$

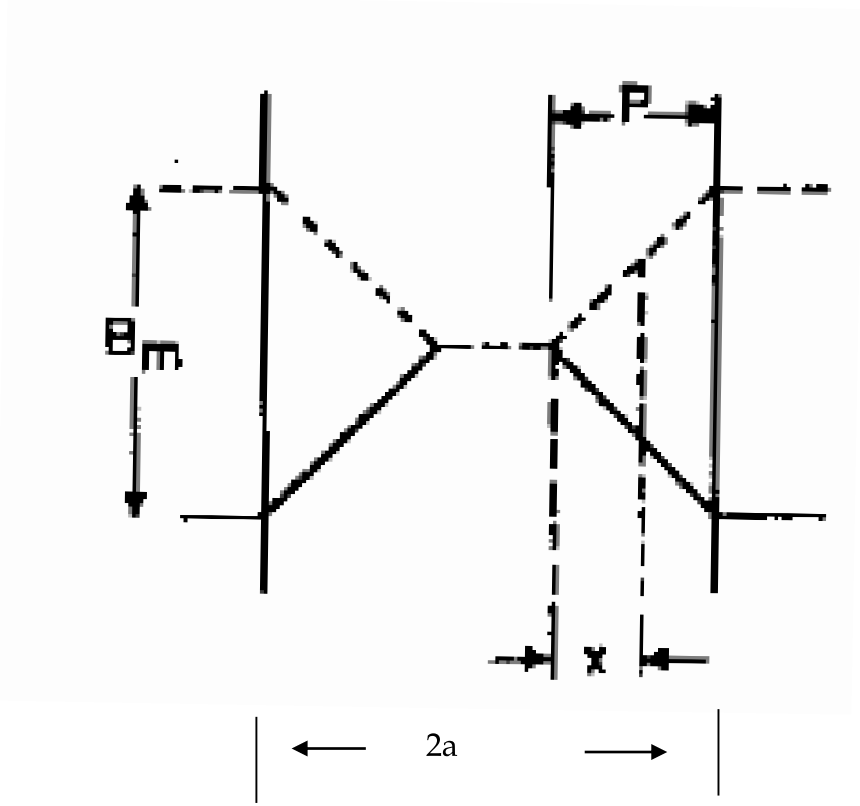
=

$$\Delta\phi(x) = \int_0^x \Delta B(\varsigma)\, d\varsigma = \int_0^x \mu_0 J_c \varsigma \, d\varsigma = \frac{\mu_0}{2} J_c x^2$$

=

$$\Rightarrow q = \frac{1}{a}\int_0^p J_c\left(\mu_0 J_c x^2\right) dx = \frac{\mu_0 J_c^{\,2} p^3}{3a}$$

- Note that this calculation assumed *p<a*; a similar analysis can be applied for the more generally case in which the sample is fully penetrated.



Hysteresis losses - general

- The hysteresis model can be developed in terms of:

$$\beta = \frac{B_m}{B_p} = \frac{B_m}{2a\mu_0 J_c}$$

The total cycle loss (for the whole slab) is then:

$$Q = \frac{B_m{}^2 \beta}{2\mu_0}\Gamma(\beta);$$ The function Γ (geometry dependent) has a maximum near 1

To reduce losses, we want
β<<1 (little field penetration, so loss/volume is small)

or

β>>1 (full flux penetration, but little overall flux movement)

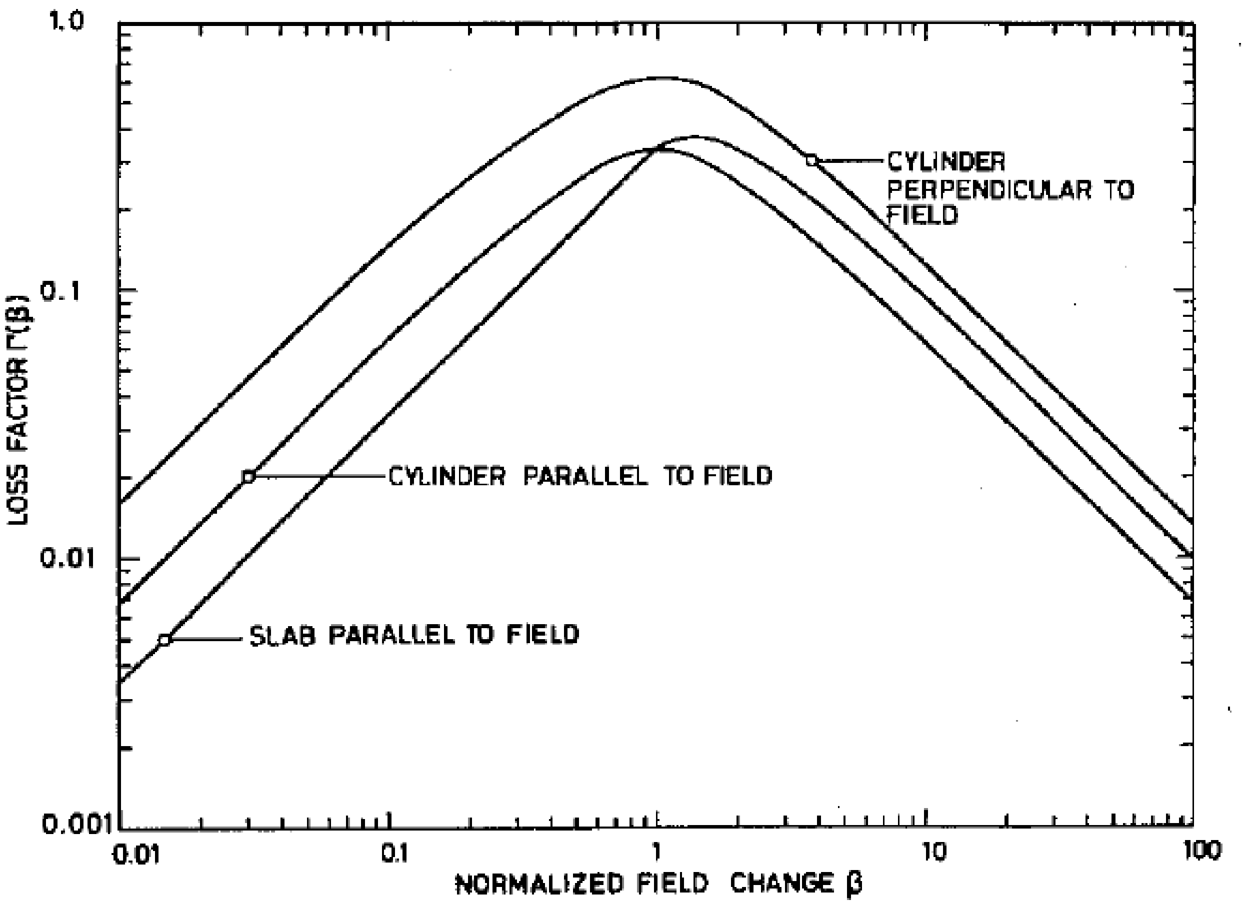


Fig. 8.5. Loss factor Γ(β) for hysteresis loss per cycle in different shapes of superconductor.

Hysteresis losses in presence of a transport current

- The **addition of transport current enhances the losses**; this can be viewed as stemming from power supply voltage compensating the system inductance voltage generated by the varying background field.

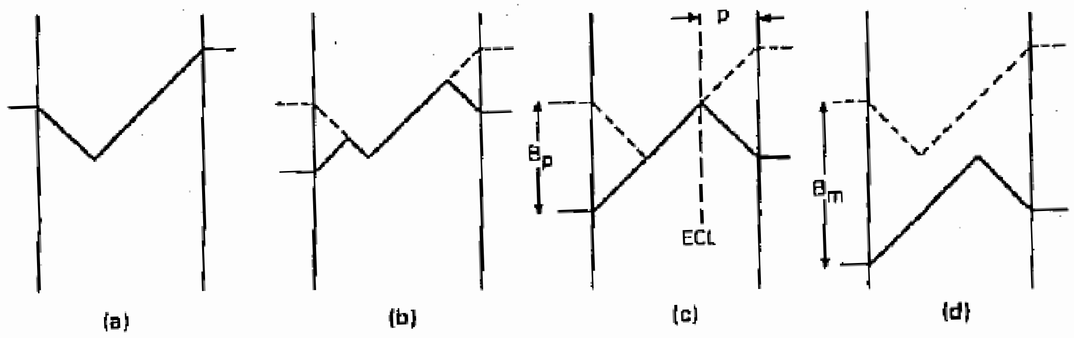


Fig. 8.11. (a) Slab carrying fixed transport current in external field; (b) as the field is reduced; (c) when the field change penetrates the entire slab; (d) when the field reaches minimum value before rising again.

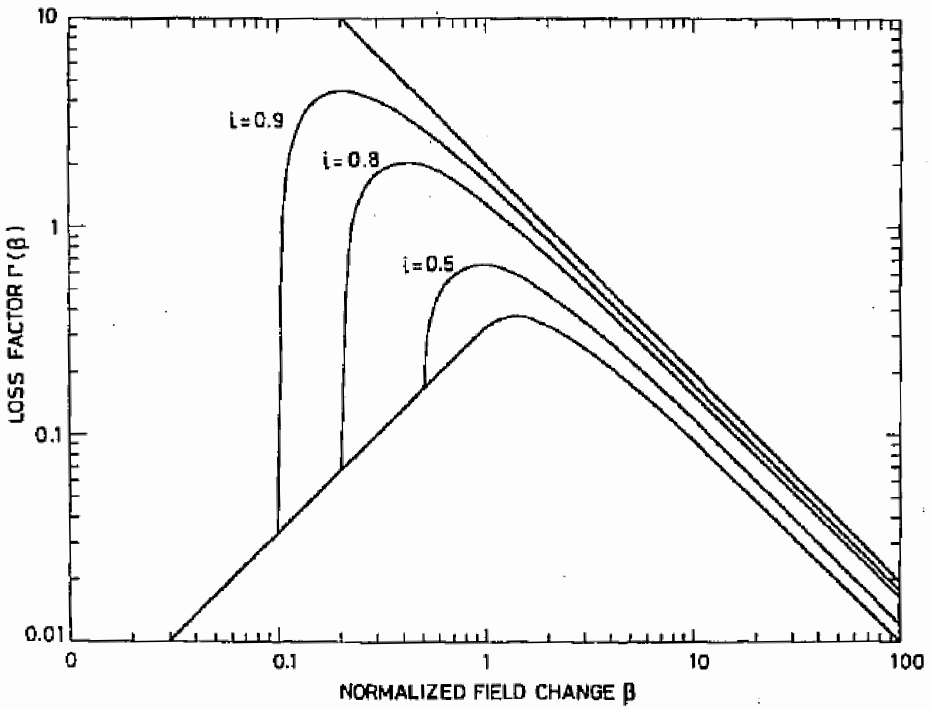
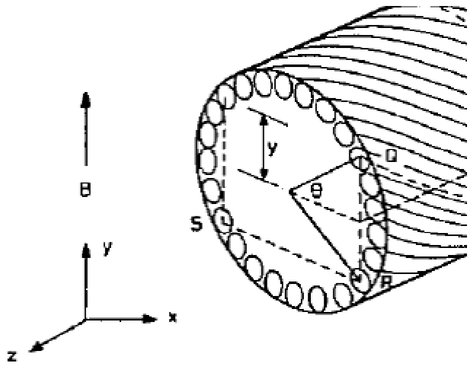


Fig. 8.12. Hysteresis loss in superconducting slab carrying fixed transport current $I_t = iI_c$ and subjected to a changing external field, calculated from eqns (8.28) and (8.7).

Coupling losses

A multifilamentary wire subjected to a transverse varying field will see an electric field generated between filaments of amplitude:

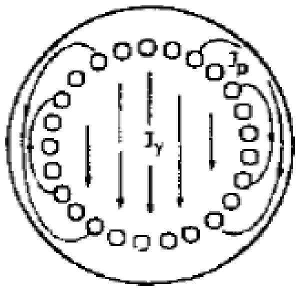
$$E = \frac{\dot{B}L}{2\pi};\ L \text{ is the twist-pitch of the filaments}$$



The metal matrix then sees a current (parallel to the applied field) of amplitude:

$$J = \frac{\dot{B}L}{2\pi\rho_t}$$

Similarly, the filaments couple via the periphery to yield a current: $J_p(\theta) = \frac{\dot{B}L\cos(\theta)}{2\pi\rho_m}$



There are also eddy currents of amplitude: $J_e(\theta) = \frac{\dot{B}a\cos(\theta)}{\rho_m}$

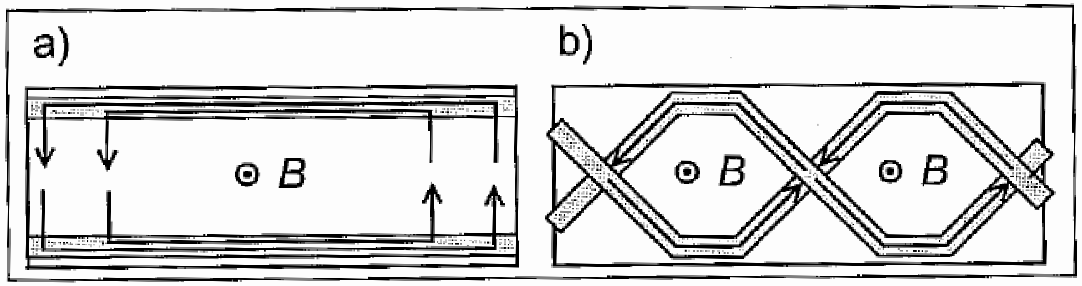


Figure 2.4 Schematic of coupling currents between two filaments in a wire or tape.

Coupling losses – time constant

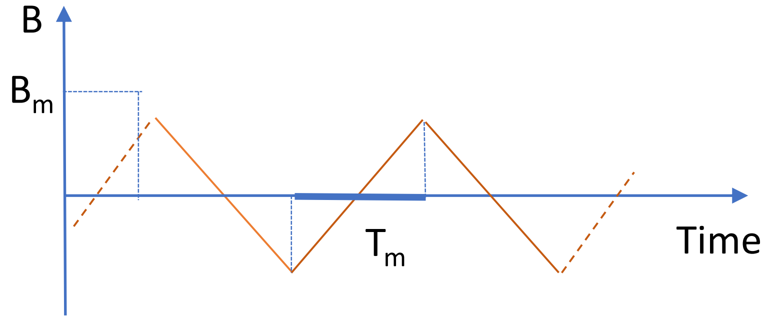
- The combined Cos(θ) coupling current distribution leads to a natural time constant (coupling time constant):

$$\tau = \frac{\mu_0}{2\rho_{eff}}\left(\frac{L}{2\pi}\right)^2$$

- The time constant τ corresponds to the natural decay time of the eddy currents when the varying field becomes stationary.
- The losses associated with these currents (per unit volume, per cycle) are:

$$Q_e = \frac{B_m{}^2}{2\mu_0}\frac{8\tau}{T_m}, \text{ where } T_m \text{ is the half-time of a full cycle}$$

Here B_m is the maximum field during the cycle.



Coupling losses – Rutherford cables

- Coupling currents also form between strands in cables

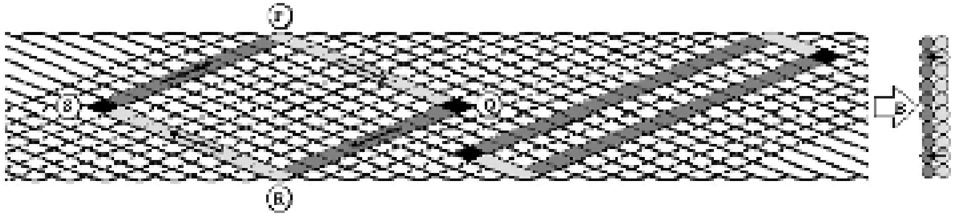


Fig. 19. Coupling currents flowing via crossover resistance R_c in transverse field (upper wires shown light grey).

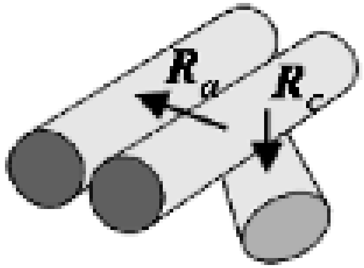
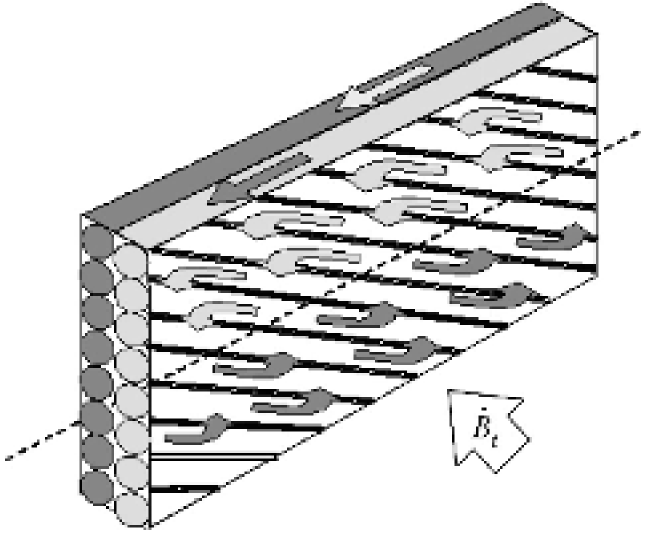


Fig. 18. Crossover resistance R_c and adjacent resistance R_a.



Add core to dramatically reduce transverse coupling, while maintaining decent Ra for current sharing

Other loss terms

- In the previous analysis, we assumed the cos(θ) longitudinal current flowed on the outer filament shell of the conductor. Depending on dB/dt, ρ, and L, the outer filaments may saturate (i.e. reach Jc), resulting in a larger zone of field penetration. The field penetration results in an additional loss term:

$$Q_p = \frac{B_m^2}{2\mu_0}\frac{4\pi^2}{T_m^2}\Gamma(\beta')$$

$$\beta' = \frac{\pi B_m}{2\mu_0 \lambda J_c a}\frac{\tau}{T_m}$$

- Self-field losses: as the transport current is varied, the self-field lines change, penetrating and exiting the conductor surface. The effect is independent of frequency, yielding a hysteresis-like energy loss:

$$Q_{sf} = \frac{B_{ms}^2}{2\mu_0}\Gamma(\beta);\ \beta = \frac{B_{ms}}{B_p} = \frac{I}{I_c}$$

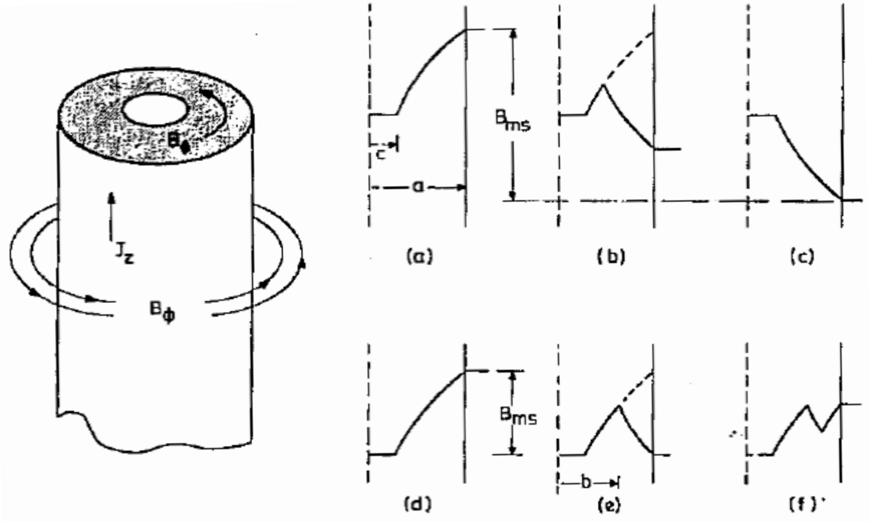


Fig. 8.24. Self-field in a superconducting cylinder or filamentary composite carrying transport current. (a), (b), and (c) show profiles of **B** within the cylinder when transport current is reversed; (d), (e), and (f) show effect of unidirectional current oscillations.

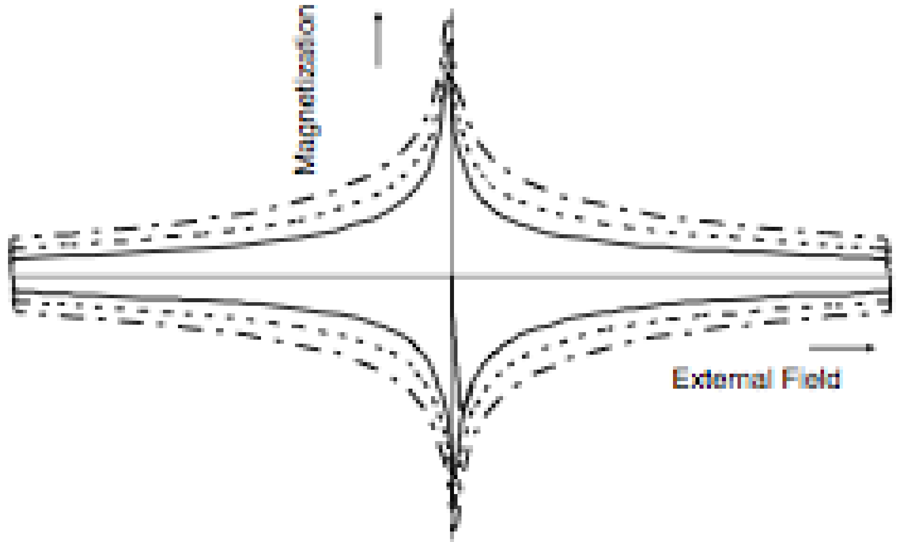
First estimate of AC losses: Hysteresis losses

$$Q_{cyc} = \int_0^{t_0} J_c(B)\frac{2D_{eff}}{3\pi}\frac{dB}{dt}dt$$ [J/m^3, per cycle]

$$Q_{hyst-tot} = Q_{cyc} * V_{sc}$$ [J, per cycle]

This has motivated the quest for fine filament wire!

Hysteresis loss reduction:
Minimize D_{eff}

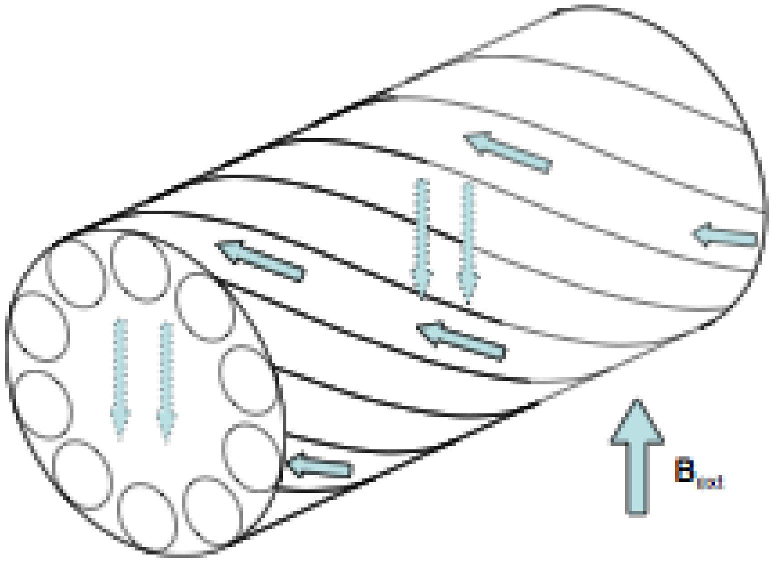


First estimate of AC losses: Coupling losses

$$q_{coupling} = \frac{(dB/dt)^2}{\mu_0} 2\tau \quad [W/m^3]$$

$$Q_{coupling-tot} = Q_{coupling} * V_{cond}$$

Coupling loss reduction:
Minimize twist pitch



"Inter-filament resistance, effective transverse resistivity and coupling loss in superconducting multi-filamentary NbTi and Nb_3Sn strands", C. Zhou et al., *Supercond. Sci. Technol.* **25** 065018
https://doi.org/10.1088/0953-2048/25/6/065018

Special case: thin superconducting strip in perpendicular field

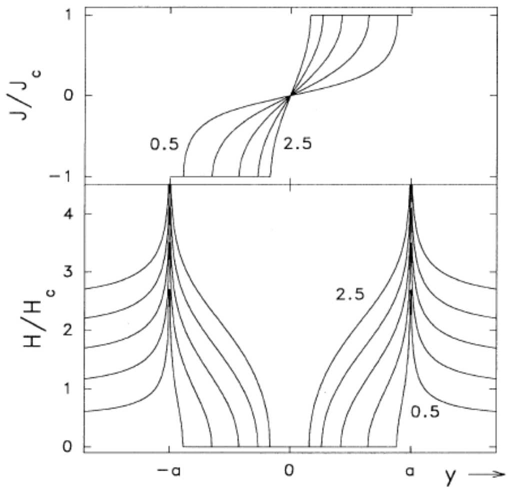
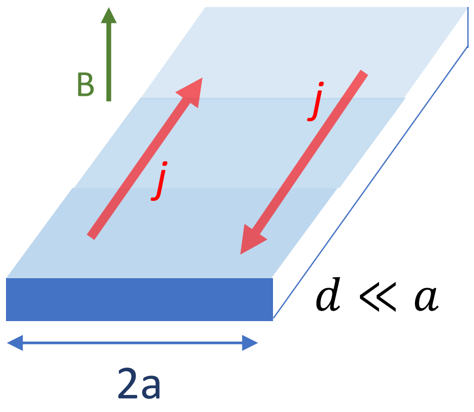


FIG. 3. Current density $J(y)$ (3.7) (top) and magnetic field $H(y)$ (3.9) (bottom) in a superconducting strip of width $2a$ in a perpendicular magnetic field H_a which is increased from zero (virgin state). The depicted profiles are for $H_a/H_c = 0.5, 1, 1.5, 2, 2.5$.

Current flows everywhere across the width of the strip!



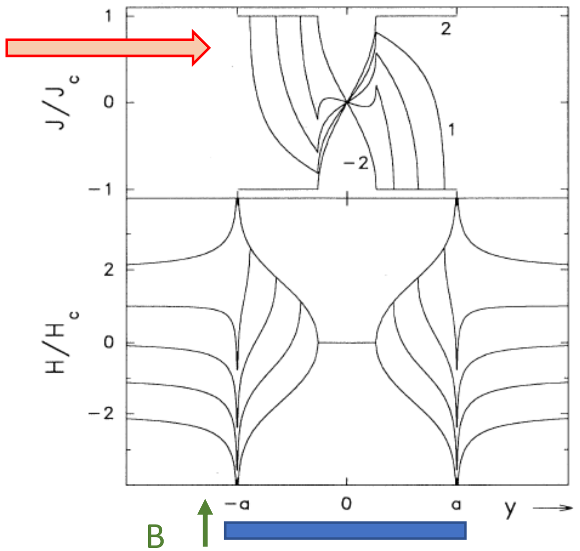


FIG. 4. Current density $J_\downarrow(y)$ (3.20) (top) and magnetic field $H_\downarrow(y)$ (3.21) (bottom) in a superconducting strip of width $2a$ when the applied field H_a is reduced from $2H_c$ to $-2H_c$. Shown are the profiles for $H_a/H_c = 2, 1, 0, -1, -2$.

$$J(y) = \begin{cases} \dfrac{2J_c}{\pi} \arctan \dfrac{cy}{(b^2 - y^2)^{1/2}}, & |y| < b \\ J_c\, y/|y|, & b < |y| < a\,, \end{cases}$$

$$H(y) = \begin{cases} 0, & |y| < b \\ H_c \operatorname{arctanh} \dfrac{(y^2 - b^2)^{1/2}}{c\,|y|}, & b < |y| < a \\ H_c \operatorname{arctanh} \dfrac{c\,|y|}{(y^2 - b^2)^{1/2}}, & |y| > a\,, \end{cases}$$

$$M = J_c a^2 c = J_c a^2 \tanh\left(H_a/H_c\right).$$

Ernst Helmut Brandt and Mikhail Indenbom, *“Type-II-superconductor strip with current in a perpendicular magnetic field”*
https://doi.org/10.1103/PhysRevB.48.12893

Magnetization losses in a thin strip

For a thin superconducting strip of width 2a, thickness d and critical current I_c in a harmonic ac field of amplitude H_0 (normal to the plane) and frequency f :

$$P = 4 f \mu_0 a^2 J_c H_0 g\left(\frac{H_0}{H_c}\right)$$

$$J_c = j_c d = \frac{I_c}{2a}$$

and $g(x) = \frac{2}{x} \ln\cosh x - \tanh x$

$$H_c = J_c / \pi$$

For small and large amplitude H_0 this gives

$$P = (2\pi\nu\mu_0 a^2/3H_c^2)\,H_0^4, \qquad H_0 \ll H_c a \;,$$
$$P = 4\nu\mu_0 a^2 J_c\,(H_0 - 1.386 H_c), \qquad H_0 \gg H_c.$$

The energy loss is thus initially very small, $P \sim H_0^4$.

Special case: thin superconducting strip with transport current

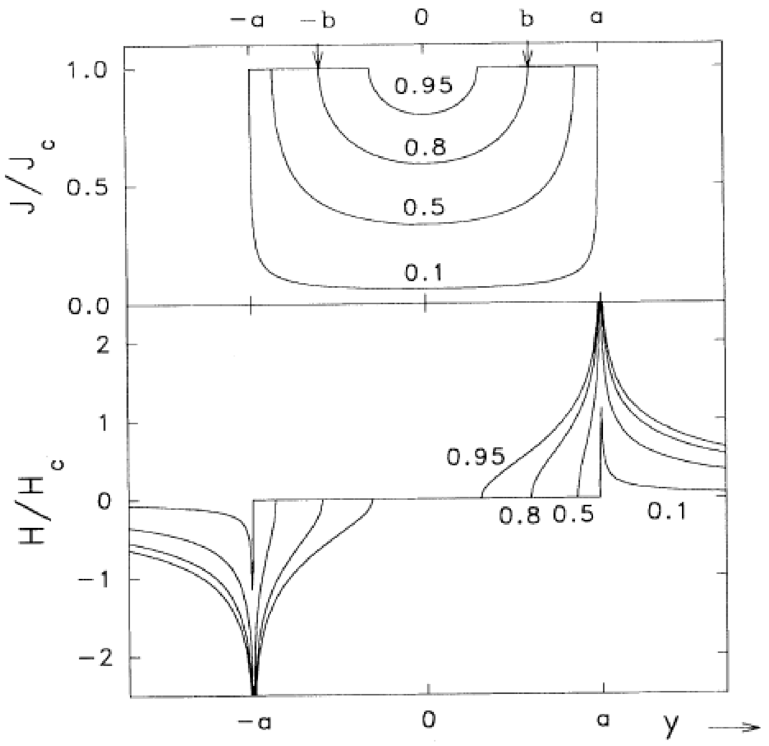


FIG. 1. Current density $J(y)$ (2.4) (top) and magnetic field $H(y)$ (2.6) (bottom) in a superconducting strip of width $2a$ carrying a transport current I which is increased from zero (virgin state). The depicted profiles are for $I/I_{\max} = 0.1$, 0.5, 0.8, and 0.95. For $I/I_{\max} = 0.8$ the penetration width b is indicated at the top.

$$J(y) = \begin{cases} \dfrac{2J_c}{\pi} \arctan\left(\dfrac{a^2-b^2}{b^2-y^2}\right)^{1/2}, & |y| < b \\ J_c, & b < |y| < a. \end{cases} \qquad (2.4)$$

Integrating this one gets the total current

$$I = 2J_c\,(a^2-b^2)^{1/2}, \quad b = a(1-I^2/I_{\max}^2)^{1/2}, \qquad (2.5)$$

where $I_{\max} = 2aJ_c$ is the maximum total current occurring at full penetration $b = 0$.

$$H(y) = \begin{cases} 0, & |y| < b \\ \dfrac{H_c y}{|y|} \operatorname{arctanh}\left[\dfrac{y^2-b^2}{a^2-b^2}\right]^{1/2}, & b < |y| < a \\ \dfrac{H_c y}{|y|} \operatorname{arctanh}\left[\dfrac{a^2-b^2}{y^2-b^2}\right]^{1/2}, & |y| > a, \end{cases} \qquad (2.6)$$

with $H_c = J_c/\pi$.

Losses due to ac transport current

The losses due to transport current are dependent only upon the critical current and transport current in the strip and not its width. According to Brandt (see ref. above) transport ac losses can be expressed as:

$$P = \frac{\mu_0 f I_c^2}{\pi}\left[\left(1-\frac{I_0}{I_c}\right)\ln\left(1-\frac{I_0}{I_c}\right)+\left(1+\frac{I_0}{I_c}\right)\ln\left(1+\frac{I_0}{I_c}\right)-\left(\frac{I_0}{I_c}\right)^2\right]$$

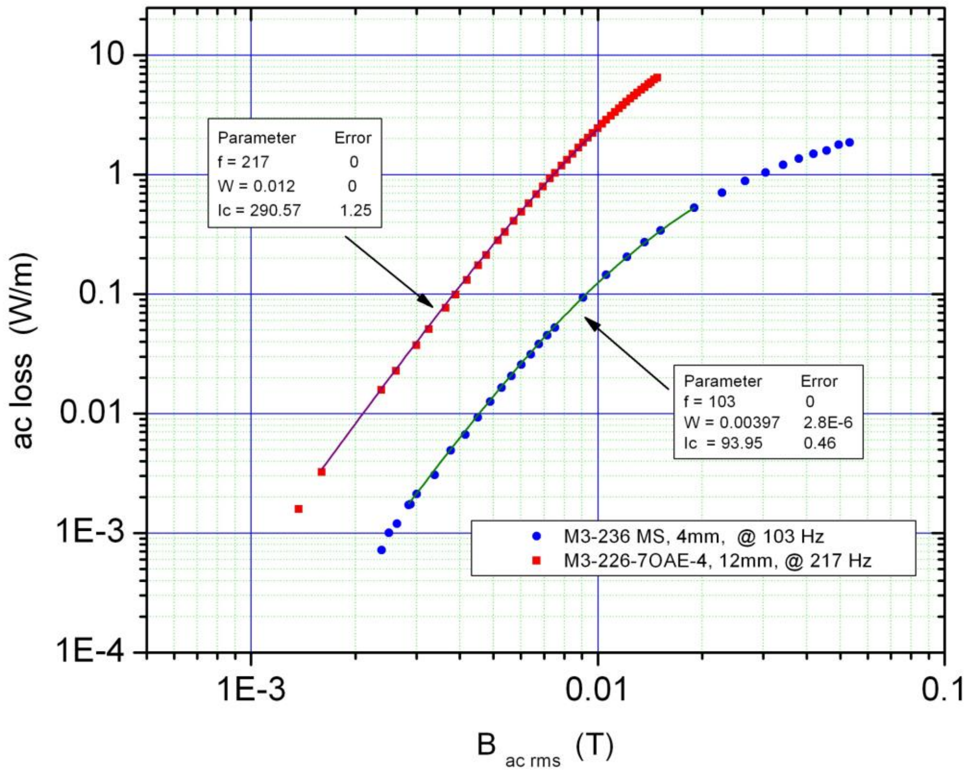
where I_0 is the amplitude of the ac transport current

For $I_0 << I_c$

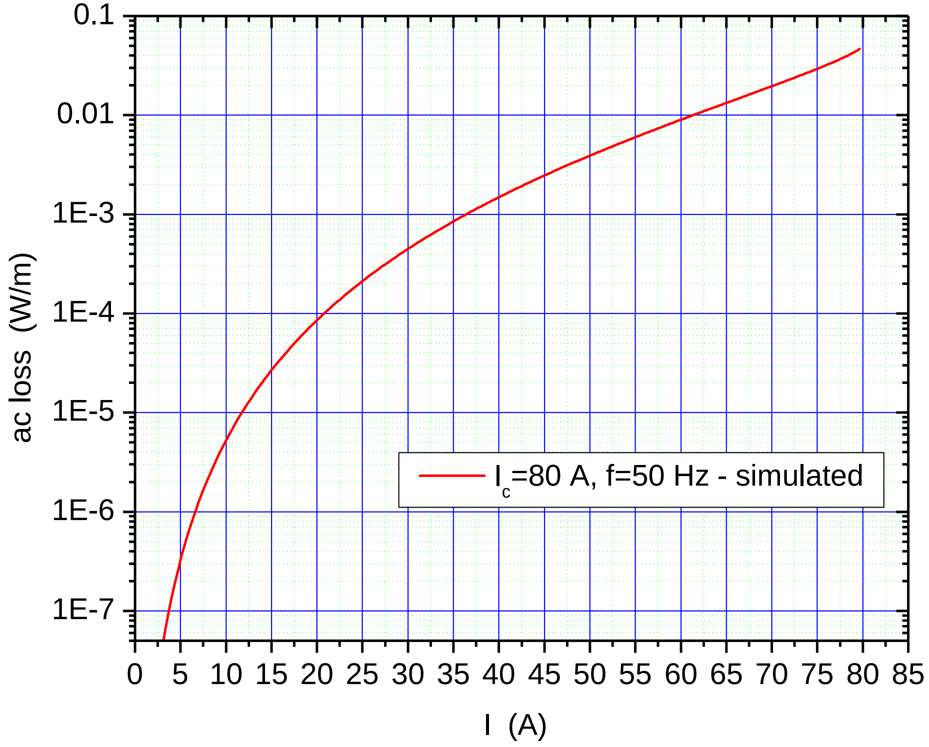
above equation simplifies to: $$P = \frac{\mu_0}{6\pi}\frac{I_0^4}{I_c^2}$$

Dividing strip into *n* filaments yields I_0 and I_c of each filament reduced by *n*, and therefore net ac loss is reduced with striation: $n\left(\frac{1}{n^4}\right)/\left(\frac{1}{n^2}\right) = n$ times

Typical ac losses in ReBCO HTS tapes (4 mm)



magnetization



transport

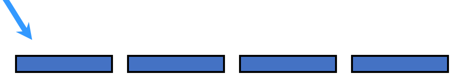
Magnetization losses are typically prevailing over transport losses

Striation: a way to reduce ac losses

Note, that P~ a^2.
So, reducing width n times will reduce ac losses n^2 times!...

If one sub-divides a strip in n equal filaments along the width, the net ac loss will be $\sim n\left(\frac{1}{n^2}\right) = 1/n$ of the original strip.

$$P = 4 f \mu_0 a^2 J_c H_0 g\left(\frac{H_0}{H_c}\right)$$



Therefore, ***striation*** is an efficient practical method of ac loss reduction

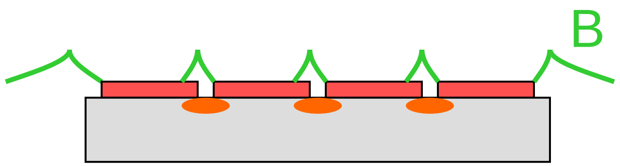
W.J. Carr and C.E. Oberley, 1999

G.A. Levin and P.N. Barnes, 2004

Coupling losses in striated coated condcutors

The alternating magnetic field penetrates through the slits between the superconducting stripes and induces electric field perpendicular to the stripes:

$$E_{\perp} = BfL$$

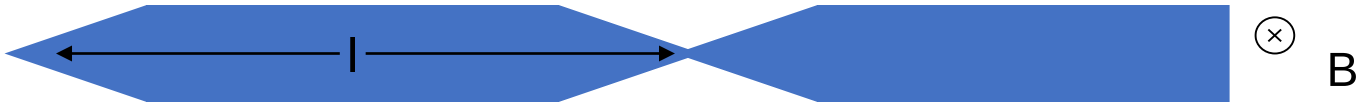


where B is field amplitude, f-frequency and L is the length of the conductor.

This electric field yields current flowing through the normal metal in between the filaments, i.e. dissipation!

$$Q = \rho^{-1} |E_{\perp}|^2 d_n W = 2\frac{(BfL)^2}{\rho} d_n W$$

where d_n is the thickness of the normal metal and ρ - resistivity



Coupling losses of the striated conductor can be reduced by **twisting** it with a short twist pitch *l*. Then *L* in the formula above becomes *l/2* !

Eddy current losses

Screening currents flowing in the normal metal (substrate, stabilizer, matrix…) also result in dissipation:

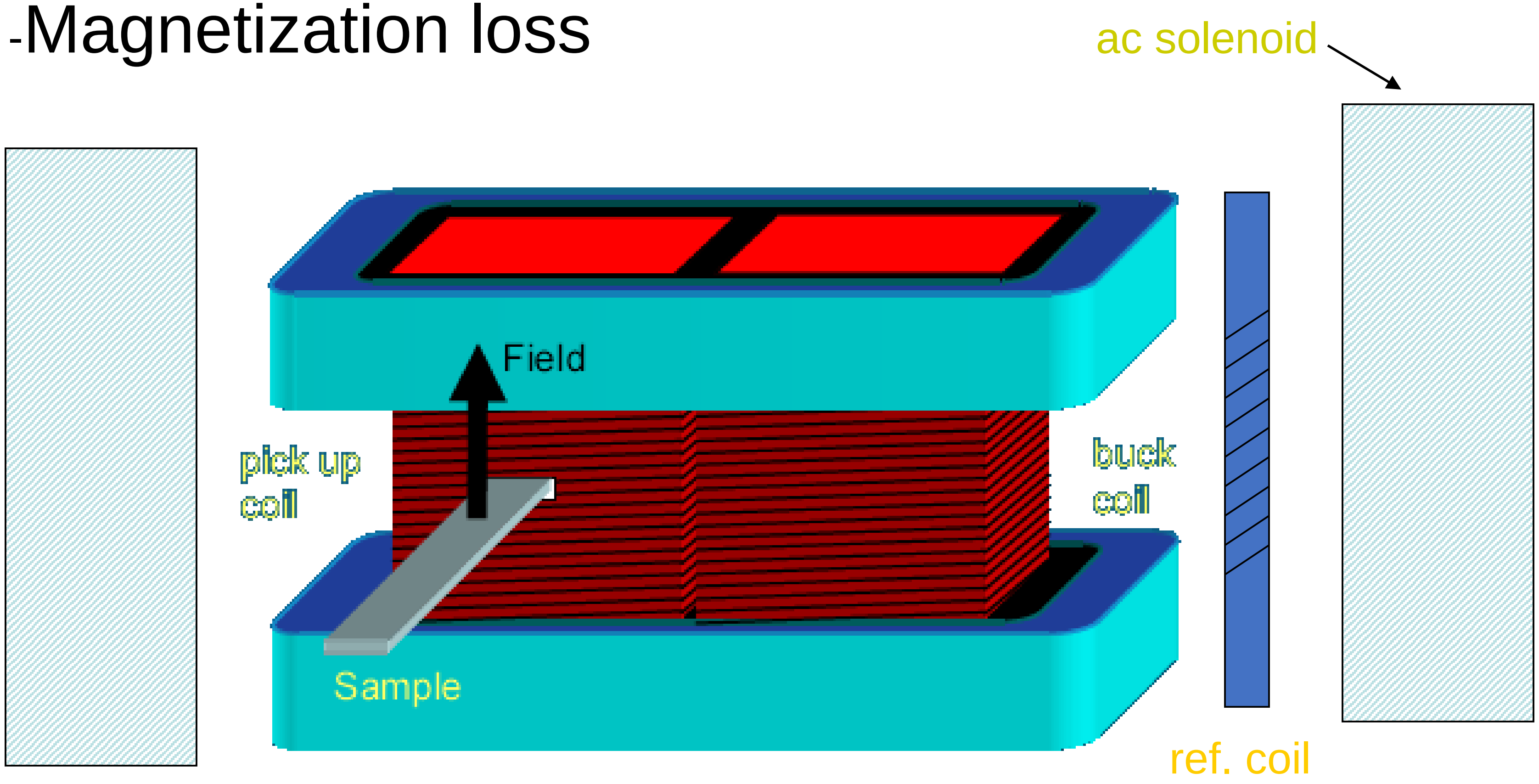
$$P = \frac{\pi}{6}\frac{B_a^2 f^2 a_\perp^2}{R_{eff}}$$

Here $a_\perp$ can be either width or thickness of the conductor, depending on the orientation with respect to the ac field

Eddy current losses in coated conductors are typically negligible compared to the hysteretic losses

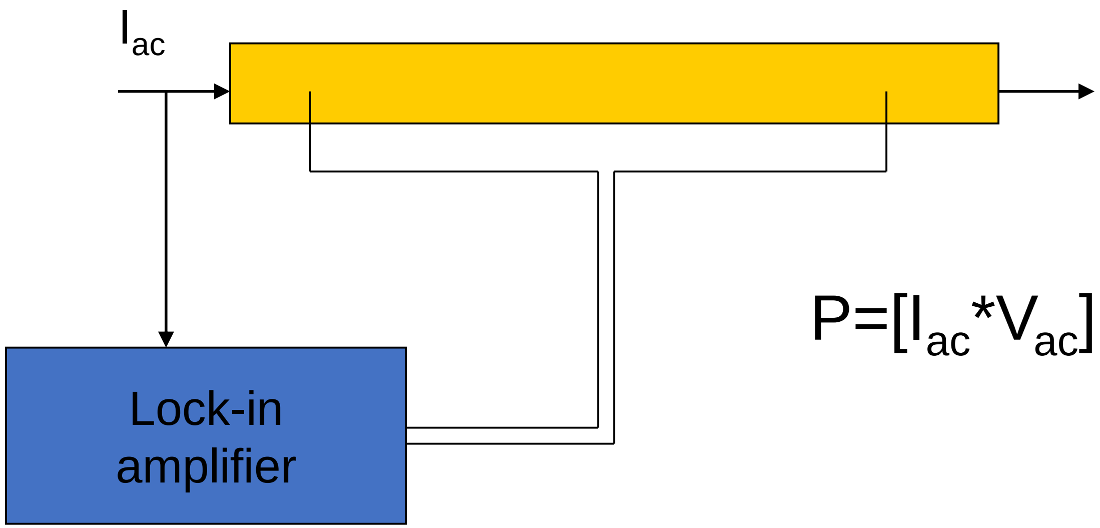
AC loss measurement techniques

-Magnetization loss



Ac loss is proportional to the out-of-phase component of the e.m.f. in the compensated pick-up coil (relative to the reference coil). Note: for small out-of-phase signals only!

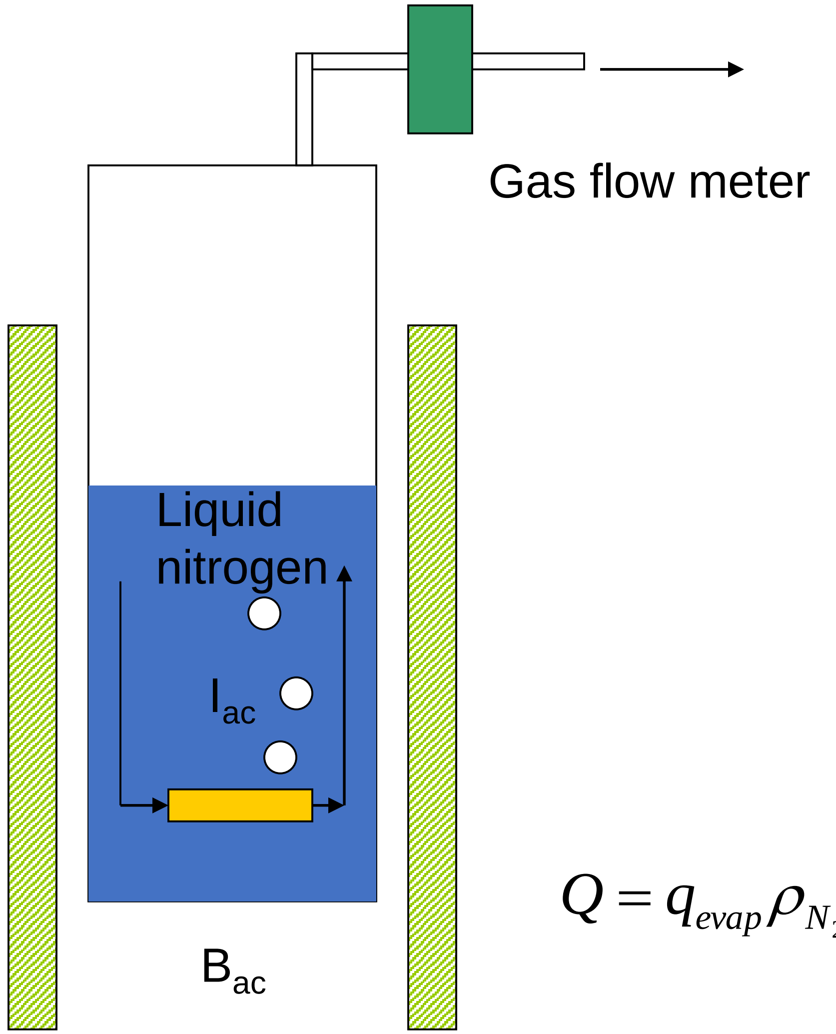
Transport ac current loss



$P=[I_{ac}*V_{ac}]$

Extreme care should be exercised in these measurements to pick up only loss-related component of the inductive voltage…

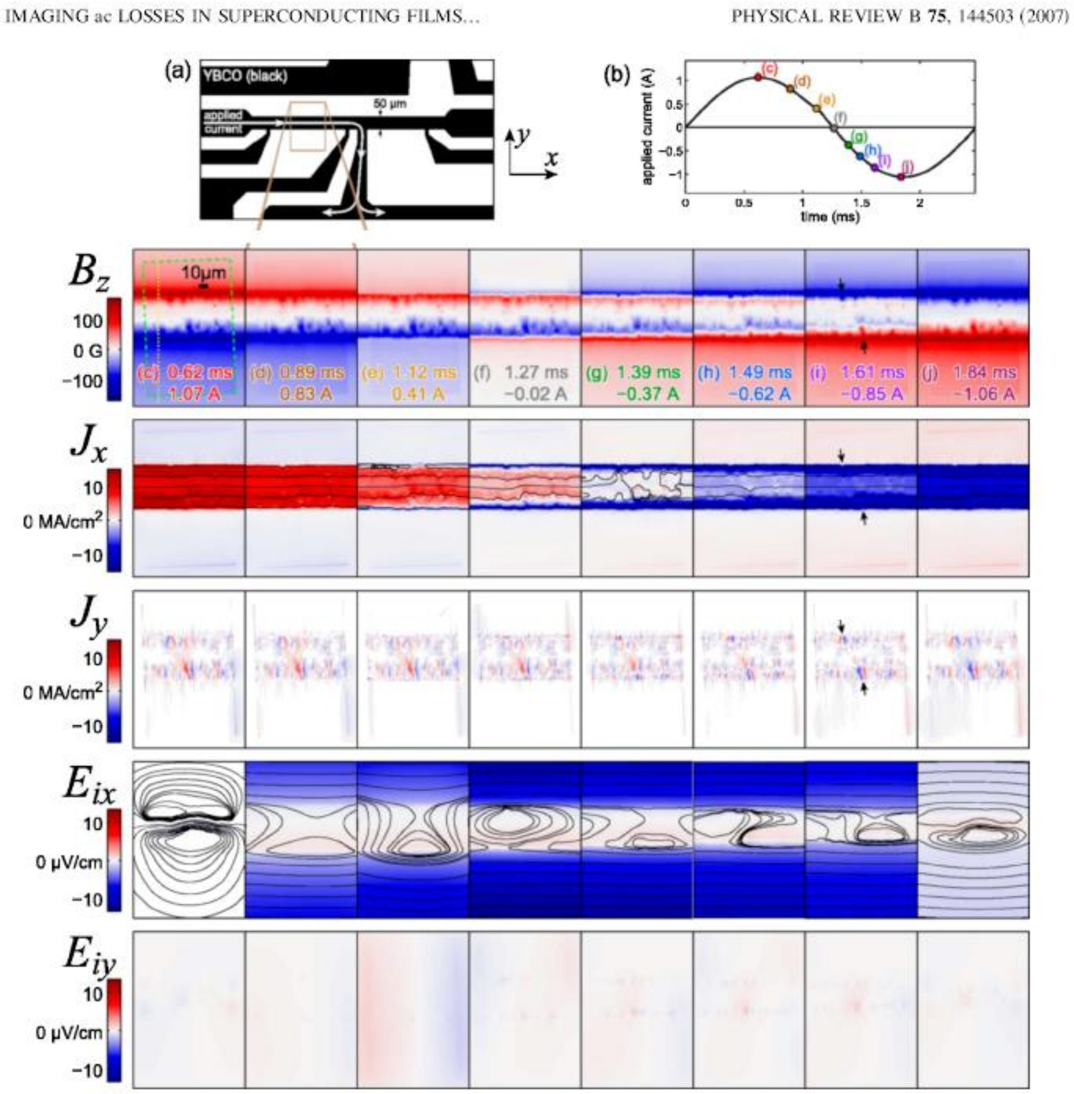
Calorimetric measurement of ac losses



Ac solenoid

$$Q = q_{evap} \rho_{N_2} V$$

Local measurement of ac losses



Rafael B. Dinner et al., "Imaging ac losses in superconducting films via scanning Hall probe microscopy ", Phys. Rev. B ***75****, 144503 (2007)*

Use of the AC-loss models

- It is common (but not necessarily correct) to add the different AC loss terms together to determine the loss budget for an conductor design and operational mode.
- AC loss calculations are "imperfect":
  - Uncertainties in effective resistivities (e.g. matrix resistivity may vary locally, e.g. based on alloy properties associated with fabrication; contact resistances between metals may vary, etc)
  - Calculations invariably assume "ideal" behavior, e.g. Bean model, homogeneous external field, etc.
- For real applications, these models usually suffice to provide grounds for conductor specifications and/or cryogenic budgeting
  - For critical applications, AC-loss measurements (non-trivial!) should be undertaken to quantify key parameters

AC losses and cryogenics

- The AC loss budget must be accounted for in the cryogenic system
  - Design must account for thermal gradients – e.g. from strand to cable, through insulation, etc. and provide sufficient temperature margin for operation
  - Typically the temperature margin needed will also depend on the cycle frequency; the ratios of the characteristic cycle time (t_w) and characteristic diffusion time (t_d) separates two regimes:
    - $t_w << t_d$: Margin determined by single cycle enthalpy
    - $t_w >> t_d$: Margin determined by thermal gradients

- The AC loss budget is critical for applications requiring controlled current rundown; if the AC losses are too large, the system may quench and the user loses control of the decay rate